

BIOLOGICAL INVASION PROBLEM WITH FREE BOUNDARY NONLOCAL DIFFUSION EQUATION

HE Yu-rong¹, ZHANG Ya-rong^{1,2,3}

(1. School of Science, Xi'an University of Architecture and Technology, Xi'an 710055, China)

(2. Xi'an Special Equipment Inspection Institute, Xi'an 710065, China)

(3. School of Power and Energy, Northwestern Polytechnical University, Xi'an 710072, China)

Abstract: In order to better describe the phenomenon of biological invasion, this paper introduces a free boundary model of biological invasion. Firstly, the right free boundary is added to the equation with logistic terms. Secondly, the existence and uniqueness of local solutions are proved by the Sobolev embedding theorem and the comparison principle. Finally, according to the relevant research data and contents of red fire ants, the diffusion area and nest number of red fire ants were simulated without external disturbance. This paper mainly simulates the early diffusion process of red fire ants. In the early diffusion stage, red fire ants grow slowly and then spread over a large area after reaching a certain number.

Keywords: Free boundary; biological invasion; differential equation

2010 MR Subject Classification: 35K55; 35R35

Document code: A **Article ID:** 0255-7797(2025)01-0048-09

1 Introduction

Biological invasion is the process by which organisms move from their original habitat to a new environment, either naturally or through human intervention. The proliferation of biological invasion involves a complex chain of events, including invasion, diffusion, settlement, adaptation, and other stages. Each of these stages is necessary for the success of the invasion. Due to the complexity of the problem of species invasion, [1, 2] attempts have been made to establish a unified biological framework to describe this issue. However, there are still many shortcomings, so it is necessary to establish a biological invasion model that is more in line with the actual situation.

As the issue of biological invasion gains importance in ecology, it is crucial to establish a mathematical model to describe it. Various mathematical models have been used to describe

* **Received date:** 2024-05-28 **Accepted date:** 2024-06-07

Foundation item: Supported by National Natural Science Foundation of China (12101482); Postdoctoral Science Foundation of China (2022M722604); General Project of Science and Technology of Shaanxi Province (2023-YBSF-372); The Natural Science Foundation of Shaan Xi Province (2023-JC-QN-0016); Shannxi Mathematical Basic Science Research Project (23JSQ042).

Biography: He Yurong(2000-), female, born at Shanxi, postgraduate, major in theory of partial differential equations and numerical. E-mail: hyrtxdi@163.com.

Corresponding author: Zhang Yarong

population growth in biology, such as Volterra integral equations [3, 4] and Logistic models [5, 6] to describe single race growth or intermediate competition. Additionally, the free boundary model has been used to describe tumor growth and biological invasion. In addition, the processing of free boundary is also a difficult problem. In order to better deal with this problem [7] use a level set approach to handle the moving boundaries to efficiently treat complicated topological changes. To investigate regional separation in population ecology, [8, 9] studied the free boundary model with local diffusion. They demonstrated the existence and uniqueness of the global solution for the model. The population expansion process was further studied by [10], who also improved the derivation of free boundary conditions. [11–13] describe a biological invasion model with free boundaries, which reduces the complexity of the problem and better simulates actual activities compared to fixed boundaries. [14] presents a numerical solution of nonlinear partial differential equations to characterize the spatial flow of a general animal population model. [15] used the He-Laplace method combined with fractional order complex transformation (FCT) to study a two-scale population growth model in a closed system.

The red fire ant (*Solenopsis invicta* Buren) has the potential to cause significant damage to local biodiversity, agriculture and public health. Consequently, it is vital to implement an effective eradication strategy that involves thorough monitoring and the destruction of the ant at the earliest stage of the invasion in order to prevent its expansion [16]. The distribution and occurrence of red fire ants in Yunnan were analyzed in [17]. [18] presents the main parameters of several red fire ant invasion events in South China, obtained through field investigation. The invasion history and propagation rules of red fire ants were studied by establishing and applying relevant models.

