

THE (p, q) -ANALOG OF MULTIVALENT BAZILEVIČ FUNCTIONS ASSOCIATED WITH A LIMACON

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Abstract: This paper studies the problem of functional inequalities for analytic functions in classical geometric function theory. Using the differential subordination principle and (p, q) -derivative operator, it introduces (p, q) -analog of a class of multivalently Bazilevič functions associated with a limaçon function, and obtains the corresponding coefficient estimates and the Fekete-Szegő inequality, which extend and improve the related results for starlike functions, even q -starlike functions.

Keywords: Fekete-Szegő inequality; symmetric Toeplitz determinant; multivalent function; Bazilevič function; (p, q) -derivative operator

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1 Introduction

Denote by $\mathcal{A}_m (m \in \mathbb{N})$ the class of multivalently analytic functions f which are expanded with the Taylor-Maclaurin's series

$$f(z) = z^m + \sum_{n=m+1}^{\infty} a_n z^n \quad (1.1)$$

in the open unit disk $\mathbb{U} = \{z \in \mathbb{C} : |z| < 1\}$, where $\mathcal{A}_1 = \mathcal{A}$ is univalent analytic function class, and by Λ the class of Schwarz functions ω with $\omega(0) = 0$ and $|\omega(z)| < 1$ for $z \in \mathbb{U}$. For two analytic functions F and G in \mathbb{U} , if there has a Schwarz function $\omega \in \Lambda$ such that $F(z) = G(\omega(z))$, then F is subordinate to G in \mathbb{U} , i.e. $F \prec G$. In addition, if G is univalent in \mathbb{U} , then the following equivalence ([1, 2])

$$F \prec G \Leftrightarrow F(0) = G(0) \text{ and } F(\mathbb{U}) \subset G(\mathbb{U})$$

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holds true. Besides, if $\omega(z) = z$, then F is majorized by G in \mathbb{U} , i.e. $F \leq G$.

The theory of quantum calculus known as q -calculus is equivalent to traditional infinitesimal calculus without the notion of limits. The q -calculus, was started by Euler and Jacobi, found many interesting applications in various areas of mathematics, physics and engineering sciences. On a recent investigation done by Sahai and Yadav in the theory of special functions by [3], quantum calculus based on two parameters (p, q) was quoted. Indeed generalization of q -calculus is the post quantum calculus, denoted (p, q) -calculus. The (p, q) -integer was introduced in order to give a generalization or to unify several forms of q -oscillator algebras, well known in the earlier physics literature related to the representation theory of single parameter quantum algebras [4]. Throughout this article, we will use basic notations and definitions of the (p, q) -calculus as follows: Let $p > 0, q > 0$. For any non-negative integer n , the (p, q) -integer number n , denoted by $[n]_{p,q}$, is defined as

$$[n]_{p,q} = \frac{p^n - q^n}{p - q}, \quad [0]_{p,q} = 0. \quad (1.2)$$

The twin-basic number is a natural generalization of the q -number, that is $[n]_q = \frac{1-q^n}{1-q}$ ($q \neq 1$). Similarly, the (p, q) -differential operator of a function f , analytic in $|z| < 1$, is defined by

$$\mathcal{D}_{p,q}f(z) = \frac{f(pz) - f(qz)}{z(p - q)} \quad (p \neq q, z \in \mathbb{U} = \{z \in \mathbb{C} : |z| < 1\}). \quad (1.3)$$

One can easily show that $\mathcal{D}_{p,q}f(z) \rightarrow f'(z)$ as $p \rightarrow 1^-$ and $q \rightarrow 1^-$. Since we cannot obtain (p, q) -integers just by replacing q by q/p in the definition of q -integers, it is clear that q -integers and (p, q) -integers differs. However, the definition (1.2) reduces to quantum calculus for the case $p = 1$. Thus, we can say that (p, q) -calculus can be taken as a generalization of q -calculus. The (p, q) -factorial is defined by $[0]_{p,q}! = 1$, $[n]_{p,q}! = \prod_{k=1}^n [k]_{p,q}!$ ($n \geq 1$). Note that for $p \rightarrow 1^-$, the (p, q) -factorial reduces to the q -factorial. Also, clearly $\lim_{p \rightarrow 1^-} \lim_{q \rightarrow 1^-} [n]_{p,q} = n$, and $\lim_{p \rightarrow 1^-} \lim_{q \rightarrow 1^-} [n]_{p,q}! = n!$. For details on q -calculus and (p, q) -calculus, one can refer to [4, 5] and [3].

For $f \in \mathcal{A}_m$, define the (p, q) -derivative or the q -difference $\mathcal{D}_{p,q}f(z)$ by

$$\mathcal{D}_{p,q}f(z) = [m]_{p,q}z^{m-1} + \sum_{n=m+1}^{\infty} [n]_{p,q}a_n z^{n-1}, \quad (0 < q \leq p \leq 1), \quad (1.4)$$

where the (p, q) -derivative operator $\mathcal{D}_{p,q}f(z)$ of f is defined by(1.3).

Recall in [6] that a limaçon function $\vartheta : \mathbb{U} \rightarrow \mathbb{C}$ is defined by

$$\vartheta(z) = 1 + \sqrt{2}z + \frac{1}{2}z^2, \quad (1.5)$$

whose image is of the bean shape(see FIG.1), that is to say, the interior of

$$\{\zeta = x + yi \in \mathbb{C} : (4x^2 + 4y^2 - 8x - 5)^2 + 8(4x^2 + 4y^2 - 12x - 3)^2 = 0\}. \quad (1.6)$$

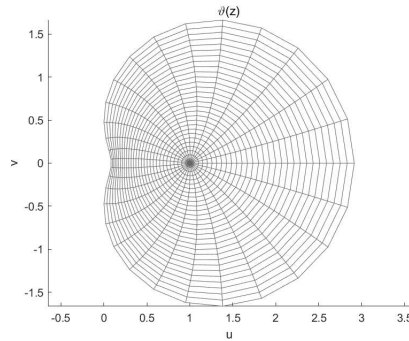


Figure 1 The image of \mathbb{U} under $\vartheta(z)$.

