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COMMON COUPLED FIXED POINT THEOREMS FOR CONTRACTIVE MAPPINGS OF MANY VARIABLES IN FUZZY METRIC SPACES

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Abstract: In this paper, we propose a notion of coincidence point between mappings in any number of variables. The main results of this paper are generalizations of the main results of fixed point theorems in partially ordered fuzzy metric spaces from low dimension to high dimension.

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1 Introduction

Since Zadeh[1] introduced the concept of fuzzy sets, many authors have extensively developed the theory of fuzzy sets and applications. George and Veeramani [2, 3] gave the concept of fuzzy metric space and defined a Hausdorff topology on this fuzzy metric space which have very important applications in quantum particle physics particularly in connection with both string and E-infinity theory.

The notion of coupled fixed points was introduced by Guo and Lakshmikantham [4] in 1987. In a recent paper, Gnana-Bhaskar and Lakshmikantham [5] introduced the concept of mixed monotone property for contractive operators of the form $F : X \times X \to X$, where X is a partially ordered metric space, and the established some coupled fixed point theorems. Lakshmikantham and Ćirić [6] discussed the mixed monotone mappings and gave some coupled fixed point theorems which can be used to discuss the existence and uniqueness of solution for a periodic boundary value problem.

Shaban Sedghi et al [7] gave a coupled fixed point theorem for contractions in fuzzy metric spaces, and Jin-xuan Fang [8] gave some common fixed point theorems under ϕ-contractions for compatible and weakly compatible mappings in Menger probabilistic metric spaces. Xin-Qi Hu [9] proved a common fixed point theorem for mappings under φ-contractive conditions in fuzzy metric spaces. B.S.Choudury et. al. [10] established coupled coincidence point and coupled fixed point results for compatible mappings in partially ordered fuzzy metric spaces and gave an example to illustrate the main theorems. In 2015,

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Jinxuan-Fang [11] generlized a crucial fixed point theorem for probabilistic φ-contraction on complete Menger space. Other more works on this topic can be found in [12-23].

Now we propose a notion of coincidence point between mappings cases of these results that are already known under some contractive conditions.

2 Mathematical Preliminaries

First we give some definitions.

Definition 2.1 (see [2]) A binary operation $* : [0,1]\times[0,1]\to[0,1]$ is continuous t-norm if $*$ satisfies the following conditions:

(1) $*$ is commutative and associative;

(2) $*$ is continuous;

(3) $a*1=a$ for all $a\in[0,1]$;

(4) $a*b\le c*d$ whenever $a\le c$ and $b\le d$ for all $a,b,c,d\in[0,1]$.

Definition 2.2 (see [7]) Let $\sup\limits_{0<t<1}\Delta(t,t)=1$. A t-norm Δ is said to be of H-type if the family of functions $\{\Delta^m(t)\}_{m=1}^{\infty}$ is equicontinuous at $t=1$, where

$$\Delta^1(t)=t\Delta t,\quad \Delta^{m+1}(t)=t\Delta(\Delta^m(t)), m=1,2,\cdots,t\in[0,1].$$

The $t-$norm $\Delta_M=\min$ is an example of t-norm of H-type, but there are some other t-norms Δ of H-type.

Obviously, Δ is a H-type t norm if and only if for any $\lambda\in(0,1)$, there exists $\delta(\lambda)\in(0,1)$ such that $\Delta^m(t)>1-\lambda$ for all $m\in\mathbb{N}$, when $t>1-\delta$.

Definition 2.3 (see [2]) A 3-tuple $(X,M,*)$ is said to be a fuzzy metric space if X is an arbitrary nonempty set, $*$ is a continuous t-norm and M is a fuzzy set on $X^2\times(0,+\infty)$ satisfying the following conditions, for each $x,y,z\in X$ and $t,s>0$,

(FM–1) $M(x,y,t)>0$;

(FM–2) $M(x,y,t)=1$ if and only if $x=y$;

(FM–3) $M(x,y,t)=M(y,x,t)$;

(FM–4) $M(x,y,t)*M(y,z,s)\le M(x,z,t+s)$;

(FM–5) $M(x,y,\cdot):(0,\infty)\to[0,1]$ is continuous.

Let $(X,M,*)$ be a fuzzy metric space. For $t>0$, the open ball $B(x,r,t)$ with a center $x\in X$ and a radius $0<r<1$ is defined by $B(x,r,t)=\{y\in X: M(x,y,t)>1-r\}$.

A subset $A\subset X$ is called open if for each $x\in A$, there exist $t>0$ and $0<r<1$ such that $B(x,r,t)\subset A$. Let τ denote the family of all open subsets of X. Then τ is called the topology on X induced by the fuzzy metric M. This topology is Hausdorff and first countable.

Example 2.4 Let (X,d) be a metric space. Define t-norm $a*b=ab$ and for all $x,y\in X$ and $t>0$, $M(x,y,t)=\frac{t}{t+d(x,y)}$. Then $(X,M,*)$ is a fuzzy metric space. We call this fuzzy metric M induced by the metric d the standard fuzzy metric.

Let n be a positive integer. X will benote a non-empty set and X^n denote the product space $X^n = \underbrace{X \times X \times \cdots \times X}_{n}$.

Definition 2.5 (see [6]) Let X be a non-empty set, $F : X \to X$ and $g : X \to X$ be two mappings. We say F and g are commutative (or that F and g commute) if $gFx = Fgx$ for all $x \in X$.

Definition 2.6 (see [6]) The mappings F and g where $F : X \to X$ and $g : X \to X$, are said to be compatible if $\lim_{n\to\infty} d(Fgx_n, gFx_n) = 0$ whenever $\{x_n\}$ is a sequence in X, such that $\lim_{n\to\infty} F(x_n) = \lim_{n\to\infty} g(x_n) = x$ for all $x \in X$ are satisfied.

Definition 2.7 (see [6]) Two mappings F and g on a metric space (X, d) are said to be weakly compatible if they commute at their coincidence points, that is, if $Fx = gx$ for some $x \in X$, then $Fgx = gFx$.