2 Model Description

The population growth of invasive species can be modelled using the logistic growth equation, which is an ordinary differential equation (ODE):

$$u'(t) = au - du - bu^2$$

The population of the invasive species is represented by u , while a and d represent the birth and natural death rates, respectively. Additionally, b represents the death rate due to intra-species competition.

The diffusive logistic equation in the \mathbb{R}^n as follow:

$$u_t - d\Delta u = u(a - bu), t > 0, x \in \mathbb{R}^n$$

Here $d\Delta u$ represents the local dispersal of species. However, scientific research has shown that local diffusion operators cannot accurately describe nonlocal actions in space. Nonlocal diffusion equations are better suited to describe the diffusion of new species in new habitats. These phenomena are well described by nonlocal diffusion equations:

$$u_t = \Delta u + u(1 - J * u), t > 0, x \in \mathbb{R}$$

Here $J(x) = J_\delta(x) = \frac{1}{\delta}J(\frac{x}{\delta})$ is kernel function and satisfies threshold condition,

$$J(x) \in L^1(\mathbb{R}), J(x) \geq 0, \int_{\mathbb{R}} J(x)dx = 1, J * u = \int_{\mathbb{R}} J(x-y)u(y)dy$$

Here $\delta > 0$ is used to represent the strength of the non-local effect, and the non-linear vector $J * u$ represents the competition for space resources between individuals within a race or with individuals of other species.

[3] proposed a model that assumes that a single population has no interaction with the outside world.

$$\begin{cases} u_t = \Delta u + au - bu^2 - uJ * u & \text{in } D = (0, \infty) \times \Omega \\ \frac{\partial u}{\partial n} = 0 & \text{on } \Gamma = (0, \infty) \times \partial\Omega \\ u(0, x) = u_0(x) & \text{for } x \in \bar{\Omega} \end{cases}$$

Here $J * u = \int_0^t J(t-s)u(s, x)ds, (t, x) \in D$ is the genetic impact of the past on the present, denotes a bounded domain in a new habitats \mathbb{R}^n with boundary $\partial\Omega$. $\partial/\partial n$ represents the exterior normal derivative to $\partial\Omega$.

[6] proposed a free boundary model based on the following diffusive logistic problem and a unique global solution is proved:

$$\begin{cases} u_t = d\Delta u + u(a - bu) & t > 0, 0 < x < h(t) \\ u_x(t, 0) = u(t, h(t)) = 0 & t > 0 \\ h'(t) = -\mu u_x(t, h(t)) & t > 0 \\ h(0) = h_0, u(0, x) = u_0(x) & 0 < x < h_0 \end{cases}$$

Here $x = h(t)$ is the spreading front of invasive species, a represents the intrinsic growth rate of the species, b is internal competition rate, d is dispersal rate. The long-term dynamic behavior of the model is either vanishing or diffusion.

Established according to the above equation, we can get:

$$\begin{cases} u_t = du_{xx} + u(a - bu - (\phi^*u)) & t > 0, 0 < x < h(t) \\ u_x(t, 0) = u(t, h(t)) = 0 & t > 0 \\ h'(t) = -\mu u_x(t, h(t)) & t > 0 \\ h(0) = h_0, u(0, x) = u_0(x) & 0 < x < h_0 \end{cases} \quad (2.1)$$

Here $x = h(t)$ is moving boundary to be determined and it denotes the spreading front, $u(t, x)$ represents the spreading of a new species in habitat, a, b, r, d and μ are given positive constants. The parameter a represents the intrinsic growth rate of the species, this is the maximum rate of growth a species can have when food and space are abundant, b measures its intraspecific competition, and d is the dispersal rate. $\mu > 0$ represents the ability of species to expand at boundaries. The free boundary $h'(t) = -\mu u_x(t, h(t))$ is a special case of Stefan condition. The integral $(\phi^*u)(t, x) = \int_{t_0}^t \phi^*u(t - \tau)u(\tau)d\tau$ is a hereditary term containing the effect of the past history on the present growth rate, $[0, h_0]$ represents species initial living space.