With the development of quantum mechanics, the q -calculus [7, 8] and (p, q) -calculus [4], which are two generalizations of the ordinary calculus without the limit symbol, have been applied into many mathematical, physical and engineering fields (refer to [9] and [10]). Since the seminal paper of Ismail et al. [11], there has a great deal of work to generalize the analytic functions in q -analysis and (p, q) -analysis; referring to Srivastava’s review paper [12] and [13–26]. For the multivalent analytic functions in q -calculus sense, we also find some related advances in [25, 27–29]. In the article, we intend to probe this object by subordinating to a limaçon [6] but not the Lemniscate of Bernoulli [30]. Specially, we introduce and study certain new subclass of analytic and multivalent Bazilevič functions related with (p, q) -derivative operator and a limaçon, and consider the corresponding estimates of the coefficients a_{m+1} and a_{m+2} as well as Fekete-Szegő functional inequalities. Meanwhile, the consequences and connections to all our results would also be pointed out.

Now, by making use of unified subordination technique by Ma-Mind[31] and (p, q) -calculus sense, we define the following subclass of multivalent Bazilevič functions associated with a limaçon function.

Definition 1.1 A function $f \in \mathcal{A}_m$ is said to be in the class $\mathcal{B}_{LC}^{p,q}(m, \lambda, \delta)$ if the following subordination

$$(1 - \lambda) \left(\frac{f(z)}{z^m} \right)^\delta + \lambda \frac{z \mathcal{D}_{p,q} f(z)}{[m]_{p,q} f(z)} \left(\frac{f(z)}{z^m} \right)^\delta \prec \vartheta(z) \tag{1.7}$$

holds for $z \in \mathbb{U}$, where $\delta > 0$ and $\lambda \in \mathbb{C}$.

Remark 1.2 In term of Definition 1.1, by choosing some special parameters, we get the following reduced versions:

- $\mathcal{B}_{LC}^{p,q}(m, \lambda, 1) \equiv \mathcal{B}_{LC}^{p,q}(m, \lambda)$ and $\mathcal{B}_{LC}^{1,1}(m, \lambda) \equiv \mathcal{B}_{LC}(m, \lambda)$;
- $\mathcal{B}_{LC}^{p,q}(m, 1, \delta) \equiv \mathcal{B}_{LC}^{p,q}(m, \delta)$ and $\mathcal{B}_{LC}^{1,1}(m, \delta) \equiv \mathcal{B}_{LC}(m, \delta)$;
- $\mathcal{B}_{LC}^{p,q}(m, 1, 0) \equiv \mathcal{S}_{LC}^{p,q}(m)$ and $\mathcal{S}_{LC}^{1,1}(m) \equiv \mathcal{S}_{LC}(m)$;
- $\lim_{p,q \rightarrow 1^-} \mathcal{B}_{LC}^{p,q}(m, \lambda, \delta) \equiv \mathcal{B}_{LC}(m, \lambda, \delta)$ and $\mathcal{B}_{LC}^{p,q}(1, \lambda, \delta) \equiv \mathcal{B}_{LC}^{p,q}(\lambda, \delta)$.

Remark 1.3 A function g belongs to the class $\mathcal{B}_{LC}(1, 1, 0)$ if and only if there exists an analytic function $q \prec \vartheta(z)$ such that

$$g(z) = z \exp \int_0^z \frac{q(t) - 1}{t} dt, \quad z \in \mathbb{U}.$$

The above intergral representation provides many examples of functions of the class $\mathcal{B}_{LC}(1, 1, 0)$. Let $q(t) = \vartheta(t^n)$, for $n = 1, 2, \dots$, and $t \in \mathbb{U}$. With with a simple calculation, we see that the function(see FIG.2)

$$\begin{aligned} h(z) &= z \exp \int_0^z \frac{\sqrt{2}t^n + \frac{1}{2}t^{2n}}{t} dt \\ &= z + \frac{\sqrt{2}}{n}z^{n+1} + \frac{1}{4n}z^{2n+1} + \dots, \end{aligned}$$

is an extreme function for the class $\mathcal{B}_{LC}(1, 1, 0)$.

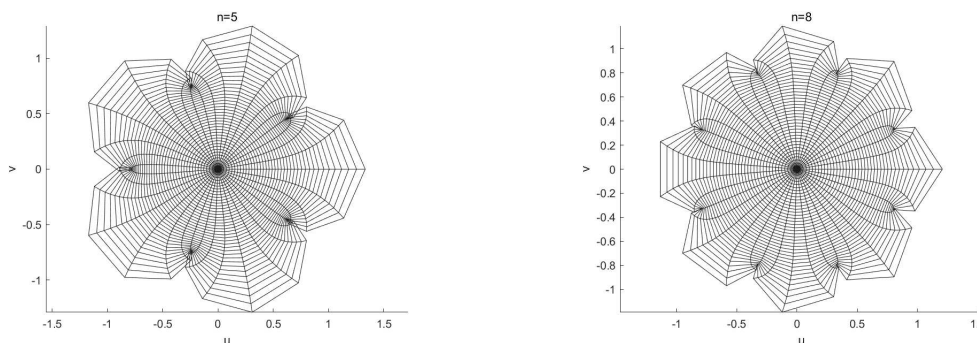


Figure 2 The image of \mathbb{U} under $h(z)$ with $n = 5, 8$.