Let $\Lambda_n = \{1, 2, \cdots, n\}$, A, B satisfy that $A \cup B = \Lambda_n$ and $A \cap B = \emptyset$. We will denote $\Omega_{A,B} = \{\sigma : \Lambda_n \to \Lambda_n,\ \sigma(A) \subseteq A$ and $\sigma(B) \subseteq B\}$, and $\Omega'_{A,B} = \{\sigma : \Lambda_n \to \Lambda_n,\ \sigma(A) \subseteq B$ and $\sigma(B) \subseteq A\}$.

Let $(X, \leq)$ be a partially ordered space, $x, y \in X$ and $i \in \Lambda_n$. We use the following notation

$$x \leq_i y \Leftrightarrow \begin{cases} x \leq y, i \in A, \\ x \geq y, i \in B. \end{cases}$$

Let $\sigma_1, \sigma_2, \cdots, \sigma_n, \tau : \Lambda_n \to \Lambda_n$ be $n+1$ mappings and let Φ be the $(n+1)$-tuple $(\sigma_1, \sigma_2, \cdots, \sigma_n, \tau)$.

Definition 2.8 (see [13]) Let $F : X^n \to X$, $g : X \to X$. A point $(x_1, x_2, \cdots, x_n) \in X^n$ is called a Φ-coincidence point of the mappings F and g if

$$F(x_{\sigma_i(1)}, x_{\sigma_i(2)}, \cdots, x_{\sigma_i(n)}) = gx_{\tau(i)} \text{ for all } i \in \Lambda_n.$$

If g is the identity mapping on X, then $(x_1, x_2, \cdots, x_n) \in X^n$ is called a Φ-fixed point of the mapping F.

Definition 2.9 Let $(X, \leq)$ be a partially ordered space. We say that F has the mixed g-monotone property if F is g-monotone non-decreasing in argument of A and g-monotone non-increasing in argument of B, i.e., for all $x_1, x_2, \cdots, x_n, y, z \in X$ and all i,

$$gy \leq gz \Rightarrow F(x_1, \cdots, x_{i-1}, y, x_{i+1}, \cdots, x_n) \leq_i F(x_1, \cdots, x_{i-1}, z, x_{i+1}, \cdots, x_n).$$

It is obvious that the above formula is equivalent to the following:

$$gy \leq_i gz \Rightarrow F(x_1, \cdots, x_{i-1}, y, x_{i+1}, \cdots, x_n) \leq F(x_1, \cdots, x_{i-1}, z, x_{i+1}, \cdots, x_n).$$

Definition 2.10 Let $F : X^n \to X$ and $g : X \to X$. F and g are called weakly compatible mappings if for $x_1, x_2, \cdots, x_n$, it satisfies

$$F(x_{\sigma_i(1)}, x_{\sigma_i(2)}, \cdots, x_{\sigma_i(n)}) = gx_{\tau(i)}, \text{ for all } i \in \Lambda_n,$$

it implies

$$F(gx_{\sigma_i(\tau(1))}, gx_{\sigma_i(\tau(2))}, \cdots, gx_{\sigma_i(\tau(n))}) = gF(x_{\sigma_{\tau(i)}(1)}, x_{\sigma_{\tau(i)}(2)}, \cdots, x_{\sigma_{\tau(i)}(n)}), \ \forall i \in \Lambda_n.$$

3 Main Results

Lemma 3.1 (see [23]) For $n \in N$, let $g_n : (0, +\infty) \to (0, +\infty)$ and $F_n : R \to [0, 1]$. Assume that $\sup\{F(t) : t > 0\} = 1$ and for any $t > 0$,

$$\lim_{n\to+\infty} g_n(t) = 0 \ and \ F_n(g_n(t)) \geq F(t).$$

If each F_n is nondecreasing, then $\lim_{n\to+\infty} F_n(t) = 1$ for any $t > 0$.

Theorem 3.2 (see [21]) Let (X, M, Δ) be a complete fuzzy metric space with Δ a triangular norm of H-type. Let $\varphi \in \Psi_\omega$, where Ψ_ω is denoted as the class of all function $\varphi : [0, +\infty) \to [0, +\infty)$ such that for each $t > 0$ there exists an $r_t \geq t$ satisfying $\lim_{n\to+\infty} \varphi^n(r_t) = 0$. Let $T : X \to X$ be a mapping, $M(Tx, Ty, \varphi(t)) \geq M(x, y, t)$ for all $x, y \in X$ and all $t > 0$. Then T has a unique fixed point x^*. In fact, for any $x_0 \in X$, $\lim_{n\to+\infty} T^n x_0 = x^*$.

Theorem 3.3 Let $(Y^*, M^*, \Delta, \preccurlyeq)$ be a complete ordered fuzzy metric space with Δ a triangular norm of H-type. Let $\varphi : [0, +\infty) \to [0, +\infty)$, $\varphi \in \Psi_\omega$, and also suppose $\widetilde{F}, \widetilde{g} : Y \to Y$ are such that $\widetilde{F}(Y) \subseteq \widetilde{g}(Y)$, $\widetilde{g}$ is continuous and $\widetilde{g}(Y)$ is complete, $\widetilde{F}$ and $\widetilde{g}$ be weakly compatible, $\widetilde{F}$ has the mixed $\widetilde{g}$-monotone property, and $M^*(\widetilde{F}x, \widetilde{F}y, \varphi(t)) \geq M^*(\widetilde{g}x, \widetilde{g}y, t)$ for each $\widetilde{g}x \preccurlyeq \widetilde{g}y$. If there exists $x_0 \in Y$ such that $\widetilde{g}x_0 \preccurlyeq \widetilde{F}x_0$, then $\widetilde{F}$ and $\widetilde{g}$ has a fixed point.

Proof Ψ is denoted as the class of all function $\varphi : [0, +\infty) \to [0, +\infty)$ be continuous with $\varphi(t) < t$ for each $t > 0$. Obviously, $\Psi \subseteq \Psi_\omega$. First we will prove Theorem 3.3 when $\varphi \in \Psi$.

