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SOME VANISHING THEOREMS FOR p-HARMONIC FORMS ON SUBMANIFOLDS IN HADAMARD MANIFOLDS

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Abstract: In this paper, we give some vanishing theorems for p-harmonic forms on a commplete submanifold M immersed in Hadamard manifold N. Firstly, assume that M satisfies the weighted Poincaré inequality and has flat normal bundle. And assume further that N has pure curvature tensor and the $(l, n-l)$-curvature of N is not less than $-k\rho(0 \leq k \leq \frac{4}{p^2})$ for $2 \leq l \leq n-2$. If the total curvature is small enough, we prove a vanishing theorem for p-harmonic l-forms, which generalizes Wang–Chao–Wu–Lv's results in [1]. Secondly, suppose that N is a Hadamard manifold with sectional curvature $-k^2 \leq K_N \leq 0$ for some constant k. If the total curvature is small enough and the first eigenvalue of Laplace satisfies a certain lower bound, we obtain a vanishing theorem for p-harmonic 1-forms, which generalizes Dung–Seo's results in [2].

Keywords: p-harmonic forms; vanishing theorems; weighted Poincaré inequality; Hadamard manifolds

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1 Introduction

An interesting problem in submanifold geometry is to study the relationship between the geometric structure and topological properties of submanifolds in various ambient space. For example, Cao–Shen–Zhu [3] proved a complete non-compact oriented stable minimal hypersurface $M^n(n \geq 3)$ in $\mathbb{R}^{n+1}$ must have only one end. Their proof mainly used a Liouville theorem for harmonic maps due to Schoen–Yau [4]. Due to this connection with harmonic functions, Li–Wang [5] showed that a complete minimal hypersurface M^n in $\mathbb{R}^{n+1}$ with finite index must have finite first L^2-Betti number, and M^n must have finitely many ends. So it is an interesting problem in geometry and topology to study vanishing theorems of harmonic forms on submanifolds in various ambient space.

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Let (M, g) be a complete Riemannian manifold of dimension n, d be the exterior differential operator on M. Then the formal dual operator of d is defined by

$$\delta = (-1)^{n(l+1)+1} * d*,$$

where $*$ is the Hodge star operator with respect to g. Then the Hodge–Laplace–Beltrami operator Δ acting on the space of smooth l-form $\Omega^l(M)$ is given by

$$\Delta = -(d\delta + \delta d).$$

A smooth l-form $\omega \in \Omega^l(M)$ is said to be harmonic if $\Delta\omega = 0$. It is well-known that ω is harmonic if and only if $d\omega = 0$ and $\delta\omega = 0$ when M is compact. Hence, for any $p \geq 2$, we say an l-form $\omega \in \Omega^l(M)$ is a p-harmonic l-form if it satisfies the following properties:

$$\begin{cases} d\omega = 0, \\ \delta(|\omega|^{p-2}\omega) = 0. \end{cases} \tag{1.1}$$

It is easy to see that when $p = 2$ and M is compact, a p-harmonic l-form is exactly a harmonic l-form. When $l = 0$, it is a p-harmonic function and the differential of a p-harmonic function is a p-harmonic 1-form.

There are various vanishing theorems for L^2 harmonic forms on complete submanifolds by assuming various geometric and analytic conditions. For example, Palmer [6] proved that there exist no nontrivial L^2 harmonic 1-forms on a complete stable minimal hypersurface in $\mathbb{R}^{n+1}$. Cavalcante–Mirandola–Vitório [7] showed some finiteness and vanishing theorems for L^2 harmonic 1-forms on submanifolds in a nonpositive curved pinching manifold with some conditions about the first eigenvalues and the total curvature. Recently, by supposing that the submanifold is stable or has sufficiently small total curvature, Chao–Lv [8] established some vanishing theorems for L^2 harmonic 1-forms on complete submanifolds with weighted Poincaré inequality. Recall that we say that M satisfies a weighted Poincaré inequality with a nonnegative weight function $\rho(x)$ if the inequality

$$\int_M \rho(x)\varphi^2 \mathrm{d}v \leq \int_M |\nabla\varphi|^2 \mathrm{d}v \tag{1.2}$$

holds true for all compactly supported smooth function $\varphi \in C_0^\infty(M)$. We would like to point out that the weighted Poincaré inequalities have appeared in many important issues of analysis and mathematical physics. When the manifolds satisfy a weighted Poincaré inequality, many works have been conduced (see [9-13] and references therein).

Furthermore, when the ambient manifold is a Hadamard manifold, there are also many interesting results. Recall that a smooth manifold is called a Hadamard manifold if it is a complete, simply connected manifold with nonpositive sectional curvature. When M^n is a complete noncompact submanifold immersed in Hadamard manifold N^{n+m} with sectional curvature satisfying $-k^2 \leq K_N \leq 0$, Cavalcante–Mirandola–VióTrio [7] obtained some vanishing theorems for L^2 harmonic 1-forms if the traceless second fundamental form is small enough and $\lambda_1(M) > \frac{(n-1)^2}{n}(k^2 - \inf|H|^2)$. Subsequently, under the same assumptions except the lower bound of the first eigenvalue of Laplacian only depends on $||\phi||^2_{L^n}$ instead of $\inf_M |H|^2$, Dung and Seo [2] obtained the similar vanishing theorems. On the other hand, assume that N^{n+m} has pure curvature tensor and the $(l, n-l)$-curvature of N^{n+m} (see Definition 2.1) is not less than $-k(k>0)$, Lin [14] proved a vanishing and finiteness theorem on M^n with flat normal bundle. Recently, under the same conditions except the $(l, n-l)$-curvature of N^{n+m} to not less than $-k\rho$ (where ρ is the weight function) and requiring M^n satisfies a weighted Poincaré inequality, Wang–Chao–Wu–Lv [1] obtained two vanishing theorems for harmonic l-forms if the total curvature is small enough or M^n has at most Euclidean volume growth.