Denote by \mathcal{P} the class of all analytic and univalent functions $\ell(z)$ with

$$\ell(z) = 1 + \sum_{n=1}^{\infty} c_n z^n, \quad (z \in \mathbb{D}) \tag{1.8}$$

satisfying $\Re [\ell(z)] > 0$ and $\ell(0) = 1$. To proceed our results, we have to be ready for some indispensable Lemmas below.

Lemma 1.4 [See [32, 33]] Let $\ell(z) \in \mathcal{P}$. Then the sharp estimates

$$|c_n| \leq 2 \quad (n \in \mathbb{N})$$

are true. In Particular, the equality holds for all n for the following function

$$\ell(z) = \frac{1+z}{1-z} = 1 + \sum_{n=1}^{\infty} 2z^n.$$

Lemma 1.5 [See [31]] If $\ell(z) \in \mathcal{P}$, then, for any complex μ ,

$$|c_2 - \mu c_1^2| \leq 2 \max \{1, |2\mu - 1|\}$$

and the result is sharp for the functions

$$\ell(z) = \frac{1+z}{1-z} \quad \text{or} \quad \ell(z) = \frac{1+z^2}{1-z^2}, \quad (z \in \mathbb{D}).$$

Lemma 1.6 [See [31]] Assume that the function $\ell(z) \in \mathcal{P}$ and $\mu \in \mathbb{R}$. Then

$$|c_2 - \mu c_1^2| \leq \begin{cases} -4\mu + 2 & \text{if } \mu \leq 0, \\ 2 & \text{if } 0 \leq \mu \leq 1, \\ 4\mu - 2 & \text{if } \mu \geq 1. \end{cases}$$

For $\mu < 0$ or $\mu > 1$, the inequality holds literally if and only if $\ell(z) = \frac{1+z}{1-z}$ or one of its rotations. If $0 < \mu < 1$, the inequality holds literally if and only if $\ell(z) = \frac{1+z^2}{1-z^2}$ or one of its rotations. In Particular, if $\mu = 0$, then the sharp result holds for the following function

$$\ell(z) = \left(\frac{1}{2} + \frac{\varsigma}{2}\right) \frac{1+z}{1-z} + \left(\frac{1}{2} - \frac{\varsigma}{2}\right) \frac{1-z}{1+z}, \quad (0 \leq \varsigma \leq 1)$$

or one of its rotations. If $\mu = 1$, then the sharp result holds for the following function

$$\frac{1}{\ell(z)} = \left(\frac{1}{2} + \frac{\varsigma}{2}\right) \frac{1+z}{1-z} + \left(\frac{1}{2} - \frac{\varsigma}{2}\right) \frac{1-z}{1+z}, \quad (0 \leq \varsigma \leq 1)$$

or one of its rotations. If $0 < \mu < 1$, then the upper bound is sharp as the followings

$$|c_2 - \mu c_1^2| + \mu |c_1|^2 \leq 2, \quad \left(0 < \mu \leq \frac{1}{2}\right)$$

and

$$|c_2 - \mu c_1^2| + (1 - \mu) |c_1|^2 \leq 2, \quad \left(\frac{1}{2} < \mu < 1\right).$$

2 Functional estimates for $\mathcal{B}_{LC}^{p,q}(m, \lambda, \delta)$

Denote the function $h \in \mathcal{P}$ by $h(z) = \frac{1+u(z)}{1-u(z)} = 1 + \sum_{n=1}^{\infty} c_n z^n$. Then, from (1.8) we derive that

$$u(z) = \frac{h(z) - 1}{h(z) + 1} = \frac{c_1}{2} z + \left(\frac{c_2}{2} - \frac{c_1^2}{4}\right) z^2 + \left(\frac{c_3}{2} - \frac{c_2 c_1}{2} + \frac{c_1^3}{8}\right) z^3 + \dots, \quad (z \in \mathbb{U}) \tag{2.1}$$

so that $u(z) \in \Lambda$. By (1.5) and (2.1), we imply that

$$\vartheta(u(z)) = 1 + \frac{\sqrt{2}c_1}{2} z + \left[\frac{\sqrt{2}c_2}{2} - \frac{(2\sqrt{2}-1)c_1^2}{8}\right] z^2 + \left[\frac{\sqrt{2}c_3}{2} - \frac{(2\sqrt{2}-1)c_2 c_1}{4} + \frac{(\sqrt{2}-1)c_1^3}{8}\right] z^3 \dots, \quad (z \in \mathbb{U}). \tag{2.2}$$

To begin with, we deal with the functional estimates for the class $\mathcal{B}_{LC}^{p,q}(m, \lambda, \delta)$ and establish the next theorems for the coefficient bounds and the corresponding Feteke-Szegö problems.

Theorem 2.1 If $f(z)$ given by (1.1) belongs to the class $\mathcal{B}_{LC}^{p,q}(m, \lambda, \delta)$, then

$$|a_{m+1}| \leq \frac{\sqrt{2}[m]_{p,q}}{|\lambda([m+1]_{p,q} - [m]_{p,q}) + \delta[m]_{p,q}|} \quad (2.3)$$

and

$$\begin{aligned} |a_{m+2}| \leq & \frac{(2\sqrt{2} - \frac{1}{2})[m]_{p,q}}{|\lambda([m+2]_{p,q} - [m]_{p,q}) + \delta[m]_{p,q}|} + \frac{2|\delta - 1|[m]_{p,q}^2}{|\lambda([m+1]_{p,q} - [m]_{p,q}) + \delta[m]_{p,q}|^2} \\ & \times \frac{|\lambda|([m+1]_{p,q} - [m]_{p,q}) + \frac{\delta}{2}[m]_{p,q}}{|\lambda([m+2]_{p,q} - [m]_{p,q}) + \delta[m]_{p,q}|}. \end{aligned} \quad (2.4)$$