The initial function $u_0(x)$ satisfies

$$u_0(x) \in C^2([0, h_0]), u_0(h_0) = u_0'(0) = 0 \text{ and } u_0 > 0 \text{ in } [0, h_0) \quad (2.2)$$

and ϕ^*u satisfies

$$\phi \in C(\mathbb{R}) \cap L^1(\mathbb{R}), \phi(x) = \phi(-x) \text{ and } \int_{\mathbb{R}} \phi(x) dx = 1. \quad (2.3)$$

3 Existence and Uniqueness

Now we prove the existence and uniqueness of the local solution of problem (2.1) by using the principle of compression mapping.

Theorem 3.1 For any given u_0 satisfying (2.2) and any $0 < \alpha < 1$, there is a $T > 0$ such that problem (2.1) admits a unique solution

$$(u, h) \in C^{(1+\alpha)/2, 1+\alpha}(D_T) \times C^{(1+\alpha)}([0, T]).$$

Moreover, $\|u\|_{C^{(1+\alpha)/2, 1+\alpha}(D_T)} + \|h\|_{C^{(1+\alpha)}} \leq k$ Here $D_T = (t, x) \in \mathbb{R}^2 : x \in [0, h(t)]$, constants K and T only depend on h_0, α and $\|u\|_{C^2([0, h_0])}$.

Proof Similar to the literature [7, 11, 12], the first step is to straighten the free boundary. This means that the free boundary problem (2.1) is reduced to a nonlinear initial boundary value problem on a fixed region through a function transformation. Let $\xi(s)$ be a function in $C^3[0, \infty]$ satisfying,

$$(t, s) \rightarrow (t, x) \text{ and } x = s + \xi(s)(h(t) - h_0).$$

Consider the transformation, it changes the boundary $x = h(t)$ to the line $s = h_0$,

$$\begin{aligned} \frac{\partial s}{\partial x} &= \frac{1}{1 + \xi'(s)(h(t) - h_0)} \equiv \sqrt{A(h(t), s)} \\ \frac{\partial^2 s}{\partial x^2} &= \frac{\xi''(s)(h(t) - h_0)}{[1 + \xi'(s)(h(t) - h_0)]^3} \equiv B(h(t), s) \\ -\frac{1}{h'(t)} \frac{\partial s}{\partial t} &= \frac{\xi(s)}{1 + \xi'(s)(h(t) - h_0)} \equiv C(h(t), s) \end{aligned}$$

Assume $u(t, x) = u(t, s + \xi(s)(h(t) - h_0)) = \varphi(t, s)$ then

$$\begin{aligned} u_t &= \varphi_t - h'(t)C(h(t), s)\varphi_s, u_x = \sqrt{A(h(t), s)}\varphi_s, \\ u_{xx} &= \sqrt{A(h(t), s)}\varphi_{ss} + B(h(t), s)\varphi_s. \end{aligned}$$

Then (2.1) becomes initial boundary value problem on fixed region

$$\left\{ \begin{array}{ll} \varphi_t - A\varphi_{ss} - (Bd + h'C)\varphi_s = \varphi(a - b\varphi - \int_0^t \phi(t - \tau)\varphi(\tau)d\tau) & t > 0, 0 < s < h_0 \\ \varphi_s(t, 0) = \varphi(t, h_0) = 0 & t > 0 \\ \varphi = 0, h'(t) = -\mu \frac{\partial \varphi}{\partial s} & t > 0, s = h_0 \\ h(0) = h_0, \varphi(0, x) = \varphi_0(s) := u_0(s) & 0 < s < h_0 \end{array} \right. \quad (3.1)$$

Here $A = A(h(t), s), B = B(h(t), s), C = C(h(t), s)$. Although the free boundary is transformed into a fixed line, the equation itself becomes more complex. The equivalent equations (3.1) still contain a free boundary.