For general p-harmonic forms, Zhang [15] proved that there is no nontrivial $L^q(q>0)$ p-harmonic 1-form on a complete manifold with nonnegative Ricci curvature. Inspired by Zhang's results, Chang–Guo–Sung [16] extended Zhang's results and obtained the compactness for any bounded set of p-harmonic 1-forms. On complete noncompact submanifolds in a Hadamard manifold, if the first eigenvalue of the Laplacian has a suitable lower bound and the total curvature is sufficiently small, Han–Pan [17] obtained a vanishing theorem for L^p p-harmonic 1-forms, which extends Cavalcante–Mirandola–Vitório's results. On complete noncompact submanifolds of N^{n+m} with pure curvature tensor, Dung–Tien [18] obtained some vanishing theorems for p-harmonic l-forms by assuming some appropriate conditions for the second fundamental form and the first eigenvalue of the Laplacian. Recently, Lin–Yang [19] proved some vanishing theorems of L^q p-harmonic l-forms on complete noncompact submanifolds in Hadamard manifold, which extends Lin's results in [14].

Inspired by these results, the main purpose of this article is to study L^p p-harmonic forms on submanifolds in Hadamard manifolds. As usual, we define the space of the L^p p-harmonic l-forms on M by

$$H^{l,p}(L^p(M)) = \left\{\omega \in \Omega^l(M) | d\omega = 0, \delta(|\omega|^{p-2}\omega) = 0, \int_M |\omega|^p \mathrm{d}v < \infty\right\}.$$

In this paper, we will prove the following theorems.

Theorem 1.1 Let M^n $(n \geq 4)$ be an n-dimensional complete submanifold immersed in a Hadamard manifold N^{n+m}. Suppose that M satisfies the weighted Poincaré inequality and has flat normal bundle. Assume further that N has pure curvature tensor and the $(l, n-l)$-curvature of N is not less than $-k\rho(0 \leq k \leq \frac{4}{p^2})$ for $2 \leq l \leq n-2$. If the traceless second fundamental form ϕ satisfies

$$||\phi||^2_{L^n(M)} < \frac{8[1 + k_p(p-1)^2] - 2kp^2}{np^2 C(n)}, \tag{1.3}$$

where $C(n) > 0$ is the Sobolev constant depending only on the dimension n. Then there is no nontrivial L^p p-harmonic l-form on M.

We should remark that our Theorem 1.1 recovers Wang–Chao–Wu–Lv's results [1, Theorem 2] when $p = 2$.

Theorem 1.2 Let $M^n(n \geq 3)$ be an n-dimensional complete noncompact submanifold in a complete simply connected Riemannian manifold N with sectional curvature $-k^2 \leq K_N \leq 0$, where k is a constant. Assume that the traceless second fundamental from ϕ satisfies

$$||\phi||_{L^n} < \frac{4}{p^2}\sqrt{\frac{1}{n(n-1)C(n)}}. \tag{1.4}$$

In the case $k \neq 0$, assume further that the first eigenvalue of the Laplacian of M satisfies

$$\lambda_1(M) > \frac{2n(n-1)p^2k^2}{8(p-1)n + 8nk_p(p-1)^2 - p^2(n-2)\sqrt{n(n-1)C(n)}||\phi||_n - 2p^2(n-1)C(n)||\phi||_n^2}, \tag{1.5}$$

where $C(n)$ is the Sobolev constant. Then there is no nontrivial L^p p-harmonic 1-form on M.

We should remark that our Theorem 1.2 also generalizes Dung–Seo's results [2, Theorem 4.1] when $p = 2$. And the upper bound of $||\phi||_{L^n(M)}$ and the lower bound of $\lambda_1(M)$ depend only on the dimension of M and the curvature of the ambient space.

The rest of this article is arranged as follows. In Section 2, we recall some preliminary knowledge and lemmas. In Section 3, we will give a detailed proof of Theorem 1.1. In Section 4, we prove the Theorem 1.2.

2 Preliminary

In this section, we will recall some terminologies and notations about geometry of submanifolds and some useful lemmas which will be adopted in the proof of our theorems.

Let $\iota : M \to N$ be an n-dimensional submanifold isometrically immersed in an $(n+m)$-dimensional Riemannian manifold $(N, \bar{g})$. Fix a point $x \in M$ and a local orthonormal frame $\{e_1, \cdots, e_{n+m}\}$ of N such that $\{e_1, \cdots, e_n\}$ are tangent fields of M at x and $\{e_{n+1}, \cdots, e_{n+m}\}$ is a local orthonormal frame of normal bundle NM. In our paper, we also adopt the following ranges of indices: $1 \leq i, j, \cdots \leq n$, $n+1 \leq \alpha, \beta, \cdots \leq n+m$.

For any $X, Y \in \Gamma(TM)$, we have the following orthogonal decomposition

$$\bar{\nabla}_X Y = \nabla_X Y + A(X, Y),$$

where $\bar{\nabla}$ is the Levi–Civita connection of the Riemannian manifold with respect to $\bar{g}$ and $A : \Gamma(TM) \times \Gamma(TM) \to \Gamma(NM)$ is the second fundamental form of the immersion. Then

$$A(X, Y) = \sum_{\alpha} \langle \bar{\nabla}_X Y, e_\alpha \rangle e_\alpha.$$

We denote by $h_{ij}^\alpha = \langle \bar{\nabla}_{e_i} e_j, e_\alpha \rangle$ the coefficients of the second fundamental form. Then the square norm $|A|^2$ of the second fundamental form and the mean curvature vector H are given by

$$|A|^2 = \sum_{\alpha} \sum_{i,j} (h_{ij}^\alpha)^2$$

and

$$H = \frac{1}{n} H^\alpha e_\alpha = \frac{1}{n} \sum_\alpha \sum_i h_{ii}^\alpha e_\alpha.$$

A submanifold M is said to be minimal if $H = 0$ identically. The traceless second fundamental form ϕ is defined by

$$\phi(X, Y) = A(X, Y) - \langle X, Y \rangle H,$$

for any vector fields $X, Y \in \Gamma(TM)$. It is easy to see check that

$$|\phi|^2 = |A|^2 - n|H|^2.$$

We say the immersion $\iota : M^n \to N^{n+m}$ has finite total curvature if the L^n-norm of the traceless second fundamental form is finite, i.e.,

$$||\phi||_{L^n(M)} = \left(\int_M |\phi|^n \mathrm{d}v \right)^{\frac{1}{n}} < +\infty.$$

Let R and $\bar{R}$ be the curvature tensors of M and N, respectively. Recall that the definition of $(l, n-l)$-curvature is given in [14], which appears naturally in the Weitzenbčk formula.

