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A NOTE OF THE CONSTANT RANK THEOREM FOR CONVEX SOLUTIONS OF SPECIAL LAGRANGIAN EQUATION

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Abstract: In this paper, we establish an inverse convexity type inequality of special Lagrangian operator, and prove a constant rank theorem for convex solutions of special Lagrangian equations with nonconstant phase under certain conditions.

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1 Introduction

In this paper, we consider fully nonlinear partial differential equation,

$$\arctan D^2u(x) = \Theta(x), \quad x \in \Omega \subset \mathbb{R}^n, \tag{1.1}$$

where $\arctan D^2u =: \arctan\lambda_1 + \arctan\lambda_2 + \cdots + \arctan\lambda_n$, and $\lambda = (\lambda_1, \lambda_2, \cdots, \lambda_n)$ are the eigenvalues of the Hessian matrix $D^2u = \{\frac{\partial^2 u}{\partial x_i \partial x_j}\}_{1\leqslant i,j\leqslant n}$. When Θ is a constant, the equation is called special Lagrangian equation, which is introduced by Harvey-Lawson [1]. Here $\Theta \in (-\frac{n}{2}\pi, \frac{n}{2}\pi)$ is called the phase, and for $\Theta \geq \frac{n-1}{2}\pi$, the solution of (1.1) is strictly convex, that is $D^2u > 0$.

Caffarelli-Friedman [2] and Singer-Wong-Yau-Yau [3] devised the constant rank theorem for semilinear equations, which is a powerful method to study the convexity for solutions of elliptic partial differential equations. For examples, the constant rank theorem for convex solutions of geometric PDEs are studied by Guan-Ma [4], Guan-Lin-Ma [5], Guan-Ma-Zhou [6] and Chen-Xu [7], the constant rank theorem for power convex solutions of fully nonlinear elliptic partial differential equations are studied by Ma-Xu [8], Liu-Ma-Xu [9], Huang [10], Zhang-Zhou [11], Chen-Jia-Xiong [12] and Chen-Ma [13], the constant rank theorem for convex solutions of fully nonlinear elliptic partial differential equations are studied by Caffarelli-Guan-Ma [14] and Bian-Guan [15].

W. J. Ogden and Y. Yuan [16] studied the constant rank theorem for saddle solutions of (1.1) with constant phase Θ. In this paper, we obtain the constant rank theorem for convex solutions of special Lagrangian equation (1.1) as follows.

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Theorem 1.1 Assume that $u \in C^4(\Omega)$ is the solution of special Lagrangian equation (1.1), and the phase $\Theta(x) \in (0, \frac{n}{2}\pi)$ is concave, that is

$$D^2\Theta \leq 0, \text{ in } \Omega. \tag{1.2}$$

If u is convex, then the rank of D^2u is of constant.

Remark 1.2 In Theorem 1.1, if there exists a point $x_0 \in \Omega$ such that $\Theta(x_0) \geq \frac{n-1}{2}\pi$ additionally, it is easy to know $D^2u(x_0) > 0$, which means that the rank of D^2u is of constant n.

The rest of the paper is organized as follows. In Section 2, we introduce some properties of the special Lagrangian operator. In Section 3, we prove Theorem 1.1 by the strong maximum principle.

2 Preliminaries

In this section, we recall the definitions and several properties of k-Hessian operators and special Lagrangian operator.

These properties are well-known and can be similarly found in [17], [18, 19], [20] and [7].

2.1 k-Hessian Operators

Definition 2.1 For any $k = 1, 2, \ldots, n$, we set

$$\sigma_k(\lambda) = \sum_{1 \leq i_1 < i_2 < \cdots < i_k \leq n} \lambda_{i_1}\lambda_{i_2}\cdots\lambda_{i_k}, \tag{2.1}$$

where $\lambda = (\lambda_1, \ldots, \lambda_n) \in \mathbb{R}^n$. For convenience, let $\sigma_0 = 1$ and $\sigma_k = 0$ for $k > n$.