Proof. Assume that $f(z) \in \mathcal{B}_{LC}^{p,q}(m, \lambda, \delta)$. Then, there exists a Schwarz function $u(z) \in \Lambda$ so that

$$(1 - \lambda) \left(\frac{f(z)}{z^m} \right)^\delta + \lambda \frac{z\mathcal{D}_{p,q}f(z)}{[m]_{p,q}f(z)} \left(\frac{f(z)}{z^m} \right)^\delta = \vartheta(u(z)). \quad (2.5)$$

Since

$$\begin{aligned} & (1 - \lambda) \left(\frac{f(z)}{z^m} \right)^\delta + \lambda \frac{z\mathcal{D}_{p,q}f(z)}{[m]_{p,q}f(z)} \left(\frac{f(z)}{z^m} \right)^\delta \\ = & 1 + \left[\lambda \left(\frac{[m+1]_{p,q}}{[m]_{p,q}} - 1 \right) + \delta \right] a_{m+1}z + \left\{ \left[\lambda \left(\frac{[m+2]_{p,q}}{[m]_{p,q}} - 1 \right) + \delta \right] a_{m+2} \right. \\ & \left. + \left[\lambda(\delta - 1) \left(\frac{[m+1]_{p,q}}{[m]_{p,q}} - 1 \right) + \frac{\delta(\delta - 1)}{2} \right] a_{m+1}^2 \right\} z^2 + \dots \end{aligned} \quad (2.6)$$

for $f \in \mathcal{A}_m(m \in \mathbb{N})$, combing (2.5) and (2.6) with (2.2) we get that

$$\frac{\sqrt{2}c_1}{2} = \left[\lambda \left(\frac{[m+1]_{p,q}}{[m]_{p,q}} - 1 \right) + \delta \right] a_{m+1}$$

and

$$\begin{aligned} \frac{\sqrt{2}c_2}{2} - \frac{(2\sqrt{2} - 1)c_1^2}{8} = & \left[\lambda \left(\frac{[m+2]_{p,q}}{[m]_{p,q}} - 1 \right) + \delta \right] a_{m+2} \\ & + \left[\lambda(\delta - 1) \left(\frac{[m+1]_{p,q}}{[m]_{p,q}} - 1 \right) + \frac{\delta(\delta - 1)}{2} \right] a_{m+1}^2, \end{aligned}$$

such that

$$a_{m+1} = \frac{\sqrt{2}[m]_{p,q}c_1}{2[\lambda([m+1]_{p,q} - [m]_{p,q}) + \delta[m]_{p,q}]} \quad (2.7)$$

and

$$\begin{aligned} a_{m+2} = & \frac{[4\sqrt{2}c_2 - (2\sqrt{2} - 1)c_1^2][m]_{p,q}}{8[\lambda([m+2]_{p,q} - [m]_{p,q}) + \delta[m]_{p,q}] - \frac{[m]_{p,q}^2c_1^2}{2[\lambda([m+1]_{p,q} - [m]_{p,q}) + \delta[m]_{p,q}]^2}} \\ & \times \frac{\lambda(\delta - 1)([m+1]_{p,q} - [m]_{p,q}) + \frac{\delta(\delta - 1)}{2}[m]_{p,q}}{[\lambda([m+2]_{p,q} - [m]_{p,q}) + \delta[m]_{p,q}]}. \end{aligned} \quad (2.8)$$

In view of Lemma 1.4 we imply that Theorem 2.1 holds true.

By taking $m = 1$ or $\lambda = 1$ or $\lambda = 1, \delta = 0$ in Theorem 2.1, we deduce the corollaries below.

Corollary 2.2 If $f(z)$ given by (1.1) belongs to the class $\mathcal{B}_{LC}^{p,q}(\lambda, \delta)$, then

$$|a_2| \leq \frac{\sqrt{2}}{|\lambda([2]_{p,q} - 1) + \delta|}$$

and

$$|a_3| \leq \frac{(2\sqrt{2} - \frac{1}{2})}{|\lambda([3]_{p,q} - 1) + \delta|} + \frac{2|\delta - 1|}{|\lambda([2]_{p,q} - 1) + \delta|^2} \times \frac{|\lambda|([2]_{p,q} - 1) + \frac{\delta}{2}}{|\lambda([3]_{p,q} - 1) + \delta|}.$$

Corollary 2.3 If $f(z)$ given by (1.1) belongs to the class $\mathcal{B}_{LC}^{p,q}(m, \delta)$, then

$$|a_{m+1}| \leq \frac{\sqrt{2}[m]_{p,q}}{([m + 1]_{p,q} - [m]_{p,q}) + \delta[m]_{p,q}}$$

and

$$|a_{m+2}| \leq \frac{(2\sqrt{2} - \frac{1}{2})[m]_{p,q}}{([m + 2]_{p,q} - [m]_{p,q}) + \delta[m]_{p,q}} + \frac{2|\delta - 1|[m]_{p,q}^2}{([m + 1]_{p,q} - [m]_{p,q}) + \delta[m]_{p,q}} \times \frac{([m + 1]_{p,q} - [m]_{p,q}) + \frac{\delta}{2}[m]_{p,q}}{([m + 2]_{p,q} - [m]_{p,q}) + \delta[m]_{p,q}}.$$