From $\widetilde{F}(Y) \subseteq \widetilde{g}(Y)$, we can choose $x_1 \in Y$ such that $\widetilde{g}x_1 = \widetilde{F}x_0$. Again we can choose $x_2 \in Y$ such that $\widetilde{g}x_2 = \widetilde{F}x_1$. Continuing this process we can construct sequence x_n in Y such that $\widetilde{g}x_{n+1} = \widetilde{F}x_n$.

Using the mathematical induction and $\widetilde{F}$ has the mixed $\widetilde{g}$-monotone property, we get

$$\widetilde{g}x_0 \preccurlyeq \widetilde{g}x_1 \preccurlyeq \widetilde{g}x_2 \preccurlyeq ... \preccurlyeq \widetilde{g}x_n \preccurlyeq \widetilde{g}x_{n+1} \preccurlyeq ...$$

and

$$\widetilde{F}x_0 \preccurlyeq \widetilde{F}x_1 \preccurlyeq \widetilde{F}x_2 \preccurlyeq ... \preccurlyeq \widetilde{F}x_n \preccurlyeq \widetilde{F}x_{n+1} \preccurlyeq ...$$

By putting $x = x_{n-1}$, $y = x_n$ in $M^*(\widetilde{F}x, \widetilde{F}y, \varphi(t)) \geq M^*(\widetilde{g}x, \widetilde{g}y, t)$, we get

$$M^*(\widetilde{F}x_{n-1}, \widetilde{F}x_n, \varphi(t)) \geq M^*(\widetilde{g}x_{n-1}, \widetilde{g}x_n, t).$$

That means $M^*(\widetilde{g}x_n, \widetilde{g}x_{n+1}, \varphi(t)) \geq M^*(\widetilde{g}x_{n-1}, \widetilde{g}x_n, t)$, thus

$$M^*(\widetilde{g}x_n, \widetilde{g}x_{n+1}, \varphi^n(t)) \geq M^*(\widetilde{g}x_{n-1}, \widetilde{g}x_n, \varphi^n(t)).$$

By Lemma 3.1, we have

$$\lim_{n\to+\infty} M^*(\widetilde{g}x_n, \widetilde{g}x_{n+1}, t) = 1, \text{ for any } \ t > 0.$$

Now let $n \in N$ and $t > 0$, we show by induction that, for any $k \in \mathbb{N}$,

$$M^*(\widetilde{g}x_n, \widetilde{g}x_{n+k}, t) \geq \Delta^k(M^*(\widetilde{g}x_n, \widetilde{g}x_{n+1}, t - \varphi(t))).$$

This is obvious for $k = 0$. Assume it holds for some k, by the monotonicity of Δ, we have

$$\begin{aligned} & M^*(\widetilde{g}x_n, \widetilde{g}x_{n+k+1}, t) = M^*(\widetilde{g}x_n, \widetilde{g}x_{n+k+1}, t - \varphi(t) + \varphi(t)) \\ \geq\ & M^*(\widetilde{g}x_n, \widetilde{g}x_{n+1}, t - \varphi(t))\Delta M^*(\widetilde{g}x_{n+1}, \widetilde{g}x_{n+k+1}, \varphi(t)) \\ \geq\ & M^*(\widetilde{g}x_n, \widetilde{g}x_{n+1}, t - \varphi(t))\Delta M^*(\widetilde{g}x_n, \widetilde{g}x_{n+k}, t) \\ \geq\ & M^*(\widetilde{g}x_n, \widetilde{g}x_{n+1}, t - \varphi(t))\Delta(\Delta^k(M^*(\widetilde{g}x_n, \widetilde{g}x_{n+1}, t - \varphi(t)))) \\ =\ & \Delta^{k+1}(M^*(\widetilde{g}x_n, \widetilde{g}x_{n+1}, t - \varphi(t))), \end{aligned}$$

which completes the induction. By $\Delta^n(1) = 1$ and Δ is a triangular norm of H-type, for any $t > 0$ and $\varepsilon > 0$, there is $\delta > 0$ such that if $s \in (1 - \delta, 1]$, then $\Delta^n(s) > 1 - \varepsilon$ for all $n \in N$.

Since, by $\lim_{n\to+\infty} M^*(\widetilde{g}x_n, \widetilde{g}x_{n+1}, t - \varphi(t)) = 1$, there is $n_0 \in N$ such that, for any $n > n_0$, $M^*(\widetilde{g}x_n, \widetilde{g}x_{n+1}, t - \varphi(t)) \in (1 - \delta, 1]$. Hence, we get $M^*(\widetilde{g}x_n, \widetilde{g}x_{n+k}, t - \varphi(t)) > 1 - \varepsilon$ for any $k \in \mathbb{N}$. This proves the Cauchy condition for $\widetilde{g}x_n$.

Thus $\widetilde{g}x_n$ is a Cauchy sequence. Since $\widetilde{g}(Y)$ is complete, there exists $x \in Y$ such that $\lim_{n\to+\infty} \widetilde{g}x_n = x$. Similarly we get $\widetilde{F}x_n$ is a Cauchy sequence, such that $\lim_{n\to+\infty} \widetilde{F}x_n = \lim_{n\to+\infty} \widetilde{g}x_{n+1} = x = \widetilde{g}a$ (notice that $\widetilde{g}$ is continuous).

By putting $x = x_n$, $y = a$ in $M^*(\widetilde{F}x, \widetilde{F}y, \varphi(t)) \geq M^*(\widetilde{g}x, \widetilde{g}y, t)$, we get

$$M^*(\widetilde{F}x_n, \widetilde{F}a, \varphi(t)) \geq M^*(\widetilde{g}x_n, \widetilde{g}a, t).$$

Letting $n \to +\infty$, we get$M^*(\widetilde{g}a, \widetilde{F}a, \varphi(t)) = 1$, that means $\widetilde{g}a = \widetilde{F}a = x$.