Property 2.2 Let $W = \{W_{ij}\}$ be an $n\times n$ symmetric matrix and $\lambda(W) = (\lambda_1, \lambda_2, \ldots, \lambda_n)$ be the eigenvalues of $\{W_{ij}\}$. If $\{W_{ij}\}$ is diagonal and $\lambda_i = W_{ii}$, then we have

$$\frac{\partial \lambda_i}{\partial W_{ii}} = 1, \quad \frac{\partial \lambda_k}{\partial W_{ij}} = 0 \text{ otherwise}, \tag{2.2}$$

$$\frac{\partial^2 \lambda_i}{\partial W_{ij}\partial W_{ji}} = \frac{1}{\lambda_i - \lambda_j}, \quad i \neq j \text{ and } \lambda_i \neq \lambda_j, \tag{2.3}$$

$$\frac{\partial^2 \lambda_i}{\partial W_{kl}\partial W_{pq}} = 0 \text{ otherwise}. \tag{2.4}$$

We will also define $\sigma_k(W|i)$ the symmetric function with W deleting the i-row and i-column and $\sigma_k(W|ij)$ the symmetric function with W deleting the i, j-rows and i, j-columns. Then we have the following identities.

Property 2.3 Suppose $W = \{W_{ij}\}$ is diagonal, and m is a positive integer, then

$$\frac{\partial \sigma_m(W)}{\partial W_{ij}} = \begin{cases} \sigma_{m-1}(W|i), & i = j, \\ 0, & i \neq j, \end{cases} \tag{2.5}$$

and

$$\frac{\partial^2 \sigma_m(W)}{\partial W_{ij} \partial W_{kl}} = \begin{cases} \sigma_{m-2}(W|ik), & i=j, \quad k=l, \quad i \neq k, \\ -\sigma_{m-2}(W|ik), & i \neq j, \quad i=l, \quad j=k, \\ 0, & \text{otherwise.} \end{cases} \tag{2.6}$$

2.2 Special Lagrangian Operator

Suppose $\lambda = (\lambda_1, \lambda_2, \cdots, \lambda_n)$ are the eigenvalues of the Hessian matrix $D^2u = \{\frac{\partial^2 u}{\partial x_i \partial x_j}\}_{1 \leqslant i,j \leqslant n}$. Denote, $\arctan D^2u =: \arctan \lambda_1 + \arctan \lambda_2 + \cdots + \arctan \lambda_n$.

For the sake of convenience, denote that

$$F(D^2u) := \arctan D^2u, \tag{2.7}$$

and

$$F^{ij} := \frac{\partial \arctan D^2u}{\partial u_{ij}}, \quad F^{ij,kl} := \frac{\partial^2 \arctan D^2u}{\partial u_{ij} \partial u_{kl}}. \tag{2.8}$$

Indeed, without loss of generality, we assume D^2u is diagonal, so is $\{F^{ij}\}$, then we have

$$\sum_{i=1}^n F^{ii} = \sum_{i=1}^n \frac{1}{1+\lambda_i^2} \in (0, n), \tag{2.9}$$

$$\sum_{i=1}^n F^{ii} u_{ii} = \sum_{i=1}^n \frac{\lambda_i}{1+\lambda_i^2} \in \left(0, \frac{n}{2}\right), \tag{2.10}$$

$$\sum_{i=1}^n F^{ii} u_{ii}^2 = \sum_{i=1}^n \frac{{\lambda_i}^2}{1+\lambda_i^2} \in (0, n). \tag{2.11}$$

Property 2.4 If $D^2u \geq 0$, then we have
(1) (**Ellipticity**) $\arctan D^2u$ is elliptic with respect to D^2u, that is

$$\left\{ \frac{\partial \arctan D^2u}{\partial u_{ij}} \right\} > 0. \tag{2.12}$$

