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APPLICATION OF THE MINIMAL NORM TENSOR PRINCIPLE IN MÖBIUS GEOMETRY

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Abstract: This paper addresses Pinching problems in Möbius geometry for hypersurfaces with Möbius isotropy in the unit sphere. By implementing the minimum norm tensor principle, we rigorously estimate the squared norm of the quadratic gradient term associated with the Möbius second fundamental form. This analysis yields a critical inequality governing the geometric configuration. Leveraging this inequality, we subsequently prove a Pinching theorem characterizing the eigenvalues of the Blaschke tensor.

Keywords: minimal norm tensor; Möbius second fundamental form; hypersurface

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1 Introduction

The Pinching point problem for submanifolds in spherical space has garnered significant attention from numerous scholars, leading to notable progress. For instance, Peng and Terng [1,2] completed the second gap problem for minimal hypersurfaces in three-dimensional space. Zhang [3] proved the second Pinching theorem for minimal hypersurfaces in 9-dimensional spherical space. Later, Zhong Dingxing [4,5] extended the study to Möbius submanifolds in spherical space, leading to a pinching theorem for the smallest eigenvalue of the Blaschke tensor.

While studying the Pinching problem, it becomes essential to estimate the squared norm of the second fundamental form, which includes the well-known inequality of Chen Qingming [6]. In 2021, Guo Zhen introduced the concept of minimal norm tensors and provided the expressions for third and fourth-order minimal norm tensors in [7]. In 2023, Jiao Lurong presented an application of the third-order minimal norm tensor in Möbius geometry [8]. Inspired by [8], this paper applies the minimal norm principle to estimate the squared norm of the Möbius second fundamental form on Möbius-oriented hypersurfaces in unit ball space, leading to the following main contents:

Proposition 1 Let $x : M^n \to S^{n+1}$ be an n-dimensional Möbius-oriented hypersurface ($n \geq 2$) without umbilical points. Then the following inequality holds

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$$\left|\nabla^2 B\right|^2 \geq \frac{3}{2}\left[\frac{n-1}{n} trB^4 - \left(trB^3\right)^2 - 4\lambda\frac{(n-1)^2}{n^2} + 4(n-1)\lambda^2\right] + \frac{3(n-1)[(n-1)-2n^2\lambda]}{2(n+4)n^3} \tag{1.1}$$

where λ is a constant, and B is the Möbius second fundamental form.

Using this inequality, we derive the following theorem.

Theorem 1 Let $x: M^n \to S^{n+1}$ $(n \geq 5)$ be a Möbius-oriented hypersurface without umbilical points, with Möbius Ricci curvature $Ric(M) \geq n\lambda$. If

$$\frac{n-1}{2n^2} - \frac{7(n-1)(n+4)}{2n^2(7(n+4)+n(2n+5))} < \lambda \leq \frac{n-1}{2n^2},$$

then $\lambda = \frac{n-1}{2n^2}$, and M^n is Möbius equivalent to a Clifford torus $S^m(\sqrt{\frac{m}{n}}) \times S^{n-m}(\sqrt{\frac{n-m}{n}})$, where $1 < m < n-1$.

2 Background Material

2.1 Minimal Norm Tensor

This section introduces some fundamental concepts of minimal norm tensors, as detailed in [3].

Definition 2.1.1 Let V be an n-dimensional vector space, and let T be a fully symmetric fourth-order tensor defined on V. For any pair $(x,y) \in \mathbb{R}^2$, the components of the fourth-order tensor $F(x,y)$ satisfy the following relations

$$\begin{aligned} F_{ijkl}(x,y) = & T_{ijkl} + x\left(T_{ij}\delta_{kl} + T_{ik}\delta_{jl} + T_{il}\delta_{jk} + T_{kl}\delta_{ij} + T_{jl}\delta_{ik} + T_{jk}\delta_{il}\right) \\ & + y \cdot t\left(\delta_{ij}\delta_{kl} + \delta_{ik}\delta_{jl} + \delta_{il}\delta_{jk}\right), \end{aligned} \tag{2.1}$$

where T_{ijkl} and $F_{ijkl}(x,y)$ are the components of tensors T and $F(x,y)$, respectively, and $T_{ij} = \sum_{k=l=1}^{n} T_{ijkl}$, $t = \sum_{i=j=1}^{n} T_{ij}$. The set $\{F(x,y) | (x,y) \in \mathbb{R}^2\}$ is called a two-parameter tensor set induced by the fully symmetric fourth-order tensor T.