Denote $h_1 = -\mu u'_0(h_0)$ and $0 < T \leq \frac{h_0}{8(1+h_1)}$, define $\mathcal{D}_T = [0, T] \times [0, h_0]$, we introduce the following metric space:

$$\mathcal{D}_1 = \left\{ \varphi \in C(\mathcal{D}_T) : \varphi(0, s) = u_0(s), \|\varphi - u_0\|_{C(\Delta T)} \leq 1 \right\},$$

$$\mathcal{D}_2 = \left\{ h \in C^1([0, T]) : h(0) = h_0, h'(0) = h_1, \|h' - h_1\|_{C([0, T])} \leq 1 \right\}.$$

$\mathcal{D} := \mathcal{D}_1 \times \mathcal{D}_2$ is a complete metric space with the metric, define a metric on \mathcal{D} :

$$d((\phi_1, h_1), (\phi_2, h_2)) = \|\phi_1 - \phi_2\|_{C(\Delta T)} + \|h'_1 - h'_2\|_{C([0, T])},$$

for any $(\phi, h) \in \mathcal{D}_1 \times \mathcal{D}_2$, we have:

$$|h(t) - h(0)| \leq \left| \frac{h(t) - h_0}{t} \cdot T \right| \leq T \|h'\|_{L^\infty} < \frac{h_0}{8(1+|h_1|)} (\|h_1\|_{L^\infty} + 1) \leq \frac{h_0}{8}.$$

then, due to $h_1, h_2 \in \mathcal{D}_1 \times \mathcal{D}_2$, $h_1(0) = h_2(0) = h_0$

$$\|h_1 - h_2\|_{C[0, T]} \leq T \|h'_1 - h'_2\|_{C[0, T]}.$$

Refer to [11] L^p theory and Sobolev embedding theorem. For any $(\phi, h_1) \in \mathcal{D}_1 \times \mathcal{D}_2$, the following initial boundary value problems

$$\begin{cases} \bar{\varphi}_t - A d \bar{\varphi}_{ss} - (Bd + h'C) \bar{\varphi}_s = \varphi(a - b\varphi - \int_0^t \phi \varphi(t - \tau) \varphi(\tau) d\tau) & t > 0, 0 < s < h_0 \\ \bar{\varphi}(t, h_0) = 0, \frac{\partial \bar{\varphi}}{\partial s}(t, 0) = 0 & t > 0 \\ \bar{\varphi}(0, s) = u_0(s) & 0 < s < h_0 \end{cases} \quad (3.2)$$

have a unique solution $\bar{\varphi} \in C^{(1+\alpha)/2, 1+\alpha}(\mathcal{D})$, and $\|\bar{\varphi}\|_{C^{(1+\alpha)/2, (1+\alpha)}(\mathcal{D}_T)} \leq K_1$, here K_1 is a constant depend on h_0, α and $\|u_0\|_{C^2[0, h_0]}$, defining $\bar{h}(t) = h_0 - \int_0^t \mu \varphi_s(\tau, h_0) d\tau$ with $\bar{h}'(t) = -\mu \phi_s(t, h_0)$, $\bar{h}(0) = h_0$, $\bar{h}'(0) = -\mu \phi_s(0, h_0)$ and hence $\bar{h}' \in C^{\alpha/2}([0, T])$, $\|\bar{h}'\|_{C^{\alpha/2}([0, T])} \leq K_2 := \mu K_1$

Define $\mathcal{F} : \mathcal{D} \rightarrow C([0, T] \times [0, h_0]) \times C^1([0, T])$ by $\mathcal{F}(\phi, h) = (\bar{\phi}, \bar{h})$ then

$$\begin{aligned} \|\bar{h}' - h_0\|_{C([0, h])} &\leq \|\bar{h}'\|_{C^{\alpha/2}([0, T])} T^{\alpha/2} \leq \mu K_1 T^{\alpha/2}, \\ \|\bar{\varphi} - u_0\|_{C(\mathcal{D}_T)} &\leq \|\bar{\varphi} - u_0\|_{C^{(1+\alpha)/2, 0}(\mathcal{D}_T)} T^{(1+\alpha)/2} \leq K_1 T^{(1+\alpha)/2}. \end{aligned}$$