(2) (**Concavity**) $\arctan D^2u$ is concave with respect to D^2u, i.e., for any $n \times n$ symmetric matrix $\{\xi_{ij}\}$,

$$\sum_{i,j,k,l=1}^n \frac{\partial^2 \arctan D^2u}{\partial u_{ij} \partial u_{kl}} \xi_{ij} \xi_{kl} \leq 0. \tag{2.13}$$

Proof Without loss of generality, we assume D^2u is diagonal and $\lambda_i = u_{ii}$. Then we have

$$F^{ij} = \frac{\partial \arctan D^2u}{\partial u_{ij}} = \begin{cases} \frac{1}{1+\lambda_i^2}, & i=j, \\ 0, & i \neq j, \end{cases} \tag{2.14}$$

and

$$F^{ij,kl}=\frac{\partial^2\arctan D^2u}{\partial u_{ij}\partial u_{kl}}=\begin{cases}\frac{-2\lambda_i}{(1+\lambda_i^2)^2}, & i=j=k=l,\\ \frac{-(\lambda_i+\lambda_j)}{(1+\lambda_i^2)(1+\lambda_j^2)}, & i=l,j=k,i\neq j,\\ 0, & \text{otherwise}.\end{cases}\tag{2.15}$$

Then combining (2.14) and (2.15), we have

$$\begin{aligned}F^{ij,kl}\xi_{ij}\xi_{kl}&=F^{ii,ii}{\xi_{ii}}^2+\sum_{i\neq j}F^{ij,ji}{\xi_{ij}}^2\\&=\frac{-2\lambda_i}{(1+\lambda_i^2)^2}{\xi_{ii}}^2+\sum_{i\neq j}\frac{-(\lambda_i+\lambda_j)}{(1+\lambda_i^2)(1+\lambda_j^2)}{\xi_{ij}}^2\leq 0.\end{aligned}\tag{2.16}$$

Property 2.5 (**Inverse convexity**) If $D^2u>0$, then $\arctan D^2u$ is "inverse convex" with respect to D^2u, that is, for any $n\times n$ symmetric matrix $\{\xi_{ij}\}$,

$$\sum_{i,j,k,l=1}^n\left[\frac{\partial^2\arctan D^2u}{\partial u_{ij}\partial u_{kl}}+2\frac{\partial\arctan D^2u}{\partial u_{ik}}u^{jl}\right]\xi_{ij}\xi_{kl}\geq 0,\tag{2.17}$$

where $\{u^{ij}\}=\{D^2u\}^{-1}$.

Proof We assume D^2u is diagonal and $\lambda_i=u_{ii}$, then we have (2.14) and (2.15). Hence

$$\begin{aligned}&\sum_{i,j,k,l=1}^n(F^{ij,kl}+2F^{ik}u^{jl})\xi_{ij}\xi_{kl}\\=&\sum_{i=1}^nF^{ii,ii}{\xi_{ii}}^2+\sum_{i\neq j}F^{ij,ji}{\xi_{ij}}^2+2\sum_{i=1}^nF^{ii}u^{ii}{\xi_{ii}}^2+2\sum_{i\neq j}F^{ii}u^{jj}{\xi_{ij}}^2\\=&\sum_{i=1}^n(F^{ii,ii}+2\frac{F^{ii}}{u_{ii}}){\xi_{ii}}^2+\sum_{i\neq j}(F^{ij,ji}+\frac{F^{ii}}{u_{jj}}+\frac{F^{jj}}{u_{ii}}){\xi_{ij}}^2\\=&\sum_{i=1}^n\left[\frac{-2\lambda_i}{(1+\lambda_i^2)^2}+2\frac{1}{(1+\lambda_i^2)\lambda_i}\right]{\xi_{ii}}^2+\sum_{i\neq j}\left[\frac{-(\lambda_i+\lambda_j)}{(1+\lambda_i^2)(1+\lambda_j^2)}+\frac{1}{(1+\lambda_i^2)\lambda_j}+\frac{1}{(1+\lambda_j^2)\lambda_i}\right]{\xi_{ij}}^2\\=&\sum_{i=1}^n\frac{-2\lambda_i^2+2(1+\lambda_i^2)}{(1+\lambda_i^2)^2\lambda_i}{\xi_{ii}}^2+\sum_{i\neq j}\frac{-(\lambda_i+\lambda_j)\lambda_i\lambda_j+(1+\lambda_i^2)\lambda_j+(1+\lambda_j^2)\lambda_i}{(1+\lambda_i^2)(1+\lambda_j^2)\lambda_j\lambda_i}{\xi_{ij}}^2\\\geq&0.\end{aligned}\tag{2.18}$$