Definition 2.1.2 For any $(x,y) \in \mathbb{R}^2$, if the function $f(x,y)$ satisfies

$$f(x,y) = \|F(x,y)\|^2 = \sum_{i,j,k,l=1}^{n} F_{ijkl}^2(x,y), \tag{2.2}$$

then $f(x,y)$ is called the modulus function of $F(x,y)$.

Given the geometric significance of the modulus function, it is clear that $f(x,y)$ is a quadratic polynomial function on $\mathbb{R}^2$ that attains a minimum. Therefore, we define the following:

Definition 2.1.3 If there exists $(x_0, y_0) \in \mathbb{R}^2$ such that

$$f\left(x_0, y_0\right) = \min_{(x,y)\in\mathbb{R}^2} f(x,y), \tag{2.3}$$

then $F(x_0, y_0)$ is called the minimal norm tensor of the two-parameter tensor set $\{F(x, y)| (x, y) \in \mathbb{R}^2\}$ induced by the fully symmetric fourth-order tensor T. In short, $F(x_0, y_0)$ is the minimal norm tensor of T.

2.2 Basic Knowledge of Möbius Geometry

Let $x : M^m \to S^n$ be an m-dimensional immersion without umbilical points. The local standard basis is denoted by $\{e_i\}$ $(1 \leq i \leq n)$, and the dual basis is $\{\omega_i\}$. We define the Möbius form $\Phi = \sum_{i,\alpha} C_i^\alpha \omega_i e_\alpha$, the Blaschke tensor $A = \sum_{ij} A_{ij}\omega_i \otimes \omega_j$, and the Möbius second fundamental form $B = \sum_{ij\alpha} B_{ij}^\alpha \omega_i\omega_j(\rho^{-1}e_\alpha)$, where $B_{ij}^\alpha = \rho^{-1}(h_{ij}^\alpha - H^\alpha\delta_{ij})$.

From these, the following structural equations hold

$$R_{ijkl} = \sum_\alpha \left(B_{ik}^\alpha B_{jl}^\alpha - B_{il}^\alpha B_{jk}^\alpha\right) + A_{ik}\delta_{jl} + \delta_{ik}A_{jl} - A_{il}\delta_{jk} - \delta_{il}A_{jk}; \tag{2.4}$$

$$R^\perp_{\alpha\beta ij} = \sum_k \left(B_{ik}^\alpha B_{kj}^\beta - B_{ik}^\beta B_{kj}^\alpha\right); \tag{2.5}$$

$$B_{ij,k}^\alpha - B_{ik,j}^\alpha = \delta_{ij}C_k^\alpha - \delta_{ik}C_j^\alpha; \tag{2.6}$$

$$C_{i,j}^\alpha - C_{j,i}^\alpha = B_{ik}^\alpha A_{kj} - B_{jk}^\alpha A_{ki}; \tag{2.7}$$

$$A_{ij,k} - A_{ik,j} = B_{ik}^\alpha C_j^\alpha - B_{ij}^\alpha C_k^\alpha. \tag{2.8}$$

Next, we define a Möbius-oriented submanifold in S^n.

Definition 2.2.1 Let $x : M^m \to S^n$ be an m-dimensional immersion without umbilical points. If the Möbius form $\Phi \equiv 0$ and there exists a constant λ such that $A = \lambda g$, then x is called a Möbius-oriented submanifold in S^n.