By the condition that $\widetilde{F}$ and $\widetilde{g}$ be weakly compatible, we get $\widetilde{g}\widetilde{F}a = \widetilde{F}\widetilde{g}a$, i.e. $\widetilde{g}x = \widetilde{F}x$. Thus we prove that $\widetilde{F}$ and $\widetilde{g}$ has a fixed point x.

Let $\varphi \in \Psi_\omega$. Put $A = \{t > 0 : \lim_{n\to+\infty} \varphi^n(t) = 0\}$, if $t \in A$, we denote by k_t the first integer number such that $\varphi^{k_t - 1}(t) \geq t > \varphi^{k_t}(t)$ ($\varphi^0(t) = t$).

If $t \in [0, +\infty)\backslash A$, take an $r_t > t$ such that $r_t \in A$, and, again, denote by k_t the first integer number such that $\varphi^{k_t - 1}(r_t) \geq t > \varphi^{k_t}(r_t)$.

Now define a function $\psi : [0, +\infty) \to [0, +\infty)$ as follows:

$$\psi(0) = 0, \psi(t) = \varphi^{k_t}(t) \,\text{if}\, t \in A, \text{ and}\, \psi(t) = \varphi^{k_t}(r_t) \,\text{if}\, t \in [0, +\infty)\backslash A.$$

It is proved that $\psi \in \Psi$(see [21]). Hence we can apply ψ and get theorem 3.3 proved by the condition that $\varphi \in \Psi_\omega$.

Theorem 3.4 Let $(X, M, \Delta, \leq)$ be a complete ordered fuzzy metric space with Δ a triangular norm of H-type. Let $\Phi = (\sigma_1, \sigma_2, \cdots, \sigma_n, \tau)$ be $(n+1)$-tuple of mappings from Λ_n into itself such that $\tau \in \Omega_{A,B}$ is a permutation and verifying that $\sigma_i \in \Omega_{A,B}$ if $i \in A$ and $\sigma_i \in \Omega'_{A,B}$ if $i \in B$. Let $\varphi : [0, +\infty) \to [0, +\infty)$, $\varphi \in \Psi_\omega$, $F : X^n \to X$ and $g : X \to X$ be two mappings, $F(X^n) \subseteq g(X)$, F is continuous and has the mixed g-monotone property, F and g are weakly compatible mappings and

$$M(F(x_1, x_2, \cdots, x_n), F(y_1, y_2, \cdots, y_n), \varphi(t)) \geq \min_{1 \leq i \leq n} M(gx_i, gy_i, t)$$

for which $gx_{\tau(i)} \leq_i gy_{\tau(i)}$ for all $i \in \Lambda_n$ and all $t > 0$. If there exists $(x_1^0, x_2^0, \cdots, x_n^0) \in X^n$ verifying $gx^0_{\tau(i)} \leq_i F(x^0_{\sigma_i(1)}, x^0_{\sigma_i(2)}, \cdots, x^0_{\sigma_i(n)})$ for all i, then F and g have at least one Φ-coincidence point.

Proof Let $Y = X^n$. For $(x_1, x_2, \cdots, x_n), (y_1, y_2, \cdots, y_n) \in X^n$, $t > 0$, M^* and binary relation $\preccurlyeq$ on Y are defined as

$$M^*((x_1, x_2, \cdots, x_n), (y_1, y_2, \cdots, y_n), t) = \min_{1 \leq i \leq n} M(x_i, y_i, t),$$

$$(x_1, x_2, \cdots, x_n) \preccurlyeq (y_1, y_2, \cdots, y_n) \ \Leftrightarrow x_i \leq_i y_i, for\ all\ i \in \Lambda_n.$$

It is easy to verify that $(Y, \preccurlyeq)$ is a partially ordered set and (Y, M^*, Δ) is a complete fuzzy metric space. Then $(Y, M^*, \Delta, \preccurlyeq)$ is a complete ordered fuzzy metric space.

For $(x_1, x_2, \cdots, x_n) \in Y$, $\widetilde{F} : Y \to Y$, $\widetilde{g} : Y \to Y$ are defined as

$$\begin{aligned} & \widetilde{F}(x_1, x_2, \cdots, x_n) \\ = \ & (F(x_{\sigma_1(1)}, x_{\sigma_1(2)}, \cdots, x_{\sigma_1(n)}), F(x_{\sigma_2(1)}, x_{\sigma_2(2)}, \cdots, x_{\sigma_2(n)}), \cdots, F(x_{\sigma_n(1)}, x_{\sigma_n(2)}, \cdots, x_{\sigma_n(n)})). \\ & \widetilde{g}(x_1, x_2, \cdots, x_n) = (gx_{\tau(1)}, gx_{\tau(2)}, \cdots, gx_{\tau(n)}). \end{aligned}$$

For $(x_1, x_2, \cdots, x_n) \in X^n$, if $\widetilde{F}(x_1, x_2, \cdots, x_n) = \widetilde{g}(x_1, x_2, \cdots, x_n)$, by definition of $\widetilde{F}$ and $\widetilde{g}$ we have

$$\begin{aligned} F(x_{\sigma_1(1)}, x_{\sigma_1(2)}, \cdots, x_{\sigma_1(n)}) &= gx_{\tau(1)}, \\ F(x_{\sigma_2(1)}, x_{\sigma_2(2)}, \cdots, x_{\sigma_2(n)}) &= gx_{\tau(2)}, \\ &\vdots \\ F(x_{\sigma_n(1)}, x_{\sigma_n(2)}, \cdots, x_{\sigma_n(n)}) &= gx_{\tau(i)}, \end{aligned}$$

which implies that

$$F(x_{\sigma_i(1)}, x_{\sigma_i(2)}, \cdots, x_{\sigma_i(n)}) = gx_{\tau(i)}, \text{ for all } i \in \Lambda_n.$$

