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二维线性化 Navier-Stokes-Poisson 方程解的逐点估计

徐红梅, 肖连慧
(河海大学数学学院, 江苏 南京 211100)

摘 要: 本文研究了二维空间线性化的等熵可压缩 Navier-Stokes-Poisson 方程柯西问题. 通过把方程组转变成关于单个函数的方程, 求解出各个函数, 得到方程组的格林函数. 利用对格林函数的详细分析, 获得了方程组解的逐点估计. 结果显示方程组中电流密度以热核的速度衰减, 动量密度衰减慢得多, 且其 L2 范数不衰减.

关 键 词: Navier-Stokes-Poisson 方程; 二维空间; 格林函数; 逐点估计
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1 引言

本文考虑二维空间线性化的等熵可压缩 Navier-Stokes-Poisson 方程, 后面简称为 N-S-P 方程柯西问题解的逐点估计. 二维 N-S-P 方程形式为

$$\begin{cases}\rho_t+divm=0,\\ m_{1t}+\partial_{x_1}\left(\frac{m_1^2}{\rho}\right)+\partial_{x_2}\left(\frac{m_1m_2}{\rho}\right)+\partial_{x_1}P\left(\rho\right)=\rho\partial_{x_1}\phi+\mu_1\Delta\frac{m_1}{\rho}+\mu_2\partial_{x_1}div\frac{m}{\rho},\\ m_{2t}+\partial_{x_1}\left(\frac{m_1m_2}{\rho}\right)+\partial_{x_2}\left(\frac{m_2^2}{\rho}\right)+\partial_{x_2}P\left(\rho\right)=\rho\partial_{x_2}\phi+\mu_1\Delta\frac{m_2}{\rho}+\mu_2\partial_{x_2}div\frac{m}{\rho},\\ \Delta\phi=\rho-\overline{\rho},\phi\rightarrow 0\text{ 当 }\sqrt{x_1^2+x_2^2}\rightarrow+\infty.\end{cases}\tag{1.1}$$

此处 $x=(x_1,x_2)\in\mathfrak{R}^2$ 是空间变量, $t>0$ 是时间变量, div 是散度算子, Δ 是通常的拉普拉斯算子, 即 $\Delta=\partial_{x_1}^2+\partial_{x_2}^2$. $\rho(x,t)$, $m(x,t)=(m_1,m_2)$, $\phi(x,t)$ 和 $P(\rho)$ 分别代表电子流体密度, 动量密度, 静电势和压力. 粘性系数 μ_1, μ_2 均大于零, $\overline{\rho}>0$ 表示正电荷背景离子的规定密度. (1.1) 是一个描述等离子体动力学行为的涉及耗散的简单模型, 它是在等熵 Navier-Stokes 方程基础上加上静电势 ϕ 的作用而得的方程. 等熵 Navier-Stokes 方程是双曲 - 抛物混合方程. Kawashima[1] 研究了一般的双曲 - 抛物系统, 得到了解的整体存在性和 L^2 衰减估计, Liu 和 Zeng[2] 得到了一维一般双曲 - 抛物系统解的逐点估计, Holf 和 Zumbrun[3] 得到高维等熵 Navier-Stokes 方程的解的 L^p 估计, Liu 和 Wang[4], Wang 和 Yang[5] 通过对格林函数的详细分析, 继续得到了此方程解的逐点估计. 对 N-S-P 方程, [6,7,8] 研究了其局部解和整体弱解的存在性. [9] 中用半群的方法得到了三维空间经典解的整体存在性, [10] 通过对其线性系统格林函数的详细分析, 得到了解的逐点估计. 相较于双曲抛物的等熵 Navier-Stokes 方程,

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作者简介: 徐红梅 (1973–), 女, 湖北英山, 副教授, 主要研究方向: 偏微分方程
E-mail: xxu_hongmei@163.com.

加了静电势的 N-S-P 方程是一个双曲 - 抛物 - 椭圆的耦合系统. 方程结构越复杂, 格林函数越难计算且形式越复杂, 做逐点估计更难. 另外, 因为有了静电势, 格林函数中含了一个非局部项, 这一项在低频时是奇异的, 因而在估计时遇到与 [4,5] 不同的新的困难. 从 [9,10] 中可看出, 方程的衰减与空间维数有关, 维数越低, 衰减越慢, 因而处理更麻烦. 本论文考虑 (1.1) 在常状态 $(\overline{\rho},\overline{m}_1,\overline{m}_2)=(1,0,0)$ 附近的线性化方程, 研究其格林函数的构造、衰减, 继而得到线性方程的衰减. 希望这些工作能给非线性方程的研究提供一个基础和支撑.

本文中, α 是多重指标 $\alpha=(\alpha_1,\alpha_2)$, $\partial_x^\alpha f(x)$ 表示 $\partial_{x_1}^{\alpha_1}\partial_{x_2}^{\alpha_2}f$, $|\alpha|$ 表示 $\alpha_1+\alpha_2$. $F(f)$ 或 $\hat{f}(\xi)$ 表示函数 $f(x)$ 关于变量 x 的傅立叶变换, 即 $F(f)(\xi,t)=\hat{f}(\xi,t)=\int_{\mathfrak{R}_n} f(x,t)\,e^{-ix\cdot\xi}dx$, 其中 i 为虚数单位. $F^{-1}\left(\hat{f}\right)(x,t)$ 或 $f(x,t)$ 表示 $\hat{f}(\xi,t)$ 关于 ξ 的逆傅立叶变换, 即 $F^{-1}\left(\hat{f}\right)=f(x,t)=(2\pi)^{-2n}\int\hat{f}(\xi,t)\,e^{ix\cdot\xi}d\xi$. $W^{s,p}(\mathfrak{R}^n)$ 是通常的索伯列夫空间, 范数为 $\|f\|_{W^{s,p}(\mathfrak{R}^n)}=\sum\limits_{|\alpha|=0}^{s}\|\partial_x^\alpha f\|_{L^p(\mathfrak{R}^n)}$. $W^{s,2}(\mathfrak{R}^n)$ 记为 $H^s(\mathfrak{R}^n)$. 任意实数 x, 定义 $x_+=\begin{cases}x, 若\ x\ge 0,\\ 0, 若\ x<0.\end{cases}$