Obviously $(\bar{\phi}, \bar{h}) \in \mathcal{D}$ is a fixed point of the operator \mathcal{F} if and only if it is a solution to the problem (3.1). If we assume $T \leq \min \left\{ (\mu K_1)^{-2/\alpha}, K_1^{-2/(1+\alpha)} \right\}$, then \mathcal{F} maps \mathcal{D} into itself.

$$\|\bar{\phi}_1 - \bar{\phi}_2\|_{C^{(1+\alpha)/2, 1+\alpha}(\mathcal{D})} \leq K_3 (\|\phi_1 - \phi_2\|_{C(\mathcal{D}_T)} + \|h_1 - h_2\|_{C^1[0, T]}).$$

Here K_3 is depend on K_1, K_2 . When $T \leq 1$, we have

$$\|\bar{\phi}_1 - \bar{\phi}_2\|_{C^{(1+\alpha)/2, 1+\alpha}(\mathcal{D}_T)} + \|\bar{h}'_1 - \bar{h}'_2\|_{C^{\alpha/2}[0, T]} \leq K_4 (\|\phi_1 - \phi_2\|_{C(\mathcal{D}_T)} + \|h'_1 - h'_2\|_{C[0, T]}).$$

Here K_4 is depend on K_3 and μ , $T := \min \left\{ 1, \left(\frac{1}{2K_4}\right)^{2/\alpha}, (\mu K_1)^{-2/\alpha}, K_1^{-2/(1+\alpha)}, \frac{h_0}{8(1+h_1)} \right\}$

$$\begin{aligned} \|\bar{\phi}_1 - \bar{\phi}_2\|_{C(\mathcal{D}_T)} + \|\bar{h}'_1 - \bar{h}'_2\|_{C([0,T])} &\leq T^{(1+\alpha)/2} \|\bar{\phi}_1 - \bar{\phi}_2\|_{C^{(1+\alpha)/2, 1+\alpha}(\mathcal{D}_T)} + T^{\alpha/2} \|\bar{h}'_1 - \bar{h}'_2\|_{C^{\alpha/2}([0,T])} \\ &\leq K_4 T^{\alpha/2} (\|\phi_1 - \phi_2\|_{C(\mathcal{D}_T)} + \|h'_1 + h'_2\|_{C([0,T])}) \\ &\leq \frac{1}{2} (\|\phi_1 - \phi_2\|_{C(\mathcal{D}_T)} + \|h'_1 + h'_2\|_{C([0,T])}). \end{aligned}$$

Therefore, the operator \mathcal{F} is a compressed map acting on space \mathcal{D} for time T and it has a unique fixed point (ϕ, h) on space \mathcal{D} . In addition, using Schauder estimate, $u \in C^{1+\alpha/2, 2+2\alpha}((0, T) \times [0, h_0])$, $h(t) \in C^{1+\alpha/2}((0, T])$. $(u(t, x), h(t))$ is the only locally classical solution to problem (3.1).

Lemma 3.2 (Comparison principle) Let $\bar{h} \in C^1([0, +\infty))$ and $\bar{h}(t) > 0$ for $t > 0$, condition (2.3) is satisfied. Assume that $\bar{u} \in C^{(1+\alpha)/2, 1+\alpha}(\mathcal{D})$, (\bar{u}, \bar{h}) satisfies

$$\begin{cases} \bar{u}_t \geq d\bar{u}_{xx} + \bar{u}(a - b\bar{u} - (\phi^*\bar{u})) & t > 0, 0 < x < \bar{h}(t) \\ \bar{u}_x(t, 0) \geq 0, \bar{u}(t, \bar{h}(t)) = 0 & t > 0 \\ \bar{h}'(t) \geq -\mu\bar{u}_x(t, \bar{h}(t)) & t > 0. \end{cases} \quad (3.3)$$