Since $\widetilde{F}\widetilde{g}(x_1, x_2, \cdots, x_n) = \widetilde{F}(gx_{\tau(1)}, gx_{\tau(2)}, \cdots, gx_{\tau(n)})$, the ith component of $\widetilde{F}\widetilde{g}(x_1, x_2, \cdots, x_n)$ is $F(gx_{\sigma_i(\tau(1))}, gx_{\sigma_i(\tau(2))}, \cdots, gx_{\sigma_i(\tau(n))})$. And

$$\widetilde{g}\widetilde{F}(x_1, x_2, \cdots, x_n) = \widetilde{g}(F(x_{\sigma_1(1)}, x_{\sigma_1(2)}, \cdots, x_{\sigma_1(n)}), \cdots, F(x_{\sigma_n(1)}, x_{\sigma_n(2)}, \cdots, x_{\sigma_n(n)})),$$

the ith component of $\widetilde{g}\widetilde{F}(x_1, x_2, \cdots, x_n)$ is $gF(x_{\sigma_{\tau(i)}(1)}, x_{\sigma_{\tau(i)}(2)}, \cdots, x_{\sigma_{\tau(i)}(n)})$.

Since F and g are weakly compatible, all the component of $\widetilde{F}\widetilde{g}(x_1, x_2, \cdots, x_n)$ and the corresponding component of $\widetilde{g}\widetilde{F}(x_1, x_2, \cdots, x_n)$ are equal, which implies that

$$\widetilde{F}\widetilde{g}(x_1, x_2, \cdots, x_n) = \widetilde{g}\widetilde{F}(x_1, x_2, \cdots, x_n).$$

That is, $\widetilde{F}$ and $\widetilde{g}$ are weakly compatible.

For $(x_1, x_2, \cdots, x_n)$, $(y_1, y_2, \cdots, y_n) \in X^n$, if $\widetilde{g}(x_1, x_2, \cdots, x_n) \preccurlyeq \widetilde{g}(y_1, y_2, \cdots, y_n)$, by definition of $\widetilde{g}$, we have

$$gx_{\tau(i)} \leq_i gy_{\tau(i)} \text{ for all } i \in \Lambda_n.$$

Now we need to prove $\widetilde{F}(x_1, x_2, \cdots, x_n) \preccurlyeq \widetilde{F}(y_1, y_2, \cdots, y_n)$. That is

$$F(x_{\sigma_i(1)}, x_{\sigma_i(2)}, \cdots, x_{\sigma_i(n)}) \preccurlyeq_i F(y_{\sigma_i(1)}, y_{\sigma_i(2)}, \cdots, y_{\sigma_i(n)}) \text{ for all } i \in \Lambda_n.$$

We use the following notation $\tau \in \Omega_{A,B}$, $\sigma_i \in \Omega_{A,B}$,

$$i \in A \Rightarrow \begin{cases} \sigma_i(t) \in A, t \in A, \\ \sigma_i(t) \in B, t \in B. \end{cases} \text{ and } x \leq_i y \Leftrightarrow \begin{cases} x \leq y, i \in A \\ x \geq y, i \in B. \end{cases}$$

For $i \in A$, if $j \in A$, then there exists $k \in A$ such that $\sigma_i(j) = \tau(k)$; if $j \in B$, then there exists $k \in B$ such that $\sigma_i(j) = \tau(k)$. So, we have

$$gx_{\tau(k)} \leq_k gy_{\tau(k)} \Rightarrow gx_{\tau(k)} \leq_j gy_{\tau(k)} \text{ for all } j \in \Lambda_n.$$

(i) If $j = 1 \in A$, we have $gx_{\tau(k_1)} \leq gy_{\tau(k_1)}$ and

$$\begin{aligned} F(x_{\sigma_i(1)}, x_{\sigma_i(2)}, \cdots, x_{\sigma_i(n)}) &= F(x_{\tau(k_1)}, x_{\sigma_i(2)}, \cdots, x_{\sigma_i(n)}) \\ \leq F(y_{\tau(k_1)}, x_{\sigma_i(2)}, \cdots, x_{\sigma_i(n)}) &= F(y_{\sigma_i(1)}, x_{\sigma_i(2)}, \cdots, x_{\sigma_i(n)}). \end{aligned}$$

(ii) If $j = 1 \in B$, we have $gx_{\tau(k_1)} \geq gy_{\tau(k_1)}$ and

$$\begin{aligned} F(x_{\sigma_i(1)}, x_{\sigma_i(2)}, \cdots, x_{\sigma_i(n)}) &= F(x_{\tau(k_1)}, x_{\sigma_i(2)}, \cdots, x_{\sigma_i(n)}) \\ \leq F(y_{\tau(k_1)}, x_{\sigma_i(2)}, \cdots, x_{\sigma_i(n)}) &= F(y_{\sigma_i(1)}, x_{\sigma_i(2)}, \cdots, x_{\sigma_i(n)}). \end{aligned}$$

That is

$$F(x_{\sigma_i(1)}, x_{\sigma_i(2)}, \cdots, x_{\sigma_i(n)}) \leq F(y_{\sigma_i(1)}, x_{\sigma_i(2)}, \cdots, x_{\sigma_i(n)}) \text{ for all } i \in A.$$

(i) If $j = 2 \in A$, we have $gx_{\tau(k_2)} \leq gy_{\tau(k_2)}$ and

$$\begin{aligned} F(y_{\sigma_i(1)}, x_{\sigma_i(2)}, \cdots, x_{\sigma_i(n)}) &= F(y_{\sigma_i(1)}, x_{\tau(k_2)}, \cdots, x_{\sigma_i(n)}) \\ \leq F(y_{\sigma_i(1)}, x_{\tau(k_2)}, \cdots, x_{\sigma_i(n)}) &= F(y_{\sigma_i(1)}, y_{\sigma_i(2)}, \cdots, x_{\sigma_i(n)}). \end{aligned}$$