2 格林函数的计算

首先把方程 (1.1) 在 $(\overline{\rho},\overline{m})=(1,0,0)$ 附近做线性化. 记 $\tilde{\rho}=\rho-\overline{\rho}$, $\tilde{m}=m-\overline{m}$, 因为

$$\partial_{x_1}\left(\frac{m_1}{\rho}\right)=\partial_{x_1}\left(\frac{\tilde{m}_1}{\tilde{\rho}+1}\right)=\frac{\partial_{x_1}\tilde{m}_1\cdot(\tilde{\rho}+1)-\tilde{m}_1\cdot\partial_{x_1}\tilde{\rho}}{(\tilde{\rho}+1)^2},$$

$$\partial_{x_1}\left(\frac{m_1^2}{\rho}\right)=\partial_{x_1}\left(\frac{\tilde{m}_1^2}{\tilde{\rho}+1}\right)=\frac{2\tilde{m}_1\cdot\partial_{x_1}\tilde{m}_1\cdot(\tilde{\rho}+1)-\tilde{m}_1^2\cdot\partial_{x_1}\tilde{\rho}}{(\tilde{\rho}+1)^2},$$

$$\partial_{x_2}\left(\frac{m_1\cdot m_2}{\rho}\right)=\partial_{x_2}\frac{\tilde{m}_1\cdot\tilde{m}_2}{\tilde{\rho}+1}=\frac{\partial_{x_2}\tilde{m}_1\cdot\tilde{m}_2(\tilde{\rho}+1)+\tilde{m}_1\cdot\partial_{x_2}\tilde{m}_2\cdot(\tilde{\rho}+1)-\tilde{m}_1\cdot\tilde{m}_2\partial_{x_2}\tilde{\rho}}{(\tilde{\rho}+1)^2},$$

$$\partial_{x_1}P(\rho)=P'(\overline{\rho})\,\partial_{x_1}\tilde{\rho}+\partial_{x_1}\left(P(\rho)-P(\overline{\rho})-P'(\overline{\rho})\,\tilde{\rho}\right),$$

所以方程 (1.1) 在 $(1,0,0)$ 附近的线性化方程为

$$\begin{cases}\tilde{\rho}_t+div\tilde{m}=0,\\ \tilde{m}_{1t}+P'(1)\,\partial_{x_1}\tilde{\rho}=\partial_{x_1}\phi+\mu_1\Delta\tilde{m}_1+\mu_2\partial_{x_1}\left(\partial_{x_1}\tilde{m}_1+\partial_{x_2}\tilde{m}_2\right),\\ \tilde{m}_{2t}+P'(1)\,\partial_{x_2}\tilde{\rho}=\partial_{x_2}\phi+\mu_1\Delta\tilde{m}_2+\mu_2\partial_{x_2}\left(\partial_{x_1}\tilde{m}_1+\partial_{x_2}\tilde{m}_2\right).\end{cases}\tag{2.1}$$

为减少记号, 不妨还是记 $\rho=\tilde{\rho}$, $m=\tilde{m}$, $P'(1)=c^2>0$, 则 (2.1) 为

$$\begin{cases}\rho_t+divm=0,\\ m_t+c^2\nabla\rho=\nabla\phi+\mu_1\Delta m+\mu_2\nabla divm,\\ \Delta\phi=\rho,\phi\to 0,\ 当\ \sqrt{x_1^2+x_2^2}\to+\infty.\end{cases}\tag{2.2}$$

下面考虑 (2.2) 的格林函数 G, 即若 (2.2) 的初值为 $(\rho,m)\,|_{t=0}=(\rho_0,m_0)$, 我们要找出矩阵 G 满足 $\begin{pmatrix}\rho\\ m\end{pmatrix}=G*\begin{pmatrix}\rho_0\\ m_0\end{pmatrix}$. 此处及后面的 $*$ 是表示对空间变量的卷积.

记 $\gamma=\mu_1+\mu_2$, 由 (2.2) 得

$$\begin{aligned}\rho_{tt}&=-\left(divm\right)_t=-divm_t=-div\left(-c^2\nabla\rho+\nabla\phi+\mu_1\Delta m+\mu_2\nabla divm\right)\\&=c^2\Delta\rho-\Delta\phi-\mu_1\Delta\left(divm\right)-\mu_2\Delta\left(divm\right)\\&=c^2\Delta\rho-\rho-\mu_1\Delta\left(-\rho_t\right)-\mu_2\Delta\left(-\rho_t\right)=c^2\Delta\rho-\rho+\gamma\Delta\rho_t.\end{aligned}\tag{2.3}$$

对空间变量 x 作傅立叶变换, 得 $\hat{\rho}_{tt}=c^2\left(-|\xi|^2\right)\hat{\rho}-\hat{\rho}+\gamma\left(-|\xi|^2\right)\hat{\rho}_t$.

解二阶常微分方程初值问题

$$\begin{cases}\hat{\rho}_{tt}+\left(c^2|\xi|^2+1\right)\hat{\rho}+\gamma|\xi|^2\hat{\rho}_t=0,\\\hat{\rho}\left(\xi,0\right)=\hat{\rho}_0\left(\xi\right),\\\hat{\rho}_t\left(\xi,0\right)=-i\xi^\tau\hat{m}_0\left(\xi\right).\end{cases}$$

得到

$$\hat{\rho}\left(\xi,t\right)=\frac{\lambda_+e^{\lambda_-t}-\lambda_-e^{\lambda_+t}}{\lambda_+-\lambda_-}\hat{\rho}_0\left(\xi\right)+\frac{\left(e^{\lambda_-t}-e^{\lambda_+t}\right)i\xi^\tau}{\lambda_+-\lambda_-}\hat{m}_0\left(\xi\right),\tag{2.4}$$

其中

$$\lambda_+=\frac{-\gamma|\xi|^2+\sqrt{\gamma^2\left|\xi\right|^4-4\left(c^2\left|\xi\right|^2+1\right)}}{2},\lambda_-=\frac{-\gamma|\xi|^2-\sqrt{\gamma^2\left|\xi\right|^4-4\left(c^2\left|\xi\right|^2+1\right)}}{2}.\tag{2.5}$$