Definition 2.1 ([14, Definition 1.1]) For any point $x \in N^{n+m}$, choose an orthonormal frame $\{e_i\}_{i=1}^{n+m}$ of the tangent space $T_x N$ and set

$$\bar{R}^{(l,n-l)}([e_{i_1}, e_{i_2}, \cdots, e_{i_n}]) = \sum_{r=1}^{l} \sum_{s=l+1}^{n} \bar{R}_{i_r i_s i_r i_s}$$

for $1 \le l \le n-1$, where the indices $1 \le i_1, i_2, \cdots, i_n \le n+m$ are distinct with each other. We call $\bar{R}^{(l,n-l)}([e_{i_1}, e_{i_2}, \cdots, e_{i_n}])$ the $(l, n-l)$-curvature of N^{n+m}.

We should remark that the $(1, n-1)$-curvature is nothing but the $(n-1)$-th Ricci curvature, which is a curvature condition between Ricci curvature and sectional curvature (see [20]).

In order to get a good estimation in Weitzenböck formula, we also need to assume that M has flat normal bundle and N has pure curvature tensor. Recall that the submanifold M is said to have flat normal bundle if the curvature of normal bundle NM is zero, i.e.,

$$h_{ij}^\alpha h_{jk}^\alpha - h_{kj}^\alpha h_{ji}^\alpha = 0.$$

In this case, there exists an orthonormal frame diagonalizing h_{ij}^α simultaneously. One can easily see that any hypersurface has flat normal bundle.

Definition 2.2 ([21, Definition 4.5]) A Riemannian manifold N^{n+m} is said to have pure curvature tensor if for every $x \in N$, there is an orthonormal basis $\{e_1, \cdots, e_{n+m}\}$ of the tangent space $T_x N$ such that

$$\langle \bar{R}(e_i, e_j)e_k, e_l \rangle = 0,$$

when the set $\{i, j, k, l\}$ contains more than two elements.

It is easy to see that all the 3-manifolds and conformally flat manifolds have pure curvature tensor (see [21]).

In the rest of this Section, we shall give some useful lemmas which play an important role in the proof of our theorems.

Lemma 2.3 ([18,22,23]) For $p \geq 2, l \geq 1$, let ω be a p-harmonic l-form on an n-dimensional complete Riemannian manifold M. Then we have

$$|\nabla(|\omega|^{p-2}\omega)|^2 \geq (1 + k_p)|\nabla|\omega|^{p-1}|^2, \tag{2.1}$$

where $k_p = \begin{cases} \frac{1}{\max\{l,n-l\}}, & \text{if } p = 2 \\ \frac{1}{(p-1)^2}\min\{1, \frac{(p-1)^2}{n-1}\}, & \text{if } p > 2 \text{ and } l = 1 \\ 0, & \text{if } \ p > 2 \text{ and } 1 < l \leq n-1 \end{cases}$.

Following the similar arguments as in [14], we shall give a Weitzenböck formula for any l-form under the assumption that M has flat normal bundle and N has pure curvature tensor.

Lemma 2.4 Let $M^n (n \geq 3)$ be a complete submanifold immersed in N^{n+m}. Assume that M has flat noral bundle and N has pure curvature tensor. Then for any l-form $\omega \in \Omega^l(M)$ with $2 \leq l \leq n-2$, we have

$$\begin{aligned} \frac{1}{2}\Delta|\omega|^2 \geq &|\nabla\omega|^2 + \langle\Delta\omega, \omega\rangle + \left(\inf_{i_1,\cdots,i_n} \sum_{r=1}^{l}\sum_{s=l+1}^{n} \bar{K}_{i_r i_s}\right)|\omega|^2 \\ &+ \left(-\frac{n}{2}|\phi|^2 + \frac{1}{2}\min\{l, n-l\}|A|^2\right)|\omega|^2, \end{aligned} \tag{2.2}$$

where $\bar{K}_{ij} = \bar{R}_{ijij} = \langle\bar{R}(e_i, e_j)e_j, e_i\rangle$.

Proof For any l-form $\omega \in \Omega^l(M)$, we have the Bochner formula [24]

$$\begin{aligned} \frac{1}{2}\Delta|\omega|^2 =&|\nabla\omega|^2 + \langle\Delta\omega, \omega\rangle + \left\langle \sum_{j,k=1}^{n} \theta^k \wedge \iota(e_j)R(e_j, e_k)\omega, \omega \right\rangle \\ =&|\nabla\omega|^2 + \langle\Delta\omega, \omega\rangle + lF_l(\omega), \end{aligned} \tag{2.3}$$

where

$$F_l(\omega) = R_{ij}\omega^{ii_2\cdots i_l}\omega^j_{i_2\cdots i_l} - \frac{l-1}{2}R_{ijkm}\omega^{iji_3\cdots i_l}\omega^{km}_{i_3\cdots i_l}.$$

From (2.8) and (2.9) in [14], we get

$$F_l \geq \frac{1}{l}\left(\inf_{i_1,\cdots,i_n} \sum_{r=1}^{l}\sum_{s=l+1}^{n} \bar{K}_{i_r i_s}\right)|\omega|^2 + \frac{1}{l}\inf_{i_1,\cdots,i_n}\{\sum_{\alpha}(h^\alpha_{i_1i_1}+\cdots+h^\alpha_{i_li_l})(h^\alpha_{i_{l+1}i_{l+1}}+\cdots+h^\alpha_{i_ni_n})\}|\omega|^2. \tag{2.4}$$