Combining the definition of a Möbius-oriented submanifold and the structural equation 2.6, we can derive that when M^m is a hypersurface, $B_{ij,k} = B_{ik,j}$, meaning that $B_{ij,k}$ is fully symmetric. Consequently, we can obtain

$$\begin{aligned} R_{ijkl} &= B_{ik}B_{jl} - B_{il}B_{jk} + A_{ik}\delta_{jl} + \delta_{ik}A_{jl} - A_{il}\delta_{jk} - \delta_{il}A_{jk} \\ &= B_{ik}B_{jl} - B_{il}B_{jk} + 2\lambda(\delta_{ik}\delta_{jl} - \delta_{il}\delta_{jk}). \end{aligned} \tag{2.9}$$

3 Proof of Proposition 1

Let $(x_0, y_0) \in \mathbb{R}^2$ be a point such that $F(x_0, y_0)$ is the minimal norm tensor. We first need to determine the corresponding minimal point. If $F(x_0, y_0)$ is the minimal norm tensor, then we have

$$\left.\frac{\partial f(x,y)}{\partial x}\right|_{(x_0,y_0)} = 0, \left.\frac{\partial f(x,y)}{\partial y}\right|_{(x_0,y_0)} = 0. \tag{3.1}$$

Solving this system of equations, we obtain

$$x_0 = -\frac{1}{n+4}, \quad y_0 = \frac{1}{(n+2)(n+4)}, \tag{3.2}$$

Thus, we have:

$$\begin{aligned} F_{ijkl} = T_{ijkl} - \frac{1}{n+4}\left(T_{ij}\delta_{kl} + T_{ik}\delta_{jl} + T_{il}\delta_{jk} + T_{kl}\delta_{ij} + T_{jl}\delta_{ik} + T_{jk}\delta_{il}\right) \\ + \frac{1}{(n+2)(n+4)} \cdot t\left(\delta_{ij}\delta_{kl} + \delta_{ik}\delta_{jl} + \delta_{il}\delta_{jk}\right). \end{aligned} \tag{3.3}$$

Combining with Equation 2.2, we can derive

$$f = \|F\|^2 = \sum_{i,j,k,l=1}^{n} T_{ijkl}^2 - \frac{6}{n+4}\sum_{i,j=1}^{n} T_{ij}^2 + \frac{3}{(n+2)(n+4)}t^2. \tag{3.4}$$

From the previous discussion, we know that on the Möbius oriented hypersurface in the unit sphere S^n, the components $B_{ij,k}$ are fully symmetric. Thus, we can define the fully symmetric four-order tensor T on the hypersurface M^n as satisfying the following component relations

$$T_{ijkl} = \frac{1}{4}\left(B_{ij,kl} + B_{jk,li} + B_{kl,ij} + B_{li,jk}\right). \tag{3.5}$$

Combining with the Ricci identity, we can obtain

$$T_{ijkl} = B_{ij,kl} + \frac{1}{4}\sum_{p}\left[B_{ip}\left(R_{pjlk} + R_{pklj}\right) + B_{jp}\left(R_{pilk} + R_{pkli}\right) + B_{kp}\left(R_{pilj} + R_{pjli}\right)\right]. \tag{3.6}$$

Furthermore, combining with Equation 2.9, we can derive

$$\begin{aligned} &T_{ijkl} \\ =&B_{ij,kl} + \frac{1}{2}\sum_{p}\left[B_{ip}B_{pl}B_{jk} + B_{jp}B_{pl}B_{ik} + B_{kp}B_{pl}B_{ij} - B_{ip}B_{pj}B_{kl} - B_{jp}B_{pk}B_{il} - B_{kp}B_{pi}B_{jl}\right] \\ &+ \lambda(B_{il}\delta_{jk} + B_{jl}\delta_{ik} + B_{kl}\delta_{ij} - B_{ij}\delta_{kl} - B_{ik}\delta_{jl} - B_{jk}\delta_{il}). \end{aligned} \tag{3.7}$$

We can sum and contract the above equation to obtain

$$T_{ij} = \sum_{k=1}^{n} B_{ij,kk} + \frac{1}{2}B_{ij}|B|^2 - n\lambda B_{ij}. \tag{3.8}$$

Using the Ricci identity again, we get $T_{ij} = (n\lambda - \frac{1}{2}|B|^2)B_{ij}$. Given that $\sum_{i=j} B_{ij} = 0$, we deduce that $t = 0$. According to Equation 3.4, to calculate the minimal norm, we need to separately compute $(T_{ijkl})^2$ and $(T_{ij})^2$.