由 (2.5) 易得

$$\begin{aligned}\lambda_+-\lambda_-&=\sqrt{\gamma^2\left|\xi\right|^4-4\left(c^2\left|\xi\right|^2+1\right)},\\\lambda_++\gamma\left|\xi\right|^2=-\lambda_-,\lambda_-+\gamma\left|\xi\right|^2&=-\lambda_+,\lambda_+\cdot\lambda_-=c^2\left|\xi\right|^2+1.\end{aligned}\tag{2.6}$$

为求解 $\hat{m}$, 将 $\hat{m}\left(\xi,t\right)$ 分解成与 ξ 平行和垂直的两个向量，即

$$\hat{m}\left(\xi,t\right)=a\left(\xi,t\right)\frac{\xi}{|\xi|}+b\left(\xi,t\right),\tag{2.7}$$

其中 $a\left(\xi,t\right)\in\Re^1$, $b\left(\xi,t\right)\in\Re^2$, 且 $\xi^\tau b=0$. 对 (2.2) 的第二个方程关于空间向量作傅立叶变换，得

$$\hat{m}_t+c^2i\xi\hat{\rho}=i\xi\hat{\phi}-\mu_1\left|\xi\right|^2\hat{m}-\mu_2\xi\xi^\tau\hat{m}.$$

于是

$$\begin{aligned}a_t\left(\xi,t\right)\frac{\xi}{|\xi|}+b_t+ic^2\xi\hat{\rho}&=i\xi\hat{\phi}-\mu_1\left|\xi\right|^2\left(a\left(\xi,t\right)\frac{\xi}{|\xi|}+b\left(\xi,t\right)\right)-\mu_2\xi\xi^\tau a\left(\xi,t\right)\frac{\xi}{|\xi|}-\mu_2\xi\xi^\tau b\\&=i\xi\hat{\phi}-\mu_1\left|\xi\right|a\left(\xi,t\right)\xi-\mu_1\left|\xi\right|^2b\left(\xi,t\right)-\mu_2\left|\xi\right|\xi a\left(\xi,t\right).\end{aligned}$$

所以得

$$b_t=-\mu_1\left|\xi\right|^2b\left(\xi,t\right),$$

$$a_t(\xi,t)=-ic^2|\xi|\hat{\rho}+i|\xi|\hat{\phi}-\mu_1|\xi|^2a(\xi,t)-\mu_2a(\xi,t)|\xi|^2=-i\frac{c^2|\xi|^2+1}{|\xi|}\hat{\rho}-\gamma a(\xi,t)|\xi|^2.$$

解常微分方程得

$$b(\xi,t)=b(\xi,0)e^{-\mu_1|\xi|^2t},\tag{2.8}$$

$$a(\xi,t)=-i\frac{1+c^2|\xi|^2}{|\xi|}e^{-\gamma|\xi|^2t}\int_0^t\hat{\rho}(\xi,s)e^{\gamma|\xi|^2s}ds+a(\xi,0)e^{-\gamma|\xi|^2t}.\tag{2.9}$$

由 (2.5), (2.6) 得

$$\begin{aligned}&\int_0^t\left[\frac{-\lambda_-}{\lambda_+-\lambda_-}e^{\left(\lambda_++\gamma|\xi|^2\right)s}+\frac{\lambda_+}{\lambda_+-\lambda_-}e^{\left(\lambda_-+\gamma|\xi|^2\right)s}\right]ds\\&=\frac{-\lambda_-}{\lambda_+-\lambda_-}\cdot\frac{1}{\lambda_++\gamma|\xi|^2}\left(e^{\left(\lambda_++\gamma|\xi|^2\right)t}-1\right)+\frac{\lambda_+}{\lambda_+-\lambda_-}\cdot\frac{1}{\lambda_-+\gamma|\xi|^2}\left(e^{\left(\lambda_-+\gamma|\xi|^2\right)t}-1\right)\\&=\frac{1}{\lambda_+-\lambda_-}\cdot e^{-\lambda_-t}-\frac{1}{\lambda_+-\lambda_-}\cdot e^{-\lambda_+t}\quad,\end{aligned}\tag{2.10}$$

$$\begin{aligned}&\int_0^t\left[\frac{-e^{\left(\lambda_++\gamma|\xi|^2\right)s}}{\lambda_+-\lambda_-}+\frac{e^{\left(\lambda_-+\gamma|\xi|^2\right)s}}{\lambda_+-\lambda_-}\right]ds\\&=\frac{-1}{\lambda_+-\lambda_-}\cdot\frac{1}{\lambda_++\gamma|\xi|^2}\cdot\left(e^{\left(\lambda_++\gamma|\xi|^2\right)t}-1\right)+\frac{1}{\lambda_+-\lambda_-}\cdot\frac{1}{\lambda_-+\gamma|\xi|^2}\cdot\left(e^{\left(\lambda_-+\gamma|\xi|^2\right)t}-1\right)\\&=\frac{1}{(\lambda_+-\lambda_-)\lambda_-}\cdot e^{-\lambda_-t}-\frac{1}{(\lambda_+-\lambda_-)\lambda_+}\cdot e^{-\lambda_+t}-\frac{1}{\lambda_+\lambda_-}\quad.\end{aligned}\tag{2.11}$$