By direct calculation, we obtain

$$
\begin{aligned}
&\sum_{\alpha}(h^{\alpha}_{i_1i_1}+\cdots+h^{\alpha}_{i_li_l})(h^{\alpha}_{i_{l+1}i_{l+1}}+\cdots+h^{\alpha}_{i_ni_n})\\
=&\frac{1}{2}\sum_{\alpha}[(h^{\alpha}_{i_1i_1}+...+h^{\alpha}_{i_ni_n})^2-(h^{\alpha}_{i_1i_1}+...+h^{\alpha}_{i_li_l})^2-(h^{\alpha}_{i_{l+1}i_{l+1}}+...+h^{\alpha}_{i_ni_n})^2]\\
\geq&\frac{1}{2}\left[n^2|H|^2-l\sum_{\alpha}\sum_{j=1}^{l}(h^{\alpha}_{i_ji_j})^2-(n-l)\sum_{\alpha}\sum_{j=1}^{l}(h^{\alpha}_{i_ji_j})^2\right]\\
\geq&\frac{1}{2}(n^2|H|^2-\max\{l,n-l\}|A|^2)\\
=&-\frac{n}{2}(|A|^2-n|H|^2)+\frac{1}{2}(n-\max\{l,n-l\})|A|^2\\
=&-\frac{n}{2}|\phi|^2+\frac{1}{2}\min\{l,n-l\}|A|^2.
\end{aligned}
\tag{2.5}
$$

Combining (2.3), (2.4) and (2.5), we have (2.2).

As is well known, the Sobolev inequality holds on a complete submanifold in a complete simply connected manifold with nonpositive sectional curvature.

Lemma 2.5 ([1,25]) Let $M^n(n\geq 3)$ be an n-dimensional complete noncompact submanifold in a complete sumply connected manifold with nonpositive sectional curvature. Then for any $\varphi\in W_0^{1,2}(M)$, we have

$$
\left(\int_M|\varphi|^{\frac{2n}{n-2}}\mathrm{d}v\right)^{\frac{n-2}{n}}\leq C(n)\int_M|\nabla\varphi|^2+|H|^2\varphi^2\mathrm{d}v,\tag{2.6}
$$

where $C(n)$ is the Sobolev constant which depends only on the dimension n.

Besides, we also need the following estimate for the differential of forms.

Lemma 2.6 ([26, Lemma 2.5]) For any closed l-form $\omega\in\Omega^l(M)$ and $f\in C^\infty(M)$, we have

$$
|d(f\omega)|=|df\wedge\omega|\leq|df|\cdot|\omega|.\tag{2.7}
$$

Finally, we also need the estimate of Ricci curvature for submanifolds, which will be used in the proof of Theorem 1.2.

Lemma 2.7 ([27,28]) Let M^n be an n-dimensional submanifold in a Riemannian manifold N with sectional curvature $-k^2\leq K_N$ for some constant k. Then the Ricci curvature Ric_M of M satisfies

$$
Ric_M\geq(n-1)(|H|^2-k^2)-\frac{(n-2)\sqrt{n(n-1)}}{n}|H||\phi|-\frac{n-1}{n}|\phi|^2.
$$

3 The Proof of Theorem 1.1

Proof Let ω be any p-harmonic l-form on M with $2\leq l\leq n-2$, then we have

$$
\begin{cases}d\omega=0,\\ \delta(|\omega|^{p-2}\omega)=0.\end{cases}
$$

Applying Lemma 2.4 to the form $|\omega|^{p-2}\omega$, we obtain

$$\begin{aligned}\frac{1}{2}\Delta|\omega|^{2(p-1)} \geq & |\nabla(|\omega|^{p-2}\omega)|^2 - \langle(d\delta+\delta d)(|\omega|^{p-2}\omega), |\omega|^{p-2}\omega\rangle \\ & + \left(\inf_{i_1,\cdots,i_n}\sum_{r=1}^{l}\sum_{s=l+1}^{n}\bar{K}_{i_r i_s}\right)|\omega|^{2(p-1)} \\ & - \frac{n}{2}|\phi|^2|\omega|^{2(p-1)} + \frac{1}{2}\min\{l,n-l\}|A|^2|\omega|^{2(p-1)}.\end{aligned} \tag{3.1}$$

By direct calculation, we have

$$\frac{1}{2}\Delta|\omega|^{2(p-1)} = |\omega|^{p-1}\Delta|\omega|^{p-1} + |\nabla|\omega|^{p-1}|^2. \tag{3.2}$$

Since the $(l, n-l)$-curvature of N is not less than $-k\rho$, we get

$$\inf_{i_1,\cdots,i_n}\sum_{r=1}^{l}\sum_{s=l+1}^{n}\bar{K}_{i_r i_s} \geq -k\rho. \tag{3.3}$$

Combining (3.1)-(3.3) and $\delta(|\omega|^{p-2}\omega)$, it follows that

$$\begin{aligned}|\omega|^{p-1}\Delta|\omega|^{p-1} \geq & (|\nabla(|\omega|^{p-2}\omega)|^2 - |\nabla|\omega|^{p-1}|^2) - \langle\delta d(|\omega|^{p-2}\omega), |\omega|^{p-2}\omega\rangle - k\rho|\omega|^{2(p-1)} \\ & - \frac{n}{2}|\phi|^2|\omega|^{2(p-1)} + \frac{1}{2}\min\{l,n-l\}|A|^2|\omega|^{2(p-1)}.\end{aligned}$$

By using the refined Kato inequality (2.1) and dividing both sides of the above inequality by $|\omega|^{p-2}$, we obtain

$$\begin{aligned}|\omega|\Delta|\omega|^{p-1} \geq & (p-1)^2k_p|\omega|^{p-2}|\nabla|\omega||^2 - \langle\delta d(|\omega|^{p-2}\omega), \omega\rangle - k\rho|\omega|^p - \frac{n}{2}|\phi|^2|\omega|^p \\ & + \frac{1}{2}\min\{l,n-l\}|A|^2|\omega|^p.\end{aligned} \tag{3.4}$$

Let $r(x)$ be the geodesic distance on M from a fixed point $x_0 \in M$ to x. Then we take a cutoff function $\varphi \in C_0^\infty(M)$ satisfying

$$\varphi(x) = \begin{cases} 1, & \text{on } B_r(x_0), \\ \in [0,1] \text{ and } |\nabla\varphi| \leq \frac{2}{r}, & \text{on } B_{2r}(x_0)\setminus B_r(x_0), \\ 0, & \text{on } M\setminus B_{2r}(x_0), \end{cases} \tag{3.5}$$

where $B_r(x_0)$ is the open geodesic ball of radius r and center at x_0 of M.