Corollary 2.4 If $f(z)$ given by (1.1) belongs to the class $\mathcal{S}_{LC}^{p,q}(m)$, then

$$|a_{m+1}| \leq \frac{\sqrt{2}[m]_{p,q}}{[m + 1]_{p,q} - [m]_{p,q}}$$

and

$$|a_{m+2}| \leq \frac{(2\sqrt{2} - \frac{1}{2})[m]_{p,q}}{[m + 2]_{p,q} - [m]_{p,q}} + \frac{2[m]_{p,q}^2}{([m + 1]_{p,q} - [m]_{p,q})([m + 2]_{p,q} - [m]_{p,q})}.$$

Theorem 2.5 If $f(z)$ given by (1.1) belongs to the class $\mathcal{B}_{LC}^{p,q}(m, \lambda, \delta)$, then

$$|a_{m+2} - \mu a_{m+1}^2| \leq \frac{\sqrt{2}[m]_{p,q} \max\{1, |2\hbar - 1|\}}{|\lambda([m + 2]_{p,q} - [m]_{p,q}) + \delta[m]_{p,q}|} \tag{2.9}$$

holds for $\mu \in \mathbb{C}$, where

$$\begin{aligned} \hbar &= \frac{(4 - \sqrt{2})}{8} + \frac{\sqrt{2}\mu[m]_{p,q}[\lambda([m + 2]_{p,q} - [m]_{p,q}) + \delta[m]_{p,q}]}{2[\lambda([m + 1]_{p,q} - [m]_{p,q}) + \delta[m]_{p,q}]^2} \\ &+ \frac{\sqrt{2}[m]_{p,q}}{2} \times \frac{\lambda(\delta - 1)([m + 1]_{p,q} - [m]_{p,q}) + \frac{\delta(\delta - 1)}{2}[m]_{p,q}}{[\lambda([m + 1]_{p,q} - [m]_{p,q}) + \delta[m]_{p,q}]^2}. \end{aligned}$$

Proof For $\mu \in \mathbb{C}$, by using (2.7) and (2.8), we infer that

$$a_{m+2} - \mu a_{m+1}^2 = \frac{\sqrt{2}[m]_{p,q}}{2[\lambda([m + 2]_{p,q} - [m]_{p,q}) + \delta[m]_{p,q}]} (c_2 - \hbar c_1^2), \tag{2.10}$$

where

$$\begin{aligned} \hbar &= \frac{(4 - \sqrt{2})}{8} + \frac{\sqrt{2}\mu[m]_{p,q}[\lambda([m+2]_{p,q} - [m]_{p,q}) + \delta[m]_{p,q}]}{2[\lambda([m+1]_{p,q} - [m]_{p,q}) + \delta[m]_{p,q}]^2} \\ &+ \frac{\sqrt{2}[m]_{p,q}}{2} \times \frac{\lambda(\delta - 1)([m+1]_{p,q} - [m]_{p,q}) + \frac{\delta(\delta-1)}{2}[m]_{p,q}}{[\lambda([m+1]_{p,q} - [m]_{p,q}) + \delta[m]_{p,q}]^2}. \end{aligned}$$

Then, we apply Lemma 1.5 into the equality (2.10) and show that Theorem 2.5 holds true.

Similarly, by fixing $\mu = 0$ or $m = 1$, or $\lambda = 1$, or $\lambda = 1, \delta = 0$ in Theorem 2.5, we obtain the next corollaries as follows.

Corollary 2.6 If $f(z)$ given by (1.1) belongs to the class $\mathcal{B}_{LC}^{p,q}(m, \lambda, \delta)$, then

$$|a_{m+2}| \leq \frac{\sqrt{2}[m]_{p,q} \max\{1, |2\hbar - 1|\}}{|\lambda([m+2]_{p,q} - [m]_{p,q}) + \delta[m]_{p,q}|},$$

where

$$\hbar = \frac{(4 - \sqrt{2})}{8} + \frac{\sqrt{2}[m]_{p,q}}{2} \times \frac{\lambda(\delta - 1)([m+1]_{p,q} - [m]_{p,q}) + \frac{\delta(\delta-1)}{2}[m]_{p,q}}{[\lambda([m+1]_{p,q} - [m]_{p,q}) + \delta[m]_{p,q}]^2}.$$

Corollary 2.7 If $f(z)$ given by (1.1) belongs to the class $\mathcal{B}_{LC}^{p,q}(\lambda, \delta)$, then

$$|a_3 - \mu a_2^2| \leq \frac{\sqrt{2} \max\{1, |2\hbar - 1|\}}{|\lambda([3]_{p,q} - 1) + \delta|}$$

holds for $\mu \in \mathbb{C}$, where

$$\hbar = \frac{(4 - \sqrt{2})}{8} + \frac{\sqrt{2}\mu[\lambda([3]_{p,q} - 1) + \delta]}{2[\lambda([2]_{p,q} - 1) + \delta]^2} + \frac{\lambda(\delta - 1)([2]_{p,q} - 1) + \frac{\delta(\delta-1)}{2}}{\sqrt{2}[\lambda([2]_{p,q} - 1) + \delta]^2}.$$

Corollary 2.8 If $f(z)$ given by (1.1) belongs to the class $\mathcal{B}_{LC}^{p,q}(m, \delta)$, then

$$|a_{m+2} - \mu a_{m+1}^2| \leq \frac{\sqrt{2}[m]_{p,q} \max\{1, |2\hbar - 1|\}}{([m+2]_{p,q} - [m]_{p,q}) + \delta[m]_{p,q}} \quad (2.11)$$

holds for $\mu \in \mathbb{C}$, where

$$\begin{aligned} \hbar &= \frac{(4 - \sqrt{2})}{8} + \frac{\sqrt{2}\mu[m]_{p,q}([\lambda([m+2]_{p,q} - [m]_{p,q}) + \delta[m]_{p,q}]}{2([\lambda([m+1]_{p,q} - [m]_{p,q}) + \delta[m]_{p,q}]^2} \\ &+ \frac{\sqrt{2}(\delta - 1)[m]_{p,q}}{2} \times \frac{([\lambda([m+1]_{p,q} - [m]_{p,q}) + \frac{\delta}{2}[m]_{p,q}]}{([\lambda([m+1]_{p,q} - [m]_{p,q}) + \delta[m]_{p,q}]^2}. \end{aligned}$$