Remark 2.6 In fact, if D^2u is diagonal, then we have

$$\sum_{i,j,k,l=1}^n\left[\frac{\partial^2\arctan D^2u}{\partial u_{ij}\partial u_{kl}}+2\frac{\partial\arctan D^2u}{\partial u_{ik}}u^{jl}\right]\xi_{ij}\xi_{kl}\geq\sum_{i=1}^n\frac{2}{(1+{u_{ii}}^2)^2u_{ii}}{\xi_{ii}}^2.\tag{2.19}$$

3 Proof of Theorem 1.1

In this section, we prove the constant rank theorem 1.1 as follows.

Proof Suppose D^2u attains its minimal rank l at some point $x_0 \in \Omega$. Then we can get $\sigma_l(D^2u)(x_0) > 0$, and $\sigma_{l+1}(D^2u)(x_0) = 0$. So there exists a small neighborhood $\mathcal{N}_{x_0} \subset \Omega$ of x_0, and a small positive constant c_0, such that $\sigma_l(D^2u)(x_0) \geq c_0 > 0$ in $\mathcal{N}_{x_0}$.

We set the test function

$$\phi(x) = \sigma_{l+1}(D^2u) \quad \text{in } \mathcal{N}_{x_0}. \tag{3.1}$$

For any fixed $x \in \mathcal{N}_{x_0}$, we choose a local orthonormal frame such that D^2u is diagonal, and $u_{11} \geq u_{22} \geq \cdots \geq u_{nn}$. We denote $\lambda = (\lambda_1, \lambda_2, \ldots, \lambda_n)$ with $\lambda_i = u_{ii}$, $G = \{1, \ldots, l\}$ and $B = \{l+1, \ldots, n\}$. When no confusion occurs, we will likewise denote $G = \{\lambda_1, \ldots, \lambda_l\}$ and $B = \{\lambda_{l+1}, \ldots, \lambda_n\}$. Then there exists a positive constant δ such that

$$u_{ii} \geq \delta > 0, \quad i \in G. \tag{3.2}$$

Moreover, we have

$$\phi = \sigma_{l+1}(D^2u) \geq \sigma_l(G) \sum_{i\in B} u_{ii}, \tag{3.3}$$

hence

$$u_{ii} = O(\phi), \quad i \in B. \tag{3.4}$$

Taking the first derivatives of ϕ, we have

$$\begin{aligned}\phi_\alpha &= \sum_{i,j=1}^n \frac{\partial \sigma_{l+1}(D^2u)}{\partial u_{ij}} u_{ij\alpha} = \sum_{i=1}^n \sigma_l(\lambda|i) u_{ii\alpha} = \sum_{i\in B} \sigma_l(\lambda|i) u_{ii\alpha} + \sum_{i\in G} \sigma_l(\lambda|i) u_{ii\alpha} \\ &= \sigma_l(G) \sum_{i\in B} u_{ii\alpha} + O(\phi),\end{aligned} \tag{3.5}$$

and then

$$\sum_{i\in B} u_{ii\alpha} = O(\phi + |D\phi|), \quad \forall\ \alpha = 1, 2, \ldots, n. \tag{3.6}$$