由 (2.4), (2.6), (2.9), (2.10), (2.11) 得

$$\begin{aligned}a(\xi,t)=&-i\frac{\left(1+c^2|\xi|^2\right)}{|\xi|}e^{-\gamma|\xi|^2t}\cdot\hat{\rho}_0\cdot\left(\frac{e^{-\lambda_-t}-e^{-\lambda_+t}}{\lambda_+-\lambda_-}\right)-\frac{i\xi^\tau\hat{m}_0}{\lambda_+\lambda_-}(-i)\frac{1+c^2|\xi|^2}{|\xi|}e^{-\gamma|\xi|^2t}\\&+a(\xi,0)e^{-\gamma|\xi|^2t}+\frac{1+c^2|\xi|^2}{|\xi|}e^{-\gamma|\xi|^2t}\xi^\tau\hat{m}_0\left(\frac{e^{-\lambda_-t}}{(\lambda_+-\lambda_-)\lambda_-}-\frac{e^{-\lambda_+t}}{(\lambda_+-\lambda_-)\lambda_+}\right)\\=&-i\frac{\left(1+c^2|\xi|^2\right)}{|\xi|}\hat{\rho}_0\cdot\frac{e^{\lambda_+t}-e^{\lambda_-t}}{\lambda_+-\lambda_-}+\frac{1}{|\xi|}\xi^\tau\hat{m}_0\left(\frac{\lambda_+e^{\lambda_+t}-\lambda_-e^{\lambda_-t}}{\lambda_+-\lambda_-}\right)\\&-\frac{\xi^\tau}{|\xi|}\left(a(\xi,0)\frac{\xi}{|\xi|}+b(\xi,0)\right)e^{-\gamma|\xi|^2t}+a(\xi,0)e^{-\gamma|\xi|^2t}\\=&-i\frac{\left(1+c^2|\xi|^2\right)}{|\xi|}\cdot\left(\frac{e^{\lambda_+t}-e^{\lambda_-t}}{\lambda_+-\lambda_-}\right)\cdot\hat{\rho}_0+\frac{1}{|\xi|}\left(\frac{\lambda_+e^{\lambda_+t}-\lambda_-e^{\lambda_-t}}{\lambda_+-\lambda_-}\right)\xi^\tau\hat{m}_0.\end{aligned}\tag{2.12}$$

由 (2.7), (2.12) 得

$$\begin{aligned} b(\xi,0) &= \hat{m}_0(\xi) - a(\xi,0)\frac{\xi}{|\xi|} = \hat{m}_0(\xi) - \frac{1}{|\xi|^2}\left(\frac{\lambda_+}{\lambda_+-\lambda_-} - \frac{\lambda_-}{\lambda_+-\lambda_-}\right)\xi^\tau \hat{m}_0 \xi \\ &= \hat{m}_0(\xi) - \frac{\xi\xi^\tau}{|\xi|^2}\hat{m}_0. \end{aligned} \tag{2.13}$$

由 (2.8), (2.13) 得

$$b(\xi,t) = e^{-\mu_1|\xi|^2 t}\left(I - \frac{\xi\xi^\tau}{|\xi|^2}\right)\hat{m}_0, \tag{2.14}$$

其中 I 为单位矩阵.

由 (2.7), (2.12), (2.14) 得

$$\begin{aligned} \hat{m}(\xi,t) =& -i\frac{\left(1+c^2|\xi|^2\right)\xi}{|\xi|^2}\hat{\rho}_0\left(\frac{e^{\lambda_+ t}}{\lambda_+-\lambda_-} - \frac{e^{\lambda_- t}}{\lambda_+-\lambda_-}\right) \\ &+ \frac{1}{|\xi|^2}\left(\frac{\lambda_+ e^{\lambda_+ t}}{\lambda_+-\lambda_-} - \frac{\lambda_- e^{\lambda_- t}}{\lambda_+-\lambda_-}\right)\xi\xi^\tau \hat{m}_0 + e^{-\mu_1|\xi|^2 t}\left(I - \frac{\xi\xi^\tau}{|\xi|^2}\right). \end{aligned} \tag{2.15}$$

若记

$$\begin{pmatrix} \hat{\rho}(\xi,t) \\ \hat{m}(\xi,t) \end{pmatrix} = \hat{G}(\xi,t)\begin{pmatrix} \hat{\rho}_0 \\ \hat{m}_0 \end{pmatrix}, \tag{2.16}$$

则由 (2.4), (2.15) 得

$$\hat{G}(\xi,t) = \begin{pmatrix} \frac{-\lambda_- e^{\lambda_+ t}}{\lambda_+-\lambda_-} + \frac{\lambda_+ e^{\lambda_- t}}{\lambda_+-\lambda_-} & \frac{e^{\lambda_+ t}-e^{\lambda_- t}}{\lambda_+-\lambda_-}(-i\xi^\tau) \\ -i\frac{\left(1+c^2|\xi|^2\right)\xi}{|\xi|^2}\left(\frac{e^{\lambda_+ t}-e^{\lambda_- t}}{\lambda_+-\lambda_-}\right) & \frac{\lambda_+ e^{\lambda_+ t}-\lambda_- e^{\lambda_- t}}{\lambda_+-\lambda_-}\frac{\xi\xi^\tau}{|\xi|^2} + e^{-\mu_1|\xi|^2 t}\left(I - \frac{\xi\xi^\tau}{|\xi|^2}\right) \end{pmatrix}. \tag{2.17}$$

我们称 $G(x,t)$ 为方程 (2.2) 的格林函数.

3 解的逐点估计

由 (2.16) 可得

$$\begin{pmatrix} \rho \\ m \end{pmatrix} = G * \begin{pmatrix} \rho_0 \\ m_0 \end{pmatrix}. \tag{3.1}$$

我们希望通过对 G 的估计及 (3.1) 式来得到方程 (2.2) 的解的逐点估计.

为方便，我们记

$$\hat{G}(\xi,t) = \begin{pmatrix} \hat{G}_{11} & \hat{G}_{12} \\ \hat{G}_{21} & \hat{G}_{22} \end{pmatrix}. \tag{3.2}$$

因为 λ_+ , λ_- 在高、低, 中频时，有不同的表现形式，所以我们定义光滑截断函数

$$\chi_1(\xi) = \begin{cases} 1, |\xi| \le \varepsilon, \\ 0, |\xi| > 2\varepsilon. \end{cases}, \quad \chi_3(\xi) = \begin{cases} 1, |\xi| \ge R, \\ 0, |\xi| < R-1. \end{cases}$$

其中 ε 充分小, R 充分大, $2\varepsilon<R$. 记 $\chi_2(\xi)=1-\chi_1(\xi)-\chi_3(\xi)$. $\chi_i(D)\,(i=1,2,3)$ 表示象征为 $\chi_i(\xi)\,(i=1,2,3)$ 的算子. 符号 $B_N(x,t)$ 表示 $B_N(x,t)=\left(1+\frac{|x|^2}{1+t}\right)^{-N}$.