Corollary 2.9 If $f(z)$ given by (1.1) belongs to the class $\mathcal{S}_{LC}^{p,q}(m)$, then

$$|a_{m+2} - \mu a_{m+1}^2| \leq \frac{\sqrt{2}[m]_{p,q} \max\{1, |2\hbar - 1|\}}{[m+2]_{p,q} - [m]_{p,q}} \quad (2.12)$$

holds for $\mu \in \mathbb{C}$, where

$$\hbar = \frac{(4 - \sqrt{2})}{8} + \frac{\sqrt{2}[m]_{p,q}[\mu([m + 2]_{p,q} - [m]_{p,q}) - ([m + 1]_{p,q} - [m]_{p,q})]}{2([m + 1]_{p,q} - [m]_{p,q})^2}.$$

If we let $\mu \in \mathbb{R}$ and $\lambda \in \mathbb{R}_+$, then we are based on the proof of Theorem 2.5 and Lemma 1.6 to establish the Fekete-Szegő functional inequality for $\mathcal{B}_{LC}^{p,q}(m, \lambda, \delta)$.

Theorem 2.10 For $\mu \in \mathbb{R}$ and $\lambda \in \mathbb{R}$, if $f(z) \in \mathcal{A}_m$ belongs to the class $\mathcal{B}_{LC}^{p,q}(m, \lambda, \delta)$, then

$$|a_{m+2} - \mu a_{m+1}^2| \leq \begin{cases} \frac{\sqrt{2}[m]_{p,q}(-2\hbar+1)}{\lambda([m+2]_{p,q} - [m]_{p,q}) + \delta[m]_{p,q}}, & (\mu \leq \Xi_1); \\ \frac{\sqrt{2}[m]_{p,q}}{\lambda([m+2]_{p,q} - [m]_{p,q}) + \delta[m]_{p,q}}, & (\Xi_1 \leq \mu \leq \Xi_2); \\ \frac{\sqrt{2}[m]_{p,q}(2\hbar-1)}{\lambda([m+2]_{p,q} - [m]_{p,q}) + \delta[m]_{p,q}}, & (\mu \geq \Xi_2), \end{cases}$$

where \hbar is the same as in Theorem 2.5, and

$$\begin{aligned} \Xi_1 &= \frac{(-2\sqrt{2} + 1)}{4} \frac{[\lambda([m + 1]_{p,q} - [m]_{p,q}) + \delta[m]_{p,q}]^2}{[m]_{p,q}[\lambda([m + 2]_{p,q} - [m]_{p,q}) + \delta[m]_{p,q}]} \\ &\quad - \frac{\lambda(\delta - 1)([m + 1]_{p,q} - [m]_{p,q}) + \frac{\delta(\delta-1)}{2}[m]_{p,q}}{[\lambda([m + 2]_{p,q} - [m]_{p,q}) + \delta[m]_{p,q}]} \end{aligned}$$

and

$$\begin{aligned} \Xi_2 &= \frac{(2\sqrt{2} + 1)}{4} \frac{[\lambda([m + 1]_{p,q} - [m]_{p,q}) + \delta[m]_{p,q}]^2}{[m]_{p,q}[\lambda([m + 2]_{p,q} - [m]_{p,q}) + \delta[m]_{p,q}]} \\ &\quad - \frac{\lambda(\delta - 1)([m + 1]_{p,q} - [m]_{p,q}) + \frac{\delta(\delta-1)}{2}[m]_{p,q}}{[\lambda([m + 2]_{p,q} - [m]_{p,q}) + \delta[m]_{p,q}]} \end{aligned}$$

In addition, we take

$$\begin{aligned} \Xi_3 &= \frac{[\lambda([m + 1]_{p,q} - [m]_{p,q}) + \delta[m]_{p,q}]^2}{[m]_{p,q}[\lambda([m + 2]_{p,q} - [m]_{p,q}) + \delta[m]_{p,q}]} \\ &\quad - \frac{\lambda(\delta - 1)([m + 1]_{p,q} - [m]_{p,q}) + \frac{\delta(\delta-1)}{2}[m]_{p,q}}{[\lambda([m + 2]_{p,q} - [m]_{p,q}) + \delta[m]_{p,q}]} \end{aligned}$$

Then, each of the following inequalities holds:

(A) For $\mu \in [\Xi_1, \Xi_3]$,

$$\begin{aligned} &|a_{m+2} - \mu a_{m+1}^2| + \frac{\sqrt{2}\hbar[\lambda([m + 1]_{p,q} - [m]_{p,q}) + \delta[m]_{p,q}]^2}{[m]_{p,q}[\lambda([m + 2]_{p,q} - [m]_{p,q}) + \delta[m]_{p,q}]} |a_{m+1}|^2 \\ &\leq \frac{\sqrt{2}[m]_{p,q}}{[\lambda([m + 2]_{p,q} - [m]_{p,q}) + \delta[m]_{p,q}]}; \end{aligned}$$

(B) For $\mu \in [\Xi_3, \Xi_2]$,

$$\begin{aligned} &|a_{m+2} - \mu a_{m+1}^2| + \frac{\sqrt{2}(1 - \hbar)[\lambda([m + 1]_{p,q} - [m]_{p,q}) + \delta[m]_{p,q}]^2}{[m]_{p,q}[\lambda([m + 2]_{p,q} - [m]_{p,q}) + \delta[m]_{p,q}]} |a_{m+1}|^2 \\ &\leq \frac{\sqrt{2}[m]_{p,q}}{[\lambda([m + 2]_{p,q} - [m]_{p,q}) + \delta[m]_{p,q}]} \end{aligned}$$

Similarly, by letting $m = 1$, or $\lambda = 1$, or $\lambda = 1, \delta = 0$ in Theorem 2.10, we provide the following corollaries.