If $\bar{h}(0) \geq h_0$, $\bar{u}(0, x) \geq 0$ in $[0, \bar{h}(0)]$ and $\bar{u}(0, x) \geq u_0(x)$ in $[0, h_0]$. Then the solution (u, h) of (2.1) satisfies $\bar{h}(t) > h(t)$ in $[0, \infty)$ and $u \leq \bar{u}$ in \mathcal{D} .

Proof According to [11, 13], the comparison principle that $u(t, x) \leq \bar{u}(t, x)$ for $t \in (0, T_0)$ and $x \in [0, h(t))$, $\bar{u}(t) = \frac{a}{b} e^{\frac{a}{b}t} (e^{-\frac{a}{b}t} - 1 + \frac{a}{b\|u_0\|_\infty})^{-1}$ is the solution of the problem: $\frac{d\bar{u}}{dt} = \bar{u}(a - b\bar{u})$, $t > 0$, $\bar{u}(0) = \|u_0\|_\infty$.

Then we get $u(t, x) \leq \sup_{t \geq 0} \bar{u}(t) = \max \left\{ \frac{a}{b}, \|u_0\|_\infty \right\} := K_1$ with $(t, x) \in (0, T_0 \times (0, h(t))$. Follows from strong maximum principle and Hopf lemma that $u(t, x) > 0$, $u_x(t, h(t)) < 0$ with $(t, x) \in (0, T) \times (0, h(t))$. When $t \in (0, T_0)$, given by Stefan condition $h'(t) > 0$, we define:

$$\begin{aligned} \Omega_K^h &:= \{(t, x) \in \mathbb{R}^2 : 0 < t < T_0, h(t) - K^{-1} < x < h(t)\} \\ \bar{u}(t, x) &= K_1 [2K(h(t) - x) - K^2(h(t) - x)^2], \end{aligned}$$

here $\bar{u}(t, x)$ is an auxiliary function, K is a given positive constant. When $(t, x) \in \Omega_K^h$

$$\begin{aligned} \bar{u}(t, x) &= 2KK_1[1 - K(h(t) - x)]h'(t) \geq 0, -\Delta\bar{u} = 2K^2K_1, \\ u(a - bu - (\phi^*u)) &\leq aK_1 \text{ and } \bar{u}_t - d\Delta\bar{u} \geq 2dK^2K_1 \geq aK_1. \end{aligned}$$

If $K \geq \sqrt{\frac{a}{2d}}$, $u(t, h(t) - K^{-1}) \leq \bar{u}(t, h(t) - K^{-1})$ and $u(t, h(t)) = \bar{u}(t, h(t)) = 0$. Besides, when $x \in [h_0 - K^{-1}, h_0]$, we have

$$\begin{aligned} \bar{u}(0, x) &= K_1 [2K(h_0 - K^2(h_0 - x)^2)] \geq KK_1(h_0 - x) \\ u_0(x) &= - \int_x^{h_0} u'_0(s) ds \leq (h_0 - x) \|u'_0\|_{C([0, h_0])}. \end{aligned}$$

According to the maximum principle and $u(t, x) \leq \bar{u}(t, x)$ with $(t, x) \in \Omega_K^h$, we can obtain

$$h'(t) = -\mu u_x(t, h(t)) \leq -\mu \bar{u}_x(t, h(t)) = 2\mu KK_1 := K_2.$$

Therefore, existence constant $K_2 > 0$ that does not depend on T_0 . The proof is complete.

4 Numerical simulation

Red fire ants (scientific name: *Solenopsis invicta* Buren) are considered one of the 100 most dangerous invasive organisms globally. Studying the diffusion law of red fire ants is crucial to prevent their large-scale spread. Transmission of red fire ants occurs naturally or through human activity. Natural transmission mainly occurs through the propagation of reproductive ants by flying or during flood flow. It can also occur when nests are moved over short distances. Artificial transmission through human activities can occur over longer distances.