定理 3.1 对任意正常数 N, 多重指标 α, 存在仅依赖于 N, α 的常数 $C_{N,\alpha}$, 有

$$\left|\partial_x^\alpha \chi_1(D)\,G_{11}\right| \le C_{N,\alpha}(1+t)^{-\frac{2+|\alpha|}{2}}B_N(x,t), \tag{3.3}$$

$$\left|\partial_x^\alpha \chi_1(D)\,G_{12}\right| \le C_{N,\alpha}(1+t)^{-\frac{3+|\alpha|}{2}}B_N(x,t), \tag{3.4}$$

$$\left|\partial_x^\alpha F^{-1}\left(\chi_1(\xi)\cdot\frac{e^{\lambda_+t}-e^{\lambda_-t}}{\lambda_+-\lambda_-}\right)\right| \le C_{N,\alpha}(1+t)^{-\frac{2+|\alpha|}{2}}B_N(x,t), \tag{3.5}$$

$$\left|\partial_x^\alpha F^{-1}\left(\chi_1(\xi)\cdot\frac{\lambda_+e^{\lambda_+t}-\lambda_-e^{\lambda_-t}}{\lambda_+-\lambda_-}\right)\right| \le C_{N,\alpha}(1+t)^{-\frac{2+|\alpha|}{2}}B_N(x,t). \tag{3.6}$$

证 当 $|\xi|$ 充分小时，由 $Taylor$ 展开有

$$\begin{aligned}\sqrt{\gamma^2|\xi|^4-4\left(c^2|\xi|^2+1\right)}&=2i+ic^2|\xi|^2+o\left(|\xi|^2\right),\\ \frac{1}{\sqrt{\gamma^2|\xi|^4-4\left(c^2|\xi|^2+1\right)}}&=\frac{1}{2i}+\frac{i}{4}c^2|\xi|^2+o\left(|\xi|^2\right).\end{aligned}\tag{3.7}$$

所以

$$\lambda_+=i+\frac{i}{2}c^2|\xi|^2-\frac{\gamma}{2}|\xi|^2+o\left(|\xi|^2\right), \tag{3.8}$$

$$\lambda_-=-i-\frac{i}{2}c^2|\xi|^2-\frac{\gamma}{2}|\xi|^2+o\left(|\xi|^2\right), \tag{3.9}$$

$$\frac{\lambda_+}{\lambda_+-\lambda_-}=\frac{1}{2}+\frac{i}{4}\gamma|\xi|^2+o\left(|\xi|^2\right), \tag{3.10}$$

$$\frac{\lambda_-}{\lambda_+-\lambda_-}=-\frac{1}{2}+\frac{i}{4}\gamma|\xi|^2+o\left(|\xi|^2\right). \tag{3.11}$$

于是对任意多重指标 α, β, 存在正常数 b, C, 有

$$\left|\partial_\xi^\beta\left(\xi^\alpha e^{\lambda_+t}\right)\right|\le C\left(|\xi|^{(|\alpha|-|\beta|)_+}+|\xi|^{|\alpha|}t^{\frac{|\beta|}{2}}\right)\left(1+|\xi|^2t\right)^{|\beta|+1}e^{-b|\xi|^2t},$$

$$\left|\partial_\xi^\beta\left(\xi^\alpha e^{\lambda_-t}\right)\right|\le C\left(|\xi|^{(|\alpha|-|\beta|)_+}+|\xi|^{|\alpha|}t^{\frac{|\beta|}{2}}\right)\left(1+|\xi|^2t\right)^{|\beta|+1}e^{-b|\xi|^2t},$$

所以

$$\left|\partial_\xi^\beta\left(\xi^\alpha\left(\frac{\lambda_+e^{\lambda_-t}-\lambda_-e^{\lambda_+t}}{\lambda_+-\lambda_-}\right)\right)\right|\le C\left(|\xi|^{(|\alpha|+|\beta|)_+}+|\xi|^{|\alpha|}t^{\frac{|\beta|}{2}}\right)\left(1+|\xi|^2t\right)^{|\beta|+1}e^{-b|\xi|^2t},$$

$$\left|\partial_\xi^\beta\left(\xi^\alpha\left(\frac{e^{\lambda_+t}-e^{\lambda_-t}}{\lambda_+-\lambda_-}\right)\right)\right|\le C\left(|\xi|^{(|\alpha|-|\beta|)_+}+|\xi|^{|\alpha|}t^{\frac{|\beta|}{2}}\right)\left(1+|\xi|^2t\right)^{|\beta|+1}e^{-b|\xi|^2t},$$

$$\left|\partial_\xi^\beta\left(\xi^\alpha\left(\frac{\lambda_+e^{\lambda_+t}-\lambda_-e^{\lambda_-t}}{\lambda_+-\lambda_-}\right)\right)\right|\le C\left(|\xi|^{(|\alpha|-|\beta|)_+}+|\xi|^{|\alpha|}t^{\frac{|\beta|}{2}}\right)\left(1+|\xi|^2t\right)^{|\beta|+1}e^{-b|\xi|^2t}.$$

由 [11] 的引理 3.1, (2.17), 本定理得证.

因为 $\hat{G}_{21}$, $\hat{G}_{22}$ 中有非局部算子 $\frac{1}{|\xi|^2}$, 所以我们需要用新的技巧来估计它们.