By direct computation, we have

$$\begin{aligned} T_{ijkl} =&3(B_{ip}B_{pl}B_{jk}B_{ij,kl} - B_{ip}B_{pk}B_{jl}B_{ij,kl}) - 6\lambda B_{ik}B_{ij,kk} \\ &+ \left|\nabla^2 B\right|^2 + \frac{3}{2}|B|^2 trB^4 + 6n\lambda^2|B|^2 - \frac{3}{2}(trB^3)^2 - 6\lambda|B|^4, \end{aligned} \tag{3.9}$$

where $|\nabla^2 B|^2 = \sum_{i,j,k,l} B_{ij,kl}^2$, $\mathrm{tr}B^3 = \sum_{i,j,k} B_{ij}B_{jk}B_{ki}$, $\mathrm{tr}B^4 = \sum_{i,j,k,l} B_{ij}B_{jk}B_{kl}B_{li}$, and $|B|^2 = \frac{n-1}{n}$.

Using the Ricci identity, we obtain

$$B_{ip}B_{pl}B_{jk}B_{ij,kl} - B_{ip}B_{pk}B_{jl}B_{ij,kl} = -|B|^2 trB^4 + (trB^3)^2 + 2\lambda|B|^4,$$

$$2n\lambda|B|^2 - |B|^4. \tag{3.10}$$

Thus, we derive

$$T_{ijkl}^2 = \left|\nabla^2 B\right|^2 - \frac{3}{2}|B|^2 trB^4 + \frac{3}{2}(trB^3)^2 + 6\lambda|B|^4 - 6n\lambda^2|B|^2. \tag{3.11}$$

Similarly, we find

$$T_{ij}^2 = n^2\lambda^2|B|^2 - n\lambda|B|^4 + \frac{1}{4}|B|^6. \tag{3.12}$$

Given the non-negativity of the minimal norm, we have thus proven proposition 1. From proposition 1, it is not difficult to see that the inequality derived here matches the famous Chen' s inequality in form. Since λ is a constant, we can also derive the following corollary.

Corollary 3.1 If the constant $\lambda \geq \frac{1}{4}$ or $\lambda \leq 0$, then the following inequality holds

$$|\nabla^2 B|^2 > \frac{3}{2}\left[\frac{n-1}{n}\mathrm{tr}B^4 - (\mathrm{tr}B^3)^2\right].$$

Proof According to proposition 1 and Equation 1.1, we know that if we set

$$A = \left[\frac{3}{2(n+4)}\left(\frac{n-1}{n}\right)^3 + \frac{12(n^2+2n)}{2(n+4)}\lambda^2\frac{n-1}{n} - \frac{12(n+2)}{n+4}\lambda\frac{(n-1)^2}{n^2}\right],$$

then we have

$$|\nabla^2 B|^2 \geq \frac{3}{2}\left[\frac{n-1}{n}\mathrm{tr}B^4 - (\mathrm{tr}B^3)^2\right] + A.$$

We only need to discuss the sign of A. We can regard A as a quadratic function of λ. By direct calculation, we know that the axis of symmetry of this function is $0 < \frac{n-1}{2n^2} \leq \frac{1}{8}$, and the function has two distinct roots λ_1 and λ_2. According to Vieta' s formulas, we have

$$\lambda_1 + \lambda_2 = \frac{n-1}{n^2}, \quad \lambda_1\lambda_2 > 0,$$

Therefore, $\lambda_1 > 0$ and $\lambda_2 > 0$, and when $n \geq 2$, $0 < \lambda_1 + \lambda_2 = \frac{n-1}{n^2} \leq \frac{1}{4}$. Thus, we have $\frac{1}{4} > \lambda_1 > 0$ or $\frac{1}{4} > \lambda_2 > 0$. This implies that when $\lambda \geq \frac{1}{4}$, $A > 0$ always holds, so the corollary is proven.

When $\lambda \leq 0$, the conclusion clearly holds.

4 Proof of Theorem 1

Before proving Theorem 1, we first need to prove the following proposition.