Based on research from [16–18], the red fire ant primarily spreads naturally in limited spaces. Using data from the aforementioned paper, we simulated the initial propagation of the red fire ant invasion, with less intermediate competition. In the absence of any influence from other species, the early spread of the red fire ant invasion was simulated. Here u represents ant nest distribution area (unit m^2), t represents propagation time (unit year), x is the spread distance (unit km). The following table lists the parameters:

Table 1 Parameter Numerical Value

Parameter	Numerical Value
a – Intrinsic growth rate	0.6346
b – Internal competition rate	0.0001
d – Diffusion coefficient	0.5551
μ – correlation index	0.8973

The equation was discretized using the finite difference method, a common numerical solution for partial differential equations. This method transforms continuous partial differential equations into discrete difference equations by approximating the differential operator, and obtains a numerical solution by solving the difference equation. The accompanying figure was generated using Matlab calculations. Figure 1 and Figure 2 show the distribution area and number of ant nests, respectively. The results indicate a slow increase in both the distribution area and number of ant nests in the early stage, followed by an acceleration with the increase of invasion time.

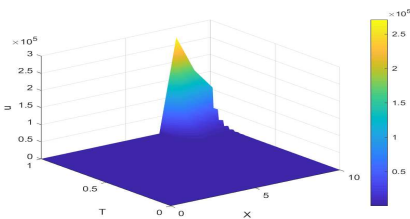


Figure 1: Ant nest distribution area

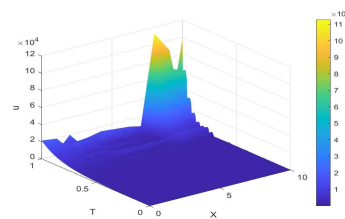


Figure 2: Ant nest number

Error analysis is a crucial aspect of numerical solutions. Errors primarily arise from

insufficient precision when using the finite difference method to approximate the analytical solution or from local truncation errors when approaching the free boundary. Additionally, rounding errors can occur when values are rounded during numerical calculations on a computer. In this paper, the time step is taken as $h_t = 0.005$, the space step is taken as $h_x = 0.05$. Based on the experimental data, the error analysis of the early population expansion of red fire ant was carried out and the results were analyzed by error and root-mean-square error (RMSE).

Table 2 Table of Error

	Error	RMSE
$n = 1$	9.37×10^{-3}	8.94×10^{-3}
$n = 2$	8.48×10^{-3}	8.26×10^{-3}

The error is calculated as the average of the absolute difference between the actual value and the experimental value. This method provides an intuitive way to observe the difference between the experiment and the actual value. The root-mean-square error, also known as the standard error, is the square root of the ratio of the sum of the squared deviations from the observed value to the number of observations (n). This measure is highly sensitive to large or small errors in a set of measurements. It is evident that the experimental data is highly accurate.

5 Discussion

The simulation data indicates that the distribution area of ant nests significantly increased when the annual transmission distance exceeded 7 km in a limited area, due to the limited autonomous transmission ability of red fire ants. The fastest transmission time was 8 km, with relatively small rest time. The number of ant nests reached its maximum when the annual dispersal distance was 8 km. Therefore, the focus of the prevention and control of red fire ants should be to strictly control the transport of goods in the red fire ant occurrence area to prevent the spread caused by human carrying.

References

- [1] Blackburn T M, Pyšek P, Bacher S, et al. A proposed unified framework for biological invasions[J]. *Trends in Ecology & Evolution*, 2011, 26(7): 333–339.
- [2] Courchamp F, Fournier A, Bellard C, et al. Invasion biology: specific problems and possible solutions[J]. *Trends in Ecology & Evolution*, 2017, 32(1): 13–22.
- [3] Redlinger R. On Volterra' s population equation with diffusion[J]. *SIAM Journal on Mathematical Analysis*, 1985, 16(1): 135–142.
- [4] Bao J, Mao X, Yin G, et al. Competitive Lotka - Volterra population dynamics with jumps[J]. *Nonlinear Analysis: Theory, Methods & Applications*, 2011, 74(17): 6601–6616.
- [5] Sakanoue S. Extended logistic model for growth of single-species populations[J]. *Ecological Modelling*, 2007, 205(1-2): 159–168.