定理 3.2 存在正常数 C, 有

$$\left|\partial_x^\alpha \chi_1(D) G_{21}\right| \le C(1+t)^{-\frac{1+|\alpha|}{2}} B_{\frac{1}{2}}(x,t), \tag{3.12}$$

$$\left|\partial_x^\alpha \chi_1(D) G_{22}\right| \le C(1+t)^{-\frac{2+|\alpha|}{2}} B_{\frac{1}{2}}(x,t). \tag{3.13}$$

证 把 $|\partial_x^\alpha \chi_1(D) G_{21}|$ 估计分成两部分,

$$|\partial_x^\alpha \chi_1(D) G_{21}| \le |\partial_x^\alpha \chi_1(D) G_{21}|_{\left\{x||x|^2\le 1+t\right\}} + |\partial_x^\alpha \chi_1(D) G_{21}|_{\left\{x||x|^2> 1+t\right\}} := I_1 + I_2.$$

由 (2.17), (3.2) (3.7), (3.8), (3.9) 得

$$\begin{aligned} I_1 &\le \int \left|\xi^\alpha \chi_1(\xi)\hat{G}_{21}\right| d\xi \le C\int |\xi|^{|\alpha|-1} e^{-\frac{\gamma|\xi|^2 t}{4}} d\xi \le C_\alpha (1+t)^{-\frac{|\alpha|+1}{2}} \\ &\le C_{N,\alpha}(1+t)^{-\frac{|\alpha|+1}{2}} B_N(x,t). \end{aligned}$$

当 $N>1$ 时,

$$\begin{aligned} \int \left(1+\frac{|x-y|^2}{1+t}\right)^{-N} dy &= \int_0^{2\pi} d\theta \int_0^{+\infty} \left(1+\frac{r^2}{1+t}\right)^{-N} r dr \\ &= \int_0^{2\pi} d\theta \int_0^{+\infty} \left(1+r^2\right)^{-N} \sqrt{1+tr}\cdot\sqrt{1+t} dr \le C(1+t). \end{aligned} \tag{3.14}$$

当 $N>\frac{1}{2}$ 时,

$$\begin{aligned} \int |y|^{-1}\left(1+\frac{|y|^2}{1+t}\right)^{-N} dy &= \int_0^{2\pi} d\theta \int_0^{+\infty} \left(1+\frac{r^2}{1+t}\right)^{-N} \frac{1}{r}\cdot r dr \\ &= 2\pi \int_0^{+\infty} \left(1+r^2\right)^{-N} \sqrt{1+t} dr \le C(1+t)^{\frac{1}{2}}. \end{aligned} \tag{3.15}$$

当 $|y| \le \frac{|x|}{2}$ 时，有

$$|y-x| \ge |x| - |y| \ge \frac{|x|}{2} \ge |y|. \tag{3.16}$$

又

$$F^{-1}\left(\frac{i\xi_j}{|\xi|^2}\right) = -\partial_{x_j}\Delta^{-1}\delta(x) = C\partial_{x_j} ln|x| = C\frac{x_j}{|x|^2}, \tag{3.17}$$

取 (3.5) 中 $N>2$, 则由 (3.5), (3.14), (3.15), (3.16), (3.17) 得

$$\begin{aligned}
I_2 \leq & \int_{\Re^2} C\frac{|y_j|}{|y|^2}\cdot(1+t)^{-\frac{2+|\alpha|}{2}}B_N(x-y,t)\,dy \\
\leq & C\int_{\Re^2\cap\left\{y||y|\geq\frac{|x|}{2}\right\}}\frac{1}{|y|}(1+t)^{-\frac{2+|\alpha|}{2}}B_N(x-y,t)\,dy \\
& +C\int_{\Re^2\cap\left\{y||y|\leq\frac{|x|}{2}\right\}}\frac{1}{|y|}(1+t)^{-\frac{2+|\alpha|}{2}}B_N(x-y,t)\,dy \\
\leq & C\left(|x|^2\right)^{-\frac{1}{2}}(1+t)^{-\frac{2+|\alpha|}{2}}\cdot(1+t)+C(1+t)^{-\frac{2+|\alpha|}{2}}(1+t)^{\frac{1}{2}}B_{\frac{1}{2}}(x,t) \\
\leq & C(1+t)^{-\frac{|\alpha|}{2}}(1+t)^{-\frac{1}{2}}B_{\frac{1}{2}}(x,t)+C(1+t)^{-\frac{1+|\alpha|}{2}}B_{\frac{1}{2}}(x,t) \\
= & C(1+t)^{-\frac{1+|\alpha|}{2}}B_{\frac{1}{2}}(x,t).
\end{aligned} \tag{3.18}$$

由 I_1, I_2 的估计, 得到 (3.12). 用同样的方法可得到 (3.13).

对中频的部分，我们有下面定理.

定理 3.3 对任意正整数 N, 存在正常数 C, b, 有 $\left|\partial_x^\alpha\chi_2(D)G(x,t)\right|\leq Ce^{-bt}B_N(x,t)$.

证 因为

$$\frac{e^{\lambda_+t}-e^{\lambda_-t}}{\lambda_+-\lambda_-}=e^{\lambda_-t}\cdot\frac{e^{(\lambda_+-\lambda_-)t}-1}{\lambda_+-\lambda_-},$$

$$\begin{aligned}
\frac{\lambda_+e^{\lambda_-t}-\lambda_-e^{\lambda_+t}}{\lambda_+-\lambda_-}= & \frac{\lambda_+e^{\lambda_+t}\left(e^{(\lambda_--\lambda_+)t}-1\right)-\lambda_-e^{\lambda_-t}\left(e^{(\lambda_+-\lambda_-)t}-1\right)}{\lambda_+-\lambda_-} \\
& -\frac{\lambda_+e^{\lambda_-t}\left(e^{(\lambda_+-\lambda_-)t}-1\right)}{\lambda_+-\lambda_-}+e^{\lambda_-t},
\end{aligned}$$

所以当 $\varepsilon\leq|\xi|\leq R$ 时, $\hat{G}$ 无奇点, 无需考虑对 ξ 的可积性问题, 又 $|Re\lambda_\pm|\leq -b$, 所以 $\left|\partial_x^\alpha\chi_2(D)G(x,t)\right|\leq\int\xi^\alpha\chi_2(\xi)\hat{G}(\xi,t)\,d\xi\leq Ce^{-bt}B_N(x,t)$.

定理 3.3 得证.