Taking the second derivatives of ϕ, we get

$$\begin{aligned}\phi_{\alpha\alpha} =& \sum_{i=1}^n \frac{\partial \sigma_{l+1}(D^2u)}{\partial u_{ii}} u_{ii\alpha\alpha} + \sum_{i,j,k,l=1}^n \frac{\partial^2 \sigma_{l+1}(D^2u)}{\partial u_{ij} \partial u_{kl}} u_{ij\alpha} u_{kl\alpha} \\ =& \sum_{i=1}^n \sigma_l(\lambda|i) u_{ii\alpha\alpha} + \sum_{i\neq k} \sigma_{l-1}(\lambda|ik) u_{ii\alpha} u_{kk\alpha} + \sum_{i\neq j} (-\sigma_{l-1}(\lambda|ij)) {u_{ij\alpha}}^2 \\ =& \Big[\sum_{i\in G} + \sum_{i\in B}\Big] \sigma_l(\lambda|i) u_{ii\alpha\alpha} + \Big[\sum_{i\in G, k\in G, i\neq k} + \sum_{i\in G, k\in B} + \sum_{i\in B, k\in G} + \sum_{i\in B, k\in B, i\neq k}\Big] \sigma_{l-1}(\lambda|ik) u_{ii\alpha} u_{kk\alpha} \\ & - \Big[\sum_{i\in G, j\in G, i\neq j} + \sum_{i\in G, j\in B} + \sum_{i\in B, j\in G} + \sum_{i\in B, j\in B, i\neq j}\Big] \sigma_{l-1}(\lambda|ij) {u_{ij\alpha}}^2\end{aligned}$$

$$
\begin{aligned}
&=\sum_{i\in B}\sigma_l(\lambda|i)u_{ii\alpha\alpha}-2\sum_{i\in G,j\in B}\sigma_{l-1}(\lambda|ij){u_{ij\alpha}}^2+O(\phi+|D\phi|)\\
&=\sigma_l(G)\sum_{i\in B}\Big(u_{ii\alpha\alpha}-2\sum_{j\in G}\frac{{u_{ij\alpha}}^2}{u_{jj}}\Big)+O(\phi+|D\phi|),
\end{aligned}\tag{3.7}
$$

where we used (3.4), (3.6), and Lemma 2.5 in [15], that is we know $|Du_{ij}|\le C(\sqrt{u_{ii}}+\sqrt{u_{jj}})$, and then

$$
u_{ii\alpha}u_{kk\alpha}=O(\phi),\quad i,k\in B,\qquad {u_{ij\alpha}}^2=O(\phi),\quad i,j\in B.\tag{3.8}
$$

Then, we have

$$
\begin{aligned}
F^{\alpha\beta}\phi_{\alpha\beta}&=F^{\alpha\alpha}\phi_{\alpha\alpha}\\
&=\sigma_l(G)\sum_{i\in B}\Big(F^{\alpha\alpha}u_{ii\alpha\alpha}-2\sum_{j\in G}\frac{F^{\alpha\alpha}{u_{ij\alpha}}^2}{u_{jj}}\Big)+O(\phi+|D\phi|)\\
&=\sigma_l(G)\sum_{i\in B}\Big(\Theta_{ii}-F^{\alpha\beta,\gamma\eta}u_{\alpha\beta i}u_{\gamma\eta i}-2\sum_{j\in G}\frac{F^{\alpha\alpha}{u_{ij\alpha}}^2}{u_{jj}}\Big)+O(\phi+|D\phi|).
\end{aligned}\tag{3.9}
$$