Multiplying both sides of the above inequality (3.4) and integrating over M, it holds

$$\begin{aligned}&\int_M \varphi^2|\omega|\Delta|\omega|^{p-1}\mathrm{d}v \\ \geq & (p-1)^2k_p\int_M \varphi^2|\omega|^{p-2}|\nabla|\omega||^2\mathrm{d}v - \int_M\langle d(|\omega|^{p-2}\omega), d(\varphi^2\omega)\rangle\mathrm{d}v \\ & - k\int_M\rho\varphi^2|\omega|^p\mathrm{d}v - \frac{n}{2}\int_M\varphi^2|\phi|^2|\omega|^p\mathrm{d}v + \frac{1}{2}\min\{l,n-l\}\int_M\varphi^2|A|^2|\omega|^p\mathrm{d}v.\end{aligned} \tag{3.6}$$

For the term in the left hand side, using integration by parts yields

$$\begin{aligned}&\int_M \varphi^2|\omega|\Delta|\omega|^{p-1}\mathrm{d}v\\ =&-\int_M \langle\nabla(\varphi^2|\omega|),\nabla|\omega|^{p-1}\rangle\mathrm{d}v\\ =&-2(p-1)\int_M \varphi|\omega|^{p-1}\langle\nabla\varphi,\nabla|\omega|\rangle\mathrm{d}v-(p-1)\int_M\varphi^2|\omega|^{p-2}|\nabla|\omega||^2\mathrm{d}v\\ \leq&2(p-1)\int_M\varphi|\omega|^{p-1}|\nabla\varphi||\nabla|\omega||\mathrm{d}v-(p-1)\int_M\varphi^2|\omega|^{p-2}|\nabla|\omega||^2\mathrm{d}v.\end{aligned} \tag{3.7}$$

By using Lemma 2.6 and $d\omega=0$ for the second term in the right hand side, we get

$$\begin{aligned}\int_M\langle d(|\omega|^{p-2}\omega),d(\varphi^2\omega)\rangle\mathrm{d}v\leq&\int_M\left|d(|\omega|^{p-2}\omega)\right|\cdot|d(\varphi^2\omega)|\mathrm{d}v\\ =&\int_M\left|d|\omega|^{p-2}\wedge\omega\right|\cdot|d\varphi^2\wedge\omega|\mathrm{d}v\\ =&2(p-2)\int_M\varphi|\omega|^{p-1}\,|\nabla|\omega||\cdot|\nabla\varphi|\mathrm{d}v.\end{aligned} \tag{3.8}$$

Combining (3.6), (3.7) and (3.8) implies that

$$\begin{aligned}0\geq&-2(2p-3)\int_M\varphi|\omega|^{p-1}|\nabla\varphi|\,|\nabla|\omega||\,\mathrm{d}v+[(p-1)+(p-1)^2k_p]\int_M\varphi^2|\omega|^{p-2}|\nabla|\omega||^2\mathrm{d}v\\ &-k\int_M\rho\varphi^2|\omega|^p\mathrm{d}v-\frac{n}{2}\int_M\varphi^2|\phi|^2|\omega|^p\mathrm{d}v+\frac{1}{2}\min\{l,n-l\}\int_M\varphi^2|A|^2|\omega|^p\mathrm{d}v.\end{aligned} \tag{3.9}$$

Next, we shall give some estimates of (3.9). Firstly, by Schwarz's inequality, we have

$$\begin{aligned}&2\int_M\varphi|\omega|^{p-1}|\nabla\varphi|\,|\nabla|\omega||\,\mathrm{d}v\\ \leq&\varepsilon_1\int_M\varphi^2|\omega|^{p-2}|\nabla|\omega||^2\mathrm{d}v+\frac{1}{\varepsilon_1}\int_M|\nabla\varphi|^2|\omega|^p\mathrm{d}v,\end{aligned} \tag{3.10}$$

where $\varepsilon_1>0$ is a constant. Secondly, by using the weighted Poincaré inequality and the Schwarz's inequality, it is easy to deduce that

$$\begin{aligned}&\int_M\rho\varphi^2|\omega|^p\mathrm{d}v\\ =&\int_M\rho\left(\varphi|\omega|^{\frac{p}{2}}\right)^2\mathrm{d}v\leq\int_M|\nabla(\varphi|\omega|^{\frac{p}{2}})|^2\mathrm{d}v\\ \leq&(1+\varepsilon_2)\left(\frac{p}{2}\right)^2\int_M\varphi^2|\omega|^{p-2}|\nabla|\omega||^2\mathrm{d}v+\left(1+\frac{1}{\varepsilon_2}\right)\int_M|\nabla\varphi|^2|\omega|^p\mathrm{d}v,\end{aligned} \tag{3.11}$$

where $\varepsilon_2>0$ is a constant. Finally, we shall give an estimate to the forth term in the right hand side. Denote by

$$E(\varphi)=C(n)\left(\int_{\mathrm{supp}\,\varphi}|\phi|^n\mathrm{d}v\right)^{\frac{2}{n}},$$

where supp φ is the compact support set of φ. Then using the Hölder inequality, Sobolev inequality and the Schwarz's inequality, we obtain

$$\begin{aligned}\int_M |\phi|^2\varphi^2|\omega|^p \mathrm{d}v \leq & \left(\int_{\mathrm{supp}\ \varphi} |\phi|^n \mathrm{d}v\right)^{\frac{2}{n}} \cdot \left(\int_M (|\varphi||\omega|^{\frac{p}{2}})^{\frac{2n}{n-2}} \mathrm{d}v\right)^{\frac{n-2}{n}} \\ \leq & E(\varphi)\left\{\int_M |\nabla(\varphi|\omega|^{\frac{p}{2}})|^2 \mathrm{d}v + \int_M |H|^2\varphi^2|\omega|^p \mathrm{d}v\right\} \\ \leq & E(\varphi)\left\{(1+\varepsilon_3)\left(\frac{p}{2}\right)^2 \int_M \varphi^2|\omega|^{p-2}|\nabla|\omega||^2 \mathrm{d}v \right. \\ & \left. + \left(1+\frac{1}{\varepsilon_3}\right)\int_M |\nabla\varphi|^2|\omega|^p \mathrm{d}v + \int_M \varphi^2|H|^2|\omega|^p \mathrm{d}v\right\},\end{aligned} \tag{3.12}$$