- [6] Du Y, Lin Z. Spreading-vanishing dichotomy in the diffusive logistic model with a free boundary[J]. *SIAM Journal on Mathematical Analysis*, 2010, 42(1): 377–405.
- [7] Liu S, Liu X. Numerical methods for a two-species competition-diffusion model with free boundaries[J]. *Mathematics*, 2018, 6(5): 72.
- [8] Mimura M, Yamada Y, Yotsutani S. A free boundary problem in ecology[J]. *Japan Journal of Applied Mathematics*, 1985, 2: 151–186.
- [9] Mimura M, Yamada Y, Yotsutani S. Stability analysis for free boundary problems in ecology[J]. *Hiroshima Mathematical Journal*, 1986, 16(3): 477–498.
- [10] Bunting G, Du Y, Krakowski K. Spreading speed revisited: Analysis of a free boundary model[J]. *Networks and Heterogeneous Media*, 2012, 7(4): 583–603.
- [11] Lei Li, Jianping Wang, Mingxin Wang. The dynamics of nonlocal diffusion systems with different free boundaries [J]. *Commun. Pure Appl. Anal.*, 2020, 19(7): 3651–3672.
- [12] Li L, Li X, Wang M. A free boundary problem with nonlocal diffusion and unbounded initial range[J]. *Zeitschrift für angewandte Mathematik und Physik*, 2022, 73(5): 192.
- [13] Li L, Li W T, Wang M. Dynamics for nonlocal diffusion problems with a free boundary[J]. *Journal of Differential Equations*, 2022, 330: 110–149.
- [14] Ul Rahman J, Lu D, Suleman M, et al. He - Elzaki method for spatial diffusion of biological population[J]. *Fractals*, 2019, 27(05): 1950069.
- [15] Anjum N, He C H, He J H. Two-scale fractal theory for the population dynamics[J]. *Fractals*, 2021, 29(07): 2150182.
- [16] Ujijama S, Tsuji K. Controlling invasive ant species: a theoretical strategy for efficient monitoring in the early stage of invasion[J]. *Scientific Reports*, 2018, 8(1): 8033.
- [17] 尹艳琼,李向永,谌爱东,等.红火蚁入侵云南10年的发生状况与防控策略探析[J]. *环境昆虫学报*, 2022, 44(06): 1356–1364.
- [18] 陆永跃,梁广文,曾玲.华南地区红火蚁局域和长距离扩散规律研究[J]. *中国农业科学*, 2008, (04): 1053–1063.

具有自由边界的非局部扩散方程的生物入侵问题

何毓容¹, 张雅荣^{1,2,3}

(1.西安建筑科技大学理学院, 西安 710055)

(2.西安特种设备检验检测院, 西安 710065)

(3.西北工业大学能动学院, 西安 710072)

摘要: 为了更好地描述生物入侵现象, 本文引入了具有自由边界的生物入侵模型. 首先, 在具有Logistic项的方程组中加入右测自由边界; 其次, 利用Sobolev嵌入定理和比较原理证明了方程局部解的存在唯一性; 最后, 根据红火蚁的相关研究数据和内容, 假设在不受外界干扰的情况下, 模拟红火蚁的扩散面积和巢数. 本文主要模拟了红火蚁的早期扩散过程. 在扩散初期, 红火蚁的传播比较缓慢, 达到一定数量后才大面积扩散.

关键词: 自由边界; 生物入侵; 微分方程

MR(2010)主题分类号: 35K55 ; 35R35

中图分类号: O175.27