Corollary 2.11 For $\mu \in \mathbb{R}$ and $\lambda \in \mathbb{R}_+$, if $f(z) \in \mathcal{A}$ belongs to the class $\mathcal{B}_{LC}^{p,q}(\lambda, \delta)$, then

$$|a_3 - \mu a_2^2| \leq \begin{cases} \frac{\sqrt{2}(-2\hbar+1)}{\lambda([3]_{p,q}-1)+\delta}, & (\mu \leq \Pi_1); \\ \frac{\sqrt{2}}{\lambda([3]_{p,q}-1)+\delta}, & (\Pi_1 \leq \mu \leq \Pi_2); \\ \frac{\sqrt{2}(2\hbar-1)}{\lambda([3]_{p,q}-1)+\delta}, & (\mu \geq \Pi_2), \end{cases}$$

where \hbar is the same as in Corollary 2.7, and

$$\Pi_1 = \frac{(-2\sqrt{2}+1)[\lambda([2]_{p,q}-1)+\delta]^2}{4[\lambda([3]_{p,q}-1)+\delta]} - \frac{\lambda(\delta-1)([2]_{p,q}-1) + \frac{\delta(\delta-1)}{2}}{\lambda([3]_{p,q}-1)+\delta}$$

and

$$\Pi_2 = \frac{(2\sqrt{2}+1)[\lambda([2]_{p,q}-1)+\delta]^2}{4[\lambda([3]_{p,q}-1)+\delta]} - \frac{\lambda(\delta-1)([2]_{p,q}-1) + \frac{\delta(\delta-1)}{2}}{\lambda([3]_{p,q}-1)+\delta}.$$

In addition, we take

$$\Pi_3 = \frac{[\lambda([2]_{p,q}-1)+\delta]^2}{\lambda([3]_{p,q}-1)+\delta} - \frac{\lambda(\delta-1)([2]_{p,q}-1) + \frac{\delta(\delta-1)}{2}}{\lambda([3]_{p,q}-1)+\delta}.$$

Then, each of the following inequalities holds:

(A) For $\mu \in [\Pi_1, \Pi_3]$,

$$|a_3 - \mu a_2^2| + \frac{\sqrt{2}\hbar[\lambda([2]_{p,q}-1)+\delta]^2}{\lambda([3]_{p,q}-1)+\delta} |a_2|^2 \leq \frac{\sqrt{2}}{\lambda([3]_{p,q}-1)+\delta};$$

(B) For $\mu \in [\Pi_3, \Pi_2]$,

$$|a_3 - \mu a_2^2| + \frac{\sqrt{2}(1-\hbar)[\lambda([2]_{p,q}-1)+\delta]^2}{\lambda([3]_{p,q}-1)+\delta} |a_2|^2 \leq \frac{\sqrt{2}}{\lambda([3]_{p,q}-1)+\delta}.$$

Corollary 2.12 For $\mu \in \mathbb{R}$, if $f(z) \in \mathcal{A}_m$ belongs to the class $\mathcal{B}_{LC}^{p,q}(m, \delta)$, then

$$|a_{m+2} - \mu a_{m+1}^2| \leq \begin{cases} \frac{\sqrt{2}[m]_{p,q}(-2\hbar+1)}{([m+2]_{p,q}-[m]_{p,q})+\delta[m]_{p,q}}, & (\mu \leq \Sigma_1); \\ \frac{\sqrt{2}[m]_{p,q}}{([m+2]_{p,q}-[m]_{p,q})+\delta[m]_{p,q}}, & (\Sigma_1 \leq \mu \leq \Sigma_2); \\ \frac{\sqrt{2}[m]_{p,q}(2\hbar-1)}{([m+2]_{p,q}-[m]_{p,q})+\delta[m]_{p,q}}, & (\mu \geq \Sigma_2), \end{cases}$$

where \hbar is the same as in Corollary 2.8, and

$$\Sigma_1 = \frac{(-2\sqrt{2}+1)}{4} \frac{[(m+1]_{p,q} - [m]_{p,q}) + \delta[m]_{p,q}}{[m]_{p,q}([m+2]_{p,q} - [m]_{p,q}) + \delta[m]_{p,q}} - \frac{(\delta-1)[(m+1]_{p,q} - [m]_{p,q}) + \frac{\delta}{2}[m]_{p,q}}{([m+2]_{p,q} - [m]_{p,q}) + \delta[m]_{p,q}}$$

and

$$\Sigma_2 = \frac{(2\sqrt{2} + 1)}{4} \frac{([m + 1]_{p,q} - [m]_{p,q}) + \delta[m]_{p,q}}{[m]_{p,q}([m + 2]_{p,q} - [m]_{p,q}) + \delta[m]_{p,q}} \frac{([m + 1]_{p,q} - [m]_{p,q}) + \delta[m]_{p,q}}{([m + 2]_{p,q} - [m]_{p,q}) + \delta[m]_{p,q}}.$$