(ii) If $j = 2 \in B$, we have $gx_{\tau(k_2)} \geq gy_{\tau(k_2)}$ and

$$\begin{aligned} F(y_{\sigma_i(1)}, x_{\sigma_i(2)}, \cdots, x_{\sigma_i(n)}) &= F(y_{\sigma_i(1)}, x_{\tau(k_2)}, \cdots, x_{\sigma_i(n)}) \\ \leq F(y_{\sigma_i(1)}, x_{\tau(k_2)}, \cdots, x_{\sigma_i(n)}) &= F(y_{\sigma_i(1)}, y_{\sigma_i(2)}, \cdots, x_{\sigma_i(n)}). \end{aligned}$$

That is,

$$F(y_{\sigma_i(1)}, x_{\sigma_i(2)}, \cdots, x_{\sigma_i(n)}) \leq F(y_{\sigma_i(1)}, y_{\sigma_i(2)}, \cdots, x_{\sigma_i(n)}) \text{ for all } i \in A.$$

Continuing in this way, we can get

$$F(x_{\sigma_i(1)}, x_{\sigma_i(2)}, \cdots, x_{\sigma_i(n)}) \leq F(y_{\sigma_i(1)}, y_{\sigma_i(2)}, \cdots, y_{\sigma_i(n)}) \text{ for all } i \in A.$$

Similarly, for $i \in B$, we can have

$$gx_{\tau(k)} \leq_k gy_{\tau(k)} \Rightarrow gx_{\tau(k)} \geq_j gy_{\tau(k)} \text{ for all } j \in \Lambda_n,$$

and

$$F(x_{\sigma_i(1)}, x_{\sigma_i(2)}, \cdots, x_{\sigma_i(n)}) \geq F(y_{\sigma_i(1)}, y_{\sigma_i(2)}, \cdots, y_{\sigma_i(n)}) \text{ for all } i \in B.$$

Then

$$F(x_{\sigma_i(1)}, x_{\sigma_i(2)}, \cdots, x_{\sigma_i(n)}) \leq_i F(y_{\sigma_i(1)}, y_{\sigma_i(2)}, \cdots, y_{\sigma_i(n)}) \text{ for all } i \in \Lambda_n.$$

That is, $\widetilde{F}$ has the mixed $\widetilde{g}$-monotone property. According to the known conditions, we have

$$\widetilde{g}(x_1, x_2, \cdots, x_n) \preccurlyeq \widetilde{g}(y_1, y_2, \cdots, y_n) \Rightarrow gx_{\tau(i)} \leq_i gx_{\tau(i)}, \text{ for all } i \in \Lambda_n.$$

Now we will prove from $gx_{\tau(\sigma_i(j))} \leq_{\sigma_i(j)} gy_{\tau(\sigma_i(j))}$ to $gx_{\tau(\sigma_i(j))} \leq_j gy_{\tau(\sigma_i(j))}$ for all $j \in \Lambda_n$.

In fact, let $i \in A$, $\sigma_i \in \Omega_{A,B}$, and $\tau \in A$, since $gx_{\tau(k)} \leq_{\sigma_i(j)} gy_{\tau(k)}$, for all $k \in \Lambda_n$,

(1) If $i \in A$, there exists $k \in A$, $\sigma_i(j) = \tau(k)$, we have $x_{\tau(\sigma_i(j))} = x_{\tau(k)}$, $y_{\tau(\sigma_i(j))} = y_{\tau(k)}$.

(2) If $i \in B$, there exists $k \in B$, $\sigma_i(j) = \tau(k)$.

Following the known conditions, we have

$$M(F(x_{\sigma_1(1)}, x_{\sigma_1(2)}, \cdots, x_{\sigma_1(n)}), F(y_{\sigma_1(1)}, y_{\sigma_1(2)}, \cdots, y_{\sigma_1(n)}), \varphi(t)) \geq \min_{1\leq j\leq n} M(gx_{\sigma_1(j)}, gy_{\sigma_1(j)}, t)$$

$$M(F(x_{\sigma_2(1)}, x_{\sigma_2(2)}, \cdots, x_{\sigma_2(n)}), F(y_{\sigma_2(1)}, y_{\sigma_2(2)}, \cdots, y_{\sigma_2(n)}), \varphi(t)) \geq \min_{1\leq j\leq n} M(gx_{\sigma_2(j)}, gy_{\sigma_2(j)}, t)$$

$$\vdots$$

$$M(F(x_{\sigma_n(1)}, x_{\sigma_n(2)}, \cdots, x_{\sigma_n(n)}), F(y_{\sigma_n(1)}, y_{\sigma_n(2)}, \cdots, y_{\sigma_n(n)}), \varphi(t)) \geq \min_{1\leq j\leq n} M(gx_{\sigma_n(j)}, gy_{\sigma_n(j)}, t)$$

which implies that

$$\begin{aligned} & \min_{1\leq i\leq n} M(F(x_{\sigma_i(1)}, x_{\sigma_i(2)}, \cdots, x_{\sigma_i(n)}), F(y_{\sigma_i(1)}, y_{\sigma_i(2)}, \cdots, y_{\sigma_i(n)}), \varphi(t)) \\ \geq & \min_{1\leq i\leq n} \{\min_{1\leq j\leq n} d(gx_{\sigma_i(j)}, gy_{\sigma_i(j)}, t)\}. \end{aligned}$$

Let i, such that the left side of the inequality gets minimum. Then we get

$$M^*(\widetilde{F}(x_1, x_2, \cdots, x_n), \widetilde{F}(y_1, y_2, \cdots, y_n), \varphi(t)) \geq \min_{1\leq j\leq n} M(gx_{\sigma_i(j)}, gy_{\sigma_i(j)}, t),$$

that is,

$$M^*(\widetilde{F}(x_1, x_2, \cdots, x_n), \widetilde{F}(y_1, y_2, \cdots, y_n), \varphi(t)) \\ \geq \min_{1\le k\le n} M(gx_{\tau(k)}, gy_{\tau(k)}, t) = M^*(\widetilde{g}(x_1, x_2, \cdots, x_n), \widetilde{g}(y_1, y_2, \cdots, y_n), t).$$

It is easy to verify that $\widetilde{F}(X^n) \subseteq \widetilde{g}(X^n)$, $\widetilde{g}$ is continuous and $\widetilde{g}(Y)$ is complete, and there exists $(x_0^1, x_0^2, \cdots, x_0^n) \in X$ verifying

$$gx^0_{\tau_k} \le_k F(x^0_{\tau_k(1))}, x^0_{\tau_k(2)}, \cdots, x^0_{\tau_k(n)}), \quad \text{for all } k.$$

That is, there exists $x_0^1, x_0^2, \cdots, x_0^n \in X$ verifying

$$\widetilde{g}(x_1^0, x_2^0, \cdots, x_n^0) \preccurlyeq \widetilde{F}(x_1^0, x_2^0, \cdots, x_n^0).$$

Following all the conditions of Theorem 3.3 and the proof, we can have F and g, at least, one Φ -coincidence point.