where $\varepsilon_3 > 0$ is a constant. Therefore, combining (3.9)-(3.12), it follows that

$$\begin{aligned}&\left\{(p-1)+(p-1)^2k_p-(2p-3)\varepsilon_1-\frac{p^2}{4}(1+\varepsilon_2)k \right.\\ &\left. -\frac{np^2}{8}(1+\varepsilon_3)E(\varphi)\right\}\int_M \varphi^2|\omega|^{p-2}|\nabla|\omega||^2 \mathrm{d}v \\ &-\frac{n}{2}E(\varphi)\int_M \varphi^2|H|^2|\omega|^p \mathrm{d}v + \frac{1}{2}\min\{l,n-l\}\int_M \varphi^2|A|^2|\omega|^p \mathrm{d}v \\ \leq & \left[\frac{2p-3}{\varepsilon_1}+\left(1+\frac{1}{\varepsilon_2}\right)+\frac{n}{2}\left(1+\frac{1}{\varepsilon_3}\right)E(\varphi)\right]\int_M |\nabla\varphi|^2|\omega|^p \mathrm{d}v.\end{aligned} \tag{3.13}$$

Since $||\phi||^2_{L^n(M)} < \frac{8[p-1+(p-1)^2k_p]-2kp^2}{np^2C(n)}$, it implies that

$$p-1+(p-1)^2k_p-\frac{kp^2}{4}-\frac{np^2}{8}E(\varphi)>0.$$

Then we can choose sufficiently small ε_1, ε_2 and ε_3 such that

$$p-1+(p-1)^2k_p-(2p-3)\varepsilon_1-\frac{p^2}{4}(1+\varepsilon_2)k-\frac{np^2}{8}(1+\varepsilon_3)E(\varphi)>0.$$

Moreover, due to

$$E(\varphi)<\frac{8[p-1+(p-1)^2k_p]-2kp^2}{np^2}<\frac{8[p-1+(p-1)^2k_p]}{np^2}<\min\{l,n-l\},$$

we have

$$\min\{l,n-l\}-E(\varphi)>0.$$

Note that $|A|^2-n|H|^2=|\phi|^2\geq 0$, then it follows from (3.13) that

$$\begin{aligned}&B\int_M \varphi^2|\omega|^{p-2}|\nabla|\omega||^2 \mathrm{d}v+\frac{1}{2}E(\varphi)\int_M (|A|^2-n|H|^2)\varphi^2|\omega|^p \mathrm{d}v + C\int_M \varphi^2|A|^2|\omega|^p \mathrm{d}v \\ \leq & D\int_M |\nabla\varphi|^2|\omega|^p \mathrm{d}v,\end{aligned} \tag{3.14}$$

where

$$\begin{aligned} B &= p-1+(p-1)^2k_p-(2p-3)\varepsilon_1-\frac{p^2}{4}(1+\varepsilon_2)k-\frac{np^2}{8}(1+\varepsilon_3)E(\varphi)>0,\\ C &= \frac{1}{2}\left(\min\{l,n-l\}-E(\varphi)\right)>0,\\ D &= \frac{2p-3}{\varepsilon_1}+\left(1+\frac{1}{\varepsilon_2}\right)+\frac{n}{2}\left(1+\frac{1}{\varepsilon_3}\right)E(\varphi). \end{aligned}$$

Since φ is a compactly supported nonnegative smooth function on M and $|\nabla\varphi|^2\leq\frac{4}{r^2}$, we have

$$\begin{aligned} &B\int_{B_r(x_0)}|\omega|^{p-2}|\nabla|\omega||^2\mathrm{d}v+\frac{1}{2}E(\varphi)\int_{B_r(x_0)}(|A|^2-n|H|^2)|\omega|^p\mathrm{d}v+C\int_{B_r(x_0)}|A|^2|\omega|^p\mathrm{d}v\\ \leq\ &D\int_M|\nabla\varphi|^2|\omega|^p\mathrm{d}v\leq\frac{4D}{r^2}\int_{B_{2r}(x_0)}|\omega|^p\mathrm{d}v. \end{aligned} \tag{3.15}$$

Since $\omega\in L^p(M)$, letting $r\to+\infty$ in (3.15) , it follows that

$$|\omega|^{p-2}|\nabla|\omega||=0,(|A|^2-n|H|^2)|\omega|^p=0,|A|^2|\omega|^p=0.$$

If $|\omega|$ is not identically zero, then $|\nabla|\omega||=0,|A|=0$, and $|H|=0$. It implies that $|\omega|$ is a constant and M is a minimal submanifold. Since $|H|=0$, the Sobolev inequality (2.6) becomes

$$\left(\int_M|\varphi|^{\frac{2n}{n-2}}\mathrm{d}v\right)^{\frac{n-2}{n}}\leq C(n)\int_M|\nabla\varphi|^2\mathrm{d}v,$$

for any $\varphi\in C_0^\infty(M)$. Therefore, M^n has infinite volume according to [29, Proposition 2.4]. This contradicts the fact that $\int_M|\omega|^p\mathrm{d}v<\infty$. Therefore, $\omega\equiv 0$ on M^n. This completes the proof.