In addition, we take

$$\Sigma_3 = \frac{([m + 1]_{p,q} - [m]_{p,q}) + \delta[m]_{p,q}}{[m]_{p,q}([m + 2]_{p,q} - [m]_{p,q}) + \delta[m]_{p,q}} \frac{([m + 1]_{p,q} - [m]_{p,q}) + \frac{\delta(\delta-1)}{2}[m]_{p,q}}{([m + 2]_{p,q} - [m]_{p,q}) + \delta[m]_{p,q}}.$$

Then, each of the following inequalities holds:

(A) For $\mu \in [\Sigma_1, \Sigma_3]$,

$$|a_{m+2} - \mu a_{m+1}^2| + \frac{\sqrt{2}h([m + 1]_{p,q} - [m]_{p,q}) + \delta[m]_{p,q}}{[m]_{p,q}([m + 2]_{p,q} - [m]_{p,q}) + \delta[m]_{p,q}} |a_{m+1}|^2 \leq \frac{\sqrt{2}[m]_{p,q}}{([m + 2]_{p,q} - [m]_{p,q}) + \delta[m]_{p,q}};$$

(B) For $\mu \in [\Sigma_3, \Sigma_2]$,

$$|a_{m+2} - \mu a_{m+1}^2| + \frac{\sqrt{2}(1-h)([m + 1]_{p,q} - [m]_{p,q}) + \delta[m]_{p,q}}{[m]_{p,q}([m + 2]_{p,q} - [m]_{p,q}) + \delta[m]_{p,q}} |a_{m+1}|^2 \leq \frac{\sqrt{2}[m]_{p,q}}{([m + 2]_{p,q} - [m]_{p,q}) + \delta[m]_{p,q}}.$$

Corollary 2.13 For $\mu \in \mathbb{R}$, if $f(z) \in \mathcal{A}_m$ belongs to the class $\mathcal{S}_{LC}^{p,q}(m)$, then

$$|a_{m+2} - \mu a_{m+1}^2| \leq \begin{cases} \frac{\sqrt{2}[m]_{p,q}(-2h+1)}{[m+2]_{p,q} - [m]_{p,q}}, & (\mu \leq \Upsilon_1); \\ \frac{\sqrt{2}[m]_{p,q}}{[m+2]_{p,q} - [m]_{p,q}}, & (\Upsilon_1 \leq \mu \leq \Upsilon_2); \\ \frac{\sqrt{2}[m]_{p,q}(2h-1)}{[m+2]_{p,q} - [m]_{p,q}}, & (\mu \geq \Upsilon_2), \end{cases}$$

where h is the same as in Corollary 2.9, and

$$\Upsilon_1 = \frac{(-2\sqrt{2} + 1)}{4} \frac{([m + 1]_{p,q} - [m]_{p,q})^2}{[m]_{p,q}([m + 2]_{p,q} - [m]_{p,q})} + \frac{[m + 1]_{p,q} - [m]_{p,q}}{[m + 2]_{p,q} - [m]_{p,q}}$$

and

$$\Upsilon_2 = \frac{(2\sqrt{2} + 1)}{4} \frac{([m + 1]_{p,q} - [m]_{p,q})^2}{[m]_{p,q}([m + 2]_{p,q} - [m]_{p,q})} + \frac{[m + 1]_{p,q} - [m]_{p,q}}{[m + 2]_{p,q} - [m]_{p,q}}.$$

In addition, we take

$$\Upsilon_3 = \frac{([m + 1]_{p,q} - [m]_{p,q})^2}{[m]_{p,q}([m + 2]_{p,q} - [m]_{p,q})} + \frac{[m + 1]_{p,q} - [m]_{p,q}}{[m + 2]_{p,q} - [m]_{p,q}}.$$

Then, each of the following inequalities holds:

(A) For $\mu \in [\Upsilon_1, \Upsilon_3]$,

$$|a_{m+2} - \mu a_{m+1}^2| + \frac{\sqrt{2}\hbar([m+1]_{p,q} - [m]_{p,q})^2}{[m]_{p,q}([m+2]_{p,q} - [m]_{p,q})} |a_{m+1}|^2 \leq \frac{\sqrt{2}[m]_{p,q}}{[m+2]_{p,q} - [m]_{p,q}};$$

(B) For $\mu \in [\Upsilon_3, \Upsilon_2]$,

$$|a_{m+2} - \mu a_{m+1}^2| + \frac{\sqrt{2}(1-\hbar)([m+1]_{p,q} - [m]_{p,q})^2}{[m]_{p,q}([m+2]_{p,q} - [m]_{p,q})} |a_{m+1}|^2 \leq \frac{\sqrt{2}[m]_{p,q}}{[m+2]_{p,q} - [m]_{p,q}}.$$

3 Conclusion

Our objective is to generalize some classical interesting results in geometric function theory from the ordinary analysis to q -analysis or (p, q) -analysis. Here, by using the (p, q) -derivative operator and Bazilevič function, certain new subclass of analytic and multivalent functions can be defined to improve the classical starlike functions and even q -starlike functions. In our main results, for this class we obtain the corresponding bound estimates of the coefficients a_{m+1} and a_{m+2} and the Fekete-Szegő functional inequalities. Of course, we may choose another special functions to replace Bazilevič function and the limaçon function, which are leaved out the interested readers to realize.

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与蚘线有关的多叶Bazilevič函数的 (p, q) -模拟

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摘要: 本文研究了经典几何函数论中解析函数泛函不等式问题. 利用微分从属原理和 (p, q) -导数算子, 引入了与limacon函数相关某类多叶Bazilevič函数的 (p, q) -模拟, 获得了相应的系数估计和Fekete-Szegö不等式. 推广和改进了星型函数甚至 q -星型函数相关结果.

关键词: Fekete-Szegö不等式; 对称Toeplitz行列式; 多叶函数; Bazilevič函数; (p, q) -导算子

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