Proposition 4.1 Let $M^n \to S^{n+1}$ be an umbilical-free Möbius oriented hypersurface. Then the following inequality holds

$$\int_M \left\{ (\frac{n-1}{n} - 2(2n+\frac{3}{2})\lambda) \sum_{i,j,k} B_{ij,k}^2 + \frac{2[(n-1)-2n^2\lambda)]trB^4}{n} - \frac{3(n-1)[(n-1)-2n^2\lambda]}{2n^3(n+4)} \right\} dM \geq 0. \tag{4.1}$$

Proof First, we can directly compute

$$\frac{1}{2}\Delta(B_{ij}^2) = \sum_{i,j,k} B_{ij,k}^2 + \sum_{i,j,k} B_{ij} B_{ij,kk}. \tag{4.2}$$

Using the relation $B_{ij,kk} = 2n\lambda B_{ij} - \frac{n-1}{n} B_{ij}$, we obtain

$$\frac{1}{2}\Delta(B_{ij}^2) = \sum_{i,j,k} B_{ij,k}^2 + \frac{(n-1)(2n^2\lambda - (n-1))}{n^2}. \tag{4.3}$$

In a similar manner, we have

$$\begin{aligned} \frac{1}{2}\Delta|\nabla B_{ij}|^2 &= \sum_{i,j,k,l} B_{ij,kl}^2 + \sum_{i,j,k,l} B_{ij,k} B_{ij,kll} \\ &= \left|\nabla^2 B\right|^2 + ((2n+3)\lambda - |B|^2) \sum_{i,j,k} B_{ij,k}^2 + 3(2B - A), \end{aligned} \tag{4.4}$$

where $A = \sum_{i,j,k,m,l} B_{ij} B_{km} B_{ij,k} B_{ij,m}$, $B = B_{mk} B_{ij} B_{k,lm} B_{jm,l}$. Next, consider $(B_{ij} B_{jk} B_{lm} B_{ik,l})_m$. By direct computation, we can derive the following separately

$$0 = \int_M (2B_{ij} B_{lm} B_{jk,m} B_{ik,l} + B_{ij} B_{jk} B_{lm} B_{ik,lm}) dM, \tag{4.5}$$

$$0 = \int_M [2B_{ij} B_{jk} B_{kl,m} B_{li,m} + B_{ij} B_{kl} B_{jk,m} B_{li,m} + (2n\lambda - |B|^2) trB^4] dM, \tag{4.6}$$

$$0 = \int_M [2B_{ij} B_{jk} B_{lm,i} B_{lm,k} + B_{ij} B_{jk} B_{lm} B_{ik,lm} + (trB^3)^2 + 2\lambda|B|^4 - |B|^2 trB^4] dM. \tag{4.7}$$

By combining Equations 4.5 and 4.7, we can derive

$$\int_M (A - 2B) dM = \int_M (|B|^2 trB^4 - (trB^3)^2 - 2\lambda|B|^4) dM. \tag{4.8}$$

Using Equations 4.6 and 4.8, we obtain

$$\int_M [(|B|^2 - 8n\lambda) trB^4 + 3(trB^3)^2 + 6\lambda|B|^4] dM \geq 0. \tag{4.9}$$

Integrating Equation 4.4, we get

$$\int_M \left|\nabla^2 B\right|^2 dM = \int_M [-((2n+3)\lambda - |B|^2) \sum_{i,j,k} B_{ij,k}^2 - 3(2B - A)] dM. \tag{4.10}$$

By combining Equations 4.8 and 4.10 with Proposition 1, we derive

$$\int_M \left\{ (|B|^2 - 2(2n+\frac{3}{2})\lambda)\sum_{i,j,k} B_{ij,k}^2 + \frac{3}{2}[|B|^2 trB^4 - (trB^3)^2 - 2\lambda|B|^2] - \frac{3|B|^2(|B|^2-2n\lambda)}{2(n+4)} \right\} dM \geq 0, \tag{4.11}$$

Finally, by combining Equations 4.9 and 4.11, we conclude

$$\int_M \left\{ (\frac{n-1}{n} - 2(2n+\frac{3}{2})\lambda)\sum_{i,j,k} B_{ij,k}^2 + \frac{2[(n-1)-2n^2\lambda)]trB^4}{n} - \frac{3(n-1)[(n-1)-2n^2\lambda]}{2n^3(n+4)} \right\} dM \geq 0. \tag{4.12}$$

Thus, we complete the proof of Proposition 4.1.

Now, we prove Theorem 1.