当 $|\xi|$ 充分大时, 由泰勒展开有

$$\lambda_+-\lambda_-=\gamma|\xi|^2\cdot\sqrt{1-4\frac{c^2}{\gamma^2}\frac{1}{|\xi|^2}-\frac{4}{\gamma^2|\xi|^4}}=\gamma|\xi|^2-\frac{2c^2}{\gamma}-\frac{2}{|\xi|^2}-\frac{2c^4}{\gamma^3}\frac{1}{|\xi|^2}+o\left(\frac{1}{|\xi|^2}\right),$$

$$\frac{1}{\lambda_+-\lambda_-}=\frac{1}{\gamma|\xi|^2}+\frac{2}{\gamma^3|\xi|^4}+o\left(\frac{1}{|\xi|^4}\right),$$

所以

$$\begin{aligned}
\frac{\lambda_+e^{\lambda_-t}-\lambda_-e^{\lambda_+t}}{\lambda_+-\lambda_-}= & \left(-\frac{1}{\gamma^2|\xi|^2}+o\left(\frac{1}{|\xi|^2}\right)\right)e^{\left(-\gamma|\xi|^2+\frac{c^2}{\gamma}+o\left(\frac{1}{|\xi|}\right)\right)t} \\
& +\left(1+\frac{1}{\gamma^2|\xi|^2}+o\left(\frac{1}{|\xi|^2}\right)\right)e^{-\frac{c^2}{\gamma}t+o\left(\frac{1}{|\xi|}\right)t},
\end{aligned} \tag{3.19}$$

$$\frac{e^{\lambda_-t}-e^{\lambda_+t}}{\lambda_+-\lambda_-}=\left(\frac{1}{\gamma|\xi|^2}+o\left(\frac{1}{|\xi|^3}\right)\right)\cdot\left(e^{\left(-\gamma|\xi|^2+\frac{c^2}{\gamma}+o\left(\frac{1}{|\xi|}\right)\right)t}-e^{\left(-\frac{c^2}{\gamma}+o\left(\frac{1}{|\xi|}\right)t\right)}\right), \tag{3.20}$$

$$\frac{\lambda_+ e^{\lambda_+ t} - \lambda_- e^{\lambda_- t}}{\lambda_+ - \lambda_-} = \left(-\frac{1}{\gamma^2 |\xi|^2} + o\left(\frac{1}{|\xi|^2}\right)\right) e^{\left(-\frac{c^2}{\gamma} + o\left(\frac{1}{|\xi|}\right)\right)t} + \left(1 + \frac{1}{\gamma^2}\frac{1}{|\xi|^2} + o\left(\frac{1}{|\xi|^2}\right)\right) e^{\left(-\gamma|\xi|^2 + \frac{c^2}{\gamma} + o\left(\frac{1}{|\xi|}\right)\right)t}. \tag{3.21}$$

所以高频时, 若 $|\alpha|$ 大, $\xi^\alpha \hat{G}$ 关于 ξ 是不可积分的, 只有 (2.2) 的初值有光滑性时, 才能得到 (2.2) 解的光滑性. 这就是下面的定理.

定理 3.4 当 $\rho_0 \in H^{l+s}$, $m_0 \in H^{l+s}$, $s > 1$, $max\left\{\|\rho_0\|_{H^{l+s}}, \|m_0\|_{H^{l+s}}\right\} = E$, 当 $|\alpha| \le l$ 时, 对任意正整数 N, 存在正常数 C_N, b, 有 $|\partial_x^\alpha \chi_3(D)\rho| + |\partial_x^\alpha \chi_3(D)m| \le CEe^{-bt}B_N(x,t)$.

证 由 (3.1), (3.19), (3.20), 当 $|\alpha| \le l$ 时, 有

$$\begin{aligned}
\left|x^\beta \partial_x^\alpha \chi_3(D)\rho\right| &\le \left|\int_{|\xi|\ge R} \partial_\xi^\beta \left(|\xi|^\alpha \hat{\rho}\right) d\xi\right| \le \int_{|\xi|\ge R} |\xi|^{|\alpha|-|\beta|}\left(\hat{G}_{11}\hat{\rho}_0 + \hat{G}_{12}\hat{m}_0\right) d\xi \\
&\le Ce^{-bt}\int_{|\xi|\ge R} |\xi|^{|\alpha|-|\beta|}\cdot\hat{\rho}_0 + |\xi|^{|\alpha|-1-|\beta|}\hat{m}_0 d\xi \le Ce^{-bt}\int_{|\xi|\ge R}|\xi|^{|\alpha|}\cdot\hat{\rho}_0 + |\xi|^{|\alpha|-1}\hat{m}_0 d\xi \\
&\le Ce^{-bt}\left(\left\||\xi|^{-s}\right\|_{L^2(|\xi|\ge R)}\cdot\left\||\xi|^{|\alpha|+s}\cdot\hat{\rho}_0\right\|_{L^2} + \left\||\xi|^{-s}\right\|_{L^2(|\xi|\ge R)}\cdot\left\||\xi|^{|\alpha|+s}\cdot\hat{m}_0\right\|_{L^2}\right) \\
&\le Ce^{-bt}\left(\|\rho_0\|_{H^{l+s}} + \|m_0\|_{H^{l+s}}\right).
\end{aligned}$$

由 (3.1), (3.19), (3.20), 当 $|\alpha| < l$, 有

$$\begin{aligned}
\left|x^\beta \partial_x^\alpha \chi_3(D)m\right| &\le \int_{|\xi|\ge R}\partial_\xi^\beta\left(\xi^\alpha \hat{m}\right)d\xi \le Ce^{-bt}\int_{|\xi|\ge R}|\xi|^{|\alpha|-|\beta|-3}\cdot\hat{\rho}_0 + |\xi|^{|\alpha|-|\beta|}\hat{m}_0 d\xi \\
&\le Ce^{-bt}\left(\left\||\xi|^{-2}\right\|_{L^2(|\xi|\ge R)}\left\||\xi|\cdot\hat{\rho}_0\right\|_{L^2} + \left\||\xi|^{-s}\right\|_{L^2(|\xi|\ge R)}\left\||\xi|^{|\alpha|+s}\cdot\hat{m}_0\right\|_{L^2}\right) \\
&\le Ce^{-bt}E.
\end{aligned}$$

所以 $|\partial_x^\alpha \rho| \le CEe^{-bt}B_N(x,t)$, $|\partial_x^\alpha m| \le CEe^{-bt}B_N(x,t)$.