4 The Proof of Theorem 1.2

Proof Let ω be a p-harmonic 1-form on M. Then the Bochner formula (2.3) becomes

$$\frac{1}{2}\Delta|\omega|^2=|\nabla\omega|^2+\langle\Delta\omega,\omega\rangle+Ric_M(\omega^\sharp,\omega^\sharp). \tag{4.1}$$

This together with Lemma 2.7 gives

$$\begin{aligned} \frac{1}{2}\Delta|\omega|^2\geq&|\nabla\omega|^2+\langle\Delta\omega,\omega\rangle\\ &+\left\{(n-1)(|H|^2-k^2)-\frac{(n-2)\sqrt{n(n-1)}}{n}|H||\phi|-\frac{n-1}{n}|\phi|^2\right\}|\omega|^2. \end{aligned} \tag{4.2}$$

Applying the Bochner formula (4.2) to the form $|\omega|^{p-2}\omega$, we obtain

$$\begin{aligned} \frac{1}{2}\Delta|\omega|^{2(p-1)}\geq&|\nabla(|\omega|^{p-2}\omega)|^2-\langle(\delta d+d\delta)(|\omega|^{p-2}\omega),|\omega|^{p-2}\omega\rangle\\ &+\left\{(n-1)(|H|^2-k^2)-\frac{(n-2)\sqrt{n(n-1)}}{n}|H||\phi|-\frac{n-1}{n}|\phi|^2\right\}|\omega|^{2(p-1)}. \end{aligned} \tag{4.3}$$

Combining (3.2) and $\delta(|\omega|^{p-2}\omega)=0$, it follows that

$$\begin{aligned}|\omega|^{p-1}\Delta|\omega|^{p-1}\geq&(|\nabla(|\omega|^{p-2}\omega)|^2-|\nabla|\omega|^{p-1}|^2)-\langle\delta d(|\omega|^{p-2}\omega),|\omega|^{p-2}\omega\rangle\\&+\left\{(n-1)(|H|^2-k^2)-\frac{(n-2)\sqrt{n(n-1)}}{n}|H||\phi|-\frac{n-1}{n}|\phi|^2\right\}|\omega|^{2(p-1)}.\end{aligned}\tag{4.4}$$

By using the refined Kato inequality (2.1) and dividing both sides of the above inequality by $|\omega|^{p-2}$, we get

$$\begin{aligned}|\omega|\Delta|\omega|^{p-1}\geq&k_p(p-1)^2|\omega|^{p-2}|\nabla|\omega||^2-\langle\delta d(|\omega|^{p-2}\omega),\omega\rangle\\&+\left\{(n-1)(|H|^2-k^2)-\frac{(n-2)\sqrt{n(n-1)}}{n}|H||\phi|-\frac{n-1}{n}|\phi|^2\right\}|\omega|^p.\end{aligned}\tag{4.5}$$

Similarly, multiplying both sides of the inequality (4.5) by φ^2 and integrating over M gives

$$\begin{aligned}&\int_M\varphi^2|\omega|\Delta|\omega|^{p-1}\mathrm{d}v\\\geq&(p-1)^2k_p\int_M\varphi^2|\omega|^{p-2}|\nabla|\omega||^2\mathrm{d}v-\int_M\langle d(|\omega|^{p-2}\omega),d(\varphi^2\omega)\rangle\mathrm{d}v\\&+(n-1)\int_M\varphi^2|H|^2|\omega|^p\mathrm{d}v-(n-1)k^2\int_M\varphi^2|\omega|^p\mathrm{d}v\\&-\frac{(n-2)\sqrt{n(n-1)}}{n}\int_M\varphi^2|\phi||H||\omega|^p\mathrm{d}v-\frac{n-1}{n}\int_M\varphi^2|\phi|^2|\omega|^p\mathrm{d}v,\end{aligned}\tag{4.6}$$

where φ is the cutoff function defined in (3.5). Applying the estimates (3.7) and (3.8), it follows that

$$\begin{aligned}0\leq&2(2p-3)\int_M\varphi|\omega|^{p-1}|\nabla\varphi||\nabla|\omega||\mathrm{d}v-[p-1+(p-1)^2k_p]\int_M\varphi^2|\omega|^{p-2}|\nabla|\omega||^2\mathrm{d}v\\&-(n-1)\int_M\varphi^2|H|^2|\omega|^p\mathrm{d}v+(n-1)k^2\int_M\varphi^2|\omega|^p\mathrm{d}v\\&+\frac{(n-2)\sqrt{n(n-1)}}{n}\int_M\varphi^2|\phi||H||\omega|^p\mathrm{d}v+\frac{n-1}{n}\int_M\varphi^2|\phi|^2|\omega|^p\mathrm{d}v.\end{aligned}\tag{4.7}$$

Then using the Young's inequality yields

$$\begin{aligned}2\int_M\varphi^2|\phi||H||\omega|^p\mathrm{d}v=&2\int_M\varphi|H||\omega|^{\frac{p}{2}}\cdot\varphi|\phi||\omega|^{\frac{p}{2}}\mathrm{d}v\\\leq&\varepsilon_4\int_M\varphi^2|H|^2|\omega|^p\mathrm{d}v+\frac{1}{\varepsilon_4}\int_M\varphi^2|\phi|^2|\omega|^p\mathrm{d}v,\end{aligned}\tag{4.8}$$

where $\varepsilon_4>0$ is a constant.

Combining the estimates (3.10), (4.7) and (4.8), we deduce that

$$\begin{aligned}0\leq&\left\{(2p-3)\varepsilon_1-[p-1+(p-1)^2k_p]\right\}\int_M\varphi^2|\omega|^{p-2}|\nabla|\omega||^2\mathrm{d}v\\&+\frac{2p-3}{\varepsilon_1}\int_M|\nabla\varphi|^2|\omega|^p\mathrm{d}v+(n-1)k^2\int_M\varphi^2|\omega|^p\mathrm{d}v\\&+\left[\frac{n-1}{n}+\frac{(n-2)\sqrt{n(n-1))}}{2n\varepsilon_4}\right]\int_M\varphi^2|\phi|^2|\omega|^p\mathrm{d}v\\&+\left[\frac{(n-2)\sqrt{n(n-1)}\varepsilon_4}{2n}-(n-1)\right]\int_M\varphi^2|H|^2|\omega|^p\mathrm{d}v.\end{aligned}\tag{4.9}$$

When $k\neq 0$, we need to estimate the second term of the second line in (4.9). More precisely, by using the monotonicity of $\lambda_1(B_r(x_0))$, it holds that

$$\lambda_1(M)\leq\lambda_1(B_r(x_0))\leq\frac{\int_{B_{r(x_0)}}|\nabla\varphi|^2\mathrm{d}v}{\int_{B_r(x_0)}\varphi^2\mathrm{d}v},\tag{4.10}$$

for any $\varphi\in C_0^\infty(M)$. Then substituting $\varphi|\omega|^{\frac{p}{2}}$ into the inequality (4.10) and using the Young's inequality again gives

$$\begin{aligned}&\lambda_1(M)\int_M\varphi^2|\omega|^p\mathrm{d}v\leq\int_M|\nabla(\varphi|\omega|^{\frac{p}{2}})|^2\mathrm{d}v\\\leq&\frac{p^2(1+\varepsilon_5)}{4}\int_M\varphi^2|\omega|^{p-2}|\nabla|\omega||^2\mathrm{d}v+\left(1+\frac{1}{\varepsilon_5}\right)\int_M|\nabla\varphi|^2|\omega|^p\mathrm{d}v,\end{aligned}\tag{4.11}$$

where $\varepsilon_5>0$ is a constant.