From Equation 4.4 and Stokes' theorem, we know that

$$\int_M \sum_{i,j,k} B_{ij,k}^2 = \int_M \frac{(n-1)((n-1)-2n^2\lambda)}{n^2}. \tag{4.13}$$

Assuming a local standard orthogonal frame $e_1, e_2, \ldots, e_n$, we choose an appropriate local standard orthogonal frame such that $B_{ij} = \lambda_i \delta_{ij}$. Then we have $|B|^2 = \sum_i \lambda_i^2 = \frac{n-1}{n}$. From Equation 2.9, we can deduce $R_{ii} = -\lambda_i^2 + 2(n-1)\lambda \geq n\lambda$. Thus,

$$\lambda_i^2 \leq (n-2)\lambda,$$

$$\sum_i \lambda_i^4 \leq (n-2)\lambda \sum_i \lambda_i^2 = \frac{(n-1)(n-2)\lambda}{n}.$$

Therefore, we obtain

$$trB^4 \leq \frac{(n-1)(n-2)\lambda}{n}. \tag{4.14}$$

From Proposition 4.1 and Equation 4.14, we can derive

$$\begin{aligned} &\int_M \left\{ (\frac{n-1}{n} - 2(2n+\frac{3}{2})\lambda)\sum_{i,j,k} B_{ij,k}^2 + \frac{2[(n-1)-2n^2\lambda)]trB^4}{n} - \frac{3(n-1)[(n-1)-2n^2\lambda]}{2n^3(n+4)} \right\} dM \\ =&\int_M \left\{ (\frac{n-1}{n} - 2(2n+\frac{3}{2})\lambda)\sum_{i,j,k} B_{ij,k}^2 + \frac{2[(n-1)-2n^2\lambda)](n-1)(n-2)\lambda}{n^2} \right\} dM \\ &-\int_M \frac{3(n-1)[(n-1)-2n^2\lambda]}{2n^3(n+4)} dM \geq 0. \end{aligned} \tag{4.15}$$

By combining Equation 4.14, we obtain

$$\begin{aligned} &\int_M \frac{(n-1)((n-1)-2n^2\lambda)}{n^2} \left[\frac{n-1}{n} - 2(2n+\frac{3}{2})\lambda + 2(n-2)\lambda - \frac{3[(n-1)-2n^2\lambda]}{2n(n+4)} \right] dM \\ =&\int_M \frac{(n-1)((n-1)-2n^2\lambda)}{n^2} \left[\frac{[(n-1)-2n^2\lambda](2n+5)}{2n(n+4)} - 7\lambda \right] dM \geq 0. \end{aligned} \tag{4.16}$$

Since we assumed

$$\frac{n-1}{2n^2} - \frac{7(n-1)(n+4)}{2n^2(7(n+4)+n(2n+5))} < \lambda \le \frac{n-1}{2n^2},$$

we have

$$\frac{(n-1)[(n-1)-2n^2\lambda]}{n^2} \ge 0, \quad \left[\frac{[(n-1)-2n^2\lambda](2n+5)}{2n(n+4)} - 7\lambda\right] < 0. \tag{4.17}$$

Thus, we can conclude

$$\int_M \frac{(n-1)[(n-1)-2n^2\lambda]}{n^2}\left[\frac{[(n-1)-2n^2\lambda](2n+5)}{2n(n+4)} - 7\lambda\right] dM = 0. \tag{4.18}$$

Therefore, $(n-1)-2n^2\lambda = 0$, which implies $\lambda = \frac{n-1}{2n^2}$. So, M^n is Möbius equivalent to a Clifford torus $S^m(\sqrt{\frac{m}{n}}) \times S^{n-m}(\sqrt{\frac{n-m}{n}})$ for $1 < m < n-1$. This completes the proof of Theorem 1.

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极小模张量原理在Möbius几何中的应用

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摘要: 本文研究了Möbius几何中的Pinching问题, 利用极小模张量原理在单位球空间的Möbius迷向超曲面上, Möbius 第二基本形式的二次梯度模平方进行估计, 得到了一个有用的不等式, 并利用这个公式等到了关于Blaschke 张量特征值的一个拼挤定理.

关键词: 极小模张量; Möbius第二基本形式; 超曲面

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