It is obvious that, if F and g are compatible, then they are weakly compatible. So, we have the following theorem.

Theorem 3.5 Let $(X, M, \Delta, \le)$ be a complete ordered fuzzy metric space with Δ a triangular norm of H-type. Let $\Phi = (\sigma_1, \sigma_2, \cdots, \sigma_n, \tau)$ be $(n+1)$-tuple of mappings from Λ_n into itself such that $\tau \in \Omega_{A,B}$ is a permutation and verifying that $\sigma_i \in \Omega_{A,B}$ if $i \in A$ and $\sigma_i \in \Omega'_{A,B}$ if $i \in B$. Let $\varphi : [0, +\infty) \to [0, +\infty)$, $\varphi \in \Psi_\omega$, $F : X^n \to X$ and $g : X \to X$ be two mappings, $F(X^n) \subseteq g(X)$, F is continuous and has the mixed g-monotone property, F and g are compatible mappings and

$$M(F(x_1, x_2, \cdots, x_n), F(y_1, y_2, \cdots, y_n), \varphi(t)) \ge \min_{1\le i\le n} M(gx_i, gy_i, t)$$

for which $gx_{\tau(i)} \le_i gy_{\tau(i)}$ for all $i \in \Lambda_n$ and all $t > 0$. If there exists $(x_1^0, x_2^0, \cdots, x_n^0) \in X^n$ verifying $gx^0_{\tau(i)} \le_i F(x^0_{\sigma_i(1)}, x^0_{\sigma_i(2)}, \cdots, x^0_{\sigma_i(n)})$ for all i, then F and g have at least one Φ-coincidence point.

In theorem 3.5, let $n = 2$, we have $\Lambda_2 = \{1, 2\}, A = \{1\}, B = \{2\}, \sigma_1 \in \Omega_{A,B}$ and $\sigma_2 \in \Omega'_{A,B}$, then $\sigma_1(1) = \{1\}, \sigma_1(2) = \{2\}$ and $\sigma_2(1) = \{2\}, \sigma_2(2) = \{1\}$. Then we have the following corollary.

Corollary 3.6 Let $(X, M, \Delta, \le)$ be a complete ordered fuzzy metric space with Δ a triangular norm of H-type. $\varphi : [0, +\infty) \to [0, +\infty)$, $\varphi \in \Psi_\omega$, Let $F : X^2 \to X$ and $g : X \to X$ be two mappings, $F(X^2) \subseteq g(X)$, F is continuous and has the mixed g-monotone property, F and g be weakly compatible mapping and

$$M(F(x_1, x_2), F(y_1, y_2), \varphi(t)) \ge \min\{M(gx_1, gy_1, t), M(gx_2, gy_2, t)\}$$

for which $gx_1 \le gy_1$ and $gx_2 \ge gy_2$. If there exists $x_1^0, x_2^0 \in X$ verifying $gx_1^0 \le F(x_1^0, x_2^0)$ and $gx_2^0 \le F(x_2^0, x_1^0)$, then F and g have a coupled fixed point in X.

Similarly, in Theorem 3.5, let $n = 3$, we have $\Lambda_3 = \{1, 2, 3\}$, $A = \{1, 3\}$, $B = \{2\}$. $\sigma_1, \sigma_3 \in \Omega_{A,B}$ and $\sigma_2 \in \Omega'_{A,B}$, then $\sigma_1(1) = \{1\}$, $\sigma_1(2) = \{2\}$, $\sigma_1(3) = \{3\}$, $\sigma_2(1) = \{2\}$, $\sigma_2(2) = \{1\}$, $\sigma_2(3) = \{2\}$ and $\sigma_3(1) = \{3\}$, $\sigma_3(2) = \{2\}$, $\sigma_3(3) = \{1\}$. Then we have the following corollary.

Corollary 3.7 Let $(X, M, \Delta, \leq)$ be a complete ordered fuzzy metric space with Δ a triangular norm of H-type. $\varphi : [0, +\infty) \to [0, +\infty)$, $\varphi \in \Psi_\omega$, Let $F : X^3 \to X$ and $g : X \to X$ be two mappings, $F(X^3) \subseteq g(X)$, F is continuous and has the mixed g-monotone property, F and g be weakly compatible mapping and

$$M(F(x_1, x_2, x_3), F(y_1, y_2, y_3), \varphi(t)) \geq \min\{M(gx_1, gy_1, t), M(gx_2, gy_2, t), M(gx_3, gy_3, t)\}$$

for which $gx_1 \leq gy_1$, $gx_2 \geq gy_2$ and $gx_3 \leq gy_3$. If there exists $x_1^0, x_2^0, x_3^0 \in X$ verifying $gx_1^0 \leq F(x_1^0, x_2^0, x_3^0)$, $gx_2^0 \geq F(x_2^0, x_1^0, x_3^0)$ and $gx_3^0 \leq F(x_3^0, x_2^0, x_1^0)$, then F and g have a tripled fixed point in X.

Remark When F and g are commutative, they are weakly compatible, so we have the following theorem.