And for $i\in B$,

$$
\begin{aligned}
&-F^{\alpha\beta,\gamma\eta}u_{\alpha\beta i}u_{\gamma\eta i}-2\sum_{j\in G}\frac{F^{\alpha\alpha}{u_{ij\alpha}}^2}{u_{jj}}\\
=&-F^{\alpha\alpha,\alpha\alpha}{u_{\alpha\alpha i}}^2-\sum_{\alpha\ne\beta}F^{\alpha\beta,\beta\alpha}{u_{\alpha\beta i}}^2-2\sum_{j\in G}\frac{F^{\alpha\alpha}{u_{ij\alpha}}^2}{u_{jj}}\\
=&-\sum_{\alpha\in G}F^{\alpha\alpha,\alpha\alpha}{u_{\alpha\alpha i}}^2-\sum_{\alpha\ne\beta\in G}F^{\alpha\beta,\beta\alpha}{u_{\alpha\beta i}}^2-2\sum_{\alpha,j\in G}\frac{F^{\alpha\alpha}{u_{ij\alpha}}^2}{u_{jj}}+O(\phi)\\
=&-\sum_{\alpha\in G}\frac{-2\lambda_\alpha}{(1+\lambda_\alpha^2)^2}{u_{\alpha\alpha i}}^2-\sum_{\alpha\ne\beta\in G}\frac{-(\lambda_\alpha+\lambda_\beta)}{(1+\lambda_\alpha^2)(1+\lambda_\beta^2)}{u_{\alpha\beta i}}^2-2\sum_{\alpha,j\in G}\frac{1}{(1+\lambda_\alpha^2)\lambda_j}{u_{ij\alpha}}^2+O(\phi)\\
=&-\sum_{\alpha\in G}\Big[\frac{-2\lambda_\alpha}{(1+\lambda_\alpha^2)^2}+\frac{2}{(1+\lambda_\alpha^2)\lambda_\alpha}\Big]{u_{\alpha\alpha i}}^2\\
&-\sum_{\alpha\ne\beta\in G}\Big[\frac{-(\lambda_\alpha+\lambda_\beta)}{(1+\lambda_\alpha^2)(1+\lambda_\beta^2)}+\frac{1}{(1+\lambda_\alpha^2)\lambda_\beta}+\frac{1}{(1+\lambda_\beta^2)\lambda_\alpha}\Big]{u_{\alpha\beta i}}^2+O(\phi)\\
=&-\sum_{\alpha\in G}\Big[\frac{2}{(1+\lambda_\alpha^2)^2\lambda_\alpha}\Big]{u_{\alpha\alpha i}}^2-\sum_{\alpha\ne\beta\in G}\Big[\frac{\lambda_\alpha+\lambda_\beta}{(1+\lambda_\alpha^2)(1+\lambda_\beta^2)\lambda_\alpha\lambda_\beta}\Big]{u_{\alpha\beta i}}^2+O(\phi)\\
\le&O(\phi).
\end{aligned}\tag{3.10}
$$

In fact, (3.10) is the inverse convexity (2.19). Hence

$$
\begin{aligned}
F^{\alpha\beta}\phi_{\alpha\beta}&\le\sigma_l(G)\sum_{i\in B}\Theta_{ii}+O(\phi+|D\phi|)\\
&\le C(\phi+|\nabla\phi|),
\end{aligned}\tag{3.11}
$$

since Θ is concave. By the strong maximum principle, we have $\phi \equiv 0$ in $\mathcal{N}_{x_0}$, and $\{x \in \Omega : \phi(x) = 0\}$ is both open and closed. Consequently, $\phi \equiv 0$ in Ω and D^2u is of constant rank l in Ω. The proof is complete.

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关于特殊拉格朗日方程凸解的常秩定理的一则注记

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摘要: 本文研究了特殊拉格朗日算子的逆凸性型不等式, 并证明了在某些条件下具有非常数相位的特殊拉格朗日方程凸解的常秩定理.

关键词: 特殊拉格朗日方程; 常秩定理; 逆凸性

MR(2010)**主题分类号**: 35J60; 35B50 **中图分类号**: O175.25