为得到解的逐点估计, 还需要下述两个引理.

引理 3.1 当 N, $s > 1$, 且 $N > s$, 有 $\int B_N(x-y,t)\left(1+|y|^2\right)^{-s}dy \le CB_s(x,t)$.

证 当 $|x|^2 \le 1+t$, 有

$$\int B_N(x-y,t)\left(1+|y|^2\right)^{-s}dy \le \int\left(1+|y|^2\right)^{-s}dy \le C \le CB_s(x,t).$$

当 $|x|^2 > 1+t$, $|y| \le \frac{|x|}{2}$, 有 $|x-y| \ge |x|-|y| \ge \frac{|x|}{2}$, 所以

$$\int B_N(x-y,t)\left(1+|y|^2\right)^{-s}dy \le B_N(x,t)\int\left(1+|y|^2\right)^{-s}dy \le CB_s(x,t).$$

当 $|x|^2 > 1+t$, $|y| > \frac{|x|}{2}$, 由 (3.14), 有

$$\begin{aligned}
\int B_N(x-y,t)\left(1+|y|^2\right)^{-s}dy &\le C(1+t)\left(1+|x|^2\right)^{-s} \\
&\le C\frac{1}{\left(\frac{1+|x|^2}{1+t}\right)^s} \le C\frac{1}{\left(\frac{|x|^2}{1+t}\right)^s} \le CB_s(x,t).
\end{aligned}$$

引理 3.1 得证.

引理 3.2 当 $s>1$, 有 $\int B_{\frac{1}{2}}(x-y,t)\left(1+|y|^2\right)^{-s}dy\le CB_{\frac{1}{2}}(x,t)$.

证 当 $|x|^2\le 1+t$, 有 $\int B_{\frac{1}{2}}(x-y,t)\left(1+|y|^2\right)^{-s}dy\le C\le CB_{\frac{1}{2}}(x,t)$.

当 $|x|^2>1+t$, $|y|\le\frac{|x|}{2}$, 有 $|x-y|\ge|x|-|y|\ge\frac{|x|}{2}$. 于是

$$\int B_{\frac{1}{2}}(x-y,t)\left(1+|y|^2\right)^{-s}dy\le CB_{\frac{1}{2}}(x,t)\int\left(1+|y|^2\right)^{-s}dy\le CB_{\frac{1}{2}}(x,t).$$

当 $|x|^2>1+t,|y|>\frac{|x|}{2}$, 有 $|x-y|\le|x|+|y|\le 3|y|$, 于是

$$\begin{aligned}\int B_{\frac{1}{2}}(x-y,t)\left(1+|y|^2\right)^{-s}dy&\le\left(1+|x|^2\right)^{-\frac{1}{2}}\int B_{\frac{1}{2}}(x-y,t)\left(1+\frac{|x-y|^2}{3}\right)^{\frac{1}{2}-s}dy\\&\le C\left(|x|^2\right)^{-\frac{1}{2}}\int_0^{+\infty}\left(1+r^2\right)^{-\frac{1}{2}}(1+t)^{\left(\frac{1}{2}-s\right)}r^{1-2s}\cdot r(1+t)^{\frac{1}{2}}dr\\&\le C\left(|x|^2\right)^{-\frac{1}{2}}(1+t)^{(1-s)}\int_0^{+\infty}\left(1+r^2\right)^{-\frac{1}{2}}r^{2-2s}dr\\&\le C\left(|x|^2\right)^{-\frac{1}{2}}\le CB_{\frac{1}{2}}(x,t).\end{aligned}$$

引理 3.2 得证.

由定理 3.1、定理 3.2、定理 3.3、引理 3.1、引理 3.2、式 (3.1) 得如下定理.

定理 3.5 当初值 $|\rho_0|+|m_0|\le C\left(1+|x|^2\right)^{-s}$, $s>1$, 有

$$|\partial_x^\alpha(1-\chi_3(D))\rho|\le C_{N,\alpha}(1+t)^{-\frac{2+|\alpha|}{2}}B_s(x,t),$$

$$|\partial_x^\alpha(1-\chi_3(D))m|\le C_{N,\alpha}(1+t)^{-\frac{1+|\alpha|}{2}}B_{\frac{1}{2}}(x,t).$$

综合定理 3.4、定理 3.5、式 (3.1), 得到本文主要结论.

定理 3.6 当初值 $|\rho_0|+|m_0|\le C\left(1+|x|^2\right)^{-s_1}$, $s_1>1$, 且 $\rho_0\in H^{l+s_2}$, $m_0\in H^{l+s_2}$, $\max\{\|\rho_0\|_{H^{l+s_2}},\quad\|m_0\|_{H^{l+s_2}}\}\le E$, $s_2>1$, 当 $|\alpha|\le l$, 有

$$|\partial_x^\alpha\rho|\le C(1+t)^{-\frac{2+|\alpha|}{2}}B_{s_1}(x,t)\quad,\quad|\partial_x^\alpha m|\le C(1+t)^{-\frac{1+|\alpha|}{2}}B_{\frac{1}{2}}(x,t).$$

注: 由 (3.14) 及定理 3.6, 知 $\|m\|_{L^2}$ 关于时间 t 不衰减, 这是非线性方程组 (1.1) 存在性做不下去的重要原因之一.

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POINTWISE ESTIMATE OF SOLUTION TO LINEARIZED NAVIER-STOKES-POISSON SYSTEM IN TWO SPACE DIMENSION

XU Hong-mei, XIAO Lian-hui

(*school of mathematics, Hohai university, Nanjing 211100, China*)

Abstract: Cauchy problem of linearized Navier-Stokes-Poisson system in two dimensional space is considered. Through changing the system into several equations for single function, we solved the function and got the Green function for the system. Using detailed analysis of the Green function, we got pointwise estimation of the solution . The result shows electron fluid density decays as fast as heat kernel, but momentum decays slower, even though its L2 norm does not decay.

Keywords: Navier-Stokes-Poisson system; two-dimensional space; Green function; pointwise estimate

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