Finally, combining these inequalities (3.12), (4.9) and (4.11), we obtain that

$$B\int_M\varphi^2|\omega|^{p-2}|\nabla|\omega||^2\mathrm{d}v+C\int_M\varphi^2|H|^2|\omega|^p\mathrm{d}v\leq D\int_M|\nabla\varphi|^2|\omega|^p\mathrm{d}v,\tag{4.12}$$

where B,C,D are constants given by

$$\begin{aligned}B:=&[p-1+(p-1)^2k_p]-(2p-3)\varepsilon_1-\frac{(n-1)k^2p^2}{4\lambda_1(M)}(1+\varepsilon_5)\\&-\left[\frac{n-1}{n}+\frac{(n-2)\sqrt{n(n-1)}}{2n\varepsilon_4}\right]E(\varphi)\cdot\frac{p^2(1+\varepsilon_3)}{4},\\C:=&(n-1)-\frac{(n-2)\sqrt{n(n-1)}}{2n}\varepsilon_4-\left[\frac{n-1}{n}+\frac{(n-2)\sqrt{n(n-1)}}{2n\varepsilon_4}\right]E(\varphi),\\D:=&\frac{2p-3}{\varepsilon_1}+\frac{(n-1)k^2}{\lambda_1(M)}\left(1+\frac{1}{\varepsilon_5}\right)+\left[\frac{n-1}{n}+\frac{(n-2)\sqrt{n(n-1)}}{2n\varepsilon_4}\right]E(\varphi)\left(1+\frac{1}{\varepsilon_3}\right).\end{aligned}\tag{4.13}$$

Since the traceless second fundamental form ϕ satisfies $||\phi||_{L^n(M)} < \frac{4}{p^2}\sqrt{\frac{1}{n(n-1)C(n)}}$, we have

$$E(\varphi) < \frac{16}{n(n-1)p^4} \leq \frac{1}{n(n-1)}.$$

Then we can choose $\varepsilon_4 = \sqrt{E(\varphi)} < \frac{1}{\sqrt{n(n-1)}}$ such that

$$\begin{aligned} C =&(n-1) - \frac{(n-2)\sqrt{n(n-1)}}{2n} \cdot 2\sqrt{E(\varphi)} - \frac{n-1}{n}E(\varphi) \\ \geq&(n-1) - \frac{n-2}{n} - \frac{1}{n^2} = (n-1)\left(1 - \frac{n-1}{n^2}\right) > 0. \end{aligned}$$

By the assumption on the first eigenvalue $\lambda_1(M)$ and $||\phi||_{L^n(M)}$, we can choose $\varepsilon_1, \varepsilon_3, \varepsilon_5$ small enough such that $B > 0$. In addition, it is obvious that $D > 0$.

On the other hand, since φ is a compactly supported nonnegative smooth function on M and $|\nabla\varphi|^2 \leq \frac{4}{r^2}$, by using (4.12), we obtain

$$\begin{aligned} &B\int_{B_r(x_0)} |\omega|^{p-2}|\nabla|\omega||^2 \mathrm{d}v + C\int_{B_r(x_0)} |H|^2|\omega|^p \mathrm{d}v \\ \leq&B\int_M \varphi^2|\omega|^p|\nabla|\omega||^2 \mathrm{d}v + C\int_M \varphi^2|H|^2|\omega|^p \mathrm{d}v \\ \leq&D\int_M |\nabla\varphi|^2|\omega|^p \mathrm{d}v \\ \leq&\frac{4D}{r^2}\int_{B_{2r}(x_0)} |\omega|^p \mathrm{d}v. \end{aligned} \tag{4.14}$$

Since $\int_M |\omega|^p \mathrm{d}v < +\infty$, taking $r \to +\infty$ in the inequality (4.14) implies that

$$|\omega|^{p-2}|\nabla|\omega||^2 \equiv 0, |H|^2|\omega|^p \equiv 0.$$

If $|\omega|$ is not identically zero, then $|\nabla|\omega|| = 0$ and $|H| = 0$. Hence, $|\omega|$ is a constant and M is a minimal submanifold. However, due to the infinite volume of complete minimal submanifold in a Riemannian manifold of nonpositive sectional curvature, we have $\int_M |\omega|^p \mathrm{d}v = +\infty$. This contradicts the fact that $\int_M |\omega|^p \mathrm{d}v < +\infty$. Therefore, $\omega \equiv 0$ on M. The proof is completed.

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Hadamard流形的子流形上的一些p-调和形式的消灭定理

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摘要：本文研究了Hadamard流形N的完备浸入子流形M上的一些p-调和形式的消灭定理. 首先, 假设M满足加权庞加莱不等式且具有平坦法丛, N具有纯曲率张量且$(l, n-l)$-曲率不小于$-k\rho(0 \le k \le \frac{4}{p^2})$, $2 \le l \le n-2$. 如果总曲率足够小, 我们得到了p-调和l-形式的消灭定理, 推广了Wang–Chao–Wu–Lv在2018年的结果. 其次, 假设N是一个截面曲率满足$-k^2 \le K_N \le 0$的Hadamard流形, 如果总曲率足够小且拉普拉斯的第一特征值满足某个下界, 我们得到了p-调和1-形式的消灭定理, 推广了Dung–Seo在2015年的结果.

关键词：p-调和形式; 消灭定理; 加权庞加莱不等式; Hadamard流形

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