Theorem 3.8 Let $(X, M, \Delta, \leq)$ be a complete ordered fuzzy metric space with Δ a triangular norm of H-type. Let $\Phi = (\sigma_1, \sigma_2, \cdots, \sigma_n, \tau)$ be $(n+1)$-tuple of mappings from Λ_n into itself such that $\tau \in \Omega_{A,B}$ is a permutation and verifying that $\sigma_i \in \Omega_{A,B}$ if $i \in A$ and $\sigma_i \in \Omega'_{A,B}$ if $i \in B$. Let $\varphi : [0, +\infty) \to [0, +\infty)$, $\varphi \in \Psi_\omega$, $F : X^n \to X$ and $g : X \to X$ be two mappings, $F(X^n) \subseteq g(X)$, F is continuous and has the mixed g-monotone property, F and g are commutative, and

$$M(F(x_1, x_2, \cdots, x_n), F(y_1, y_2, \cdots, y_n), \varphi(t)) \geq \min_{1 \leq i \leq n} M(gx_i, gy_i, t)$$

for which $gx_{\tau(i)} \leq_i gy_{\tau(i)}$ for all $i \in \Lambda_n$ and all $t > 0$. If there exists $(x_1^0, x_2^0, \cdots, x_n^0) \in X^n$ verifying $gx^0_{\tau(i)} \leq_i F(x^0_{\sigma_i(1)}, x^0_{\sigma_i(2)}, \cdots, x^0_{\sigma_i(n)})$ for all i, then F and g have at least one Φ-coincidence point.

Remark Let $k \in [0, 1)$, taking $\varphi(t) = kt$ in Theorem 3.4, 3.5, 3.8, we obtain the following corollaries.

Corollary 3.9 Let $(X, M, \Delta, \leq)$ be a complete ordered fuzzy metric space with Δ a triangular norm of H-type. Let $\Phi = (\sigma_1, \sigma_2, \cdots, \sigma_n, \tau)$ be $(n+1)$-tuple of mappings from Λ_n into itself such that $\tau \in \Omega_{A,B}$ is a permutation and verifying that $\sigma_i \in \Omega_{A,B}$ if $i \in A$ and $\sigma_i \in \Omega'_{A,B}$ if $i \in B$. Let $\varphi : [0, +\infty) \to [0, +\infty)$, $\varphi \in \Psi_\omega$, Let $F : X^n \to X$ and $g : X \to X$ be two mappings, $F(X^n) \subseteq g(X)$, F is continuous and has the mixed g-monotone property, F and g are weakly compatible mappings and

$$M(F(x_1, x_2, \cdots, x_n), F(y_1, y_2, \cdots, y_n), kt) \geq \min_{1 \leq i \leq n} M(gx_i, gy_i, t)$$

for which $gx_{\tau(i)} \leq_i gy_{\tau(i)}$ for all $i \in \Lambda_n$ and all $t > 0$. If there exists $(x_1^0, x_2^0, \cdots, x_n^0) \in X^n$ verifying $gx^0_{\tau(i)} \leq_i F(x^0_{\sigma_i(1)}, x^0_{\sigma_i(2)}, \cdots, x^0_{\sigma_i(n)})$ for all i, then F and g have at least one Φ-coincidence point.

Corollary 3.10 Let $(X, M, \Delta, \leq)$ be a complete ordered fuzzy metric space with Δ a triangular norm of H-type. Let $\Phi = (\sigma_1, \sigma_2, \cdots, \sigma_n, \tau)$ be $(n+1)$-tuple of mappings from Λ_n into itself such that $\tau \in \Omega_{A,B}$ is a permutation and verifying that $\sigma_i \in \Omega_{A,B}$ if $i \in A$ and $\sigma_i \in \Omega'_{A,B}$ if $i \in B$. Let $\varphi : [0, +\infty) \to [0, +\infty)$, $\varphi \in \Psi_\omega$, Let $F : X^n \to X$ and $g : X \to X$ be two mappings, $F(X^n) \subseteq g(X)$, F is continuous and has the mixed g-monotone property, F and g are compatible mappings and

$$M(F(x_1, x_2, \cdots, x_n), F(y_1, y_2, \cdots, y_n), kt) \geq \min_{1 \leq i \leq n} M(gx_i, gy_i, t)$$

for which $gx_{\tau(i)} \leq_i gy_{\tau(i)}$ for all $i \in \Lambda_n$ and all $t > 0$. If there exists $(x_1^0, x_2^0, \cdots, x_n^0) \in X^n$ verifying $gx^0_{\tau(i)} \leq_i F(x^0_{\sigma_i(1)}, x^0_{\sigma_i(2)}, \cdots, x^0_{\sigma_i(n)})$ for all i, then F and g have at least one Φ-coincidence point.

Corollary 3.11 Let $(X, M, \Delta, \leq)$ be a complete ordered fuzzy metric space with Δ a triangular norm of H-type. Let $\Phi = (\sigma_1, \sigma_2, \cdots, \sigma_n, \tau)$ be $(n+1)$-tuple of mappings from Λ_n into itself such that $\tau \in \Omega_{A,B}$ is a permutation and verifying that $\sigma_i \in \Omega_{A,B}$ if $i \in A$ and $\sigma_i \in \Omega'_{A,B}$ if $i \in B$. Let $\varphi : [0, +\infty) \to [0, +\infty)$, $\varphi \in \Psi_\omega$, $F : X^n \to X$ and $g : X \to X$ be two mappings, $F(X^n) \subseteq g(X)$, F is continuous and has the mixed g-monotone property, F and g are commutative, and

$$M(F(x_1, x_2, \cdots, x_n), F(y_1, y_2, \cdots, y_n), kt) \geq \min_{1 \leq i \leq n} M(gx_i, gy_i, t)$$

for which $gx_{\tau(i)} \leq_i gy_{\tau(i)}$ for all $i \in \Lambda_n$ and all $t > 0$. If there exists $(x_1^0, x_2^0, \cdots, x_n^0) \in X^n$ verifying $gx^0_{\tau(i)} \leq_i F(x^0_{\sigma_i(1)}, x^0_{\sigma_i(2)}, \cdots, x^0_{\sigma_i(n)})$ for all i, then F and g have at least one Φ-coincidence point.

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模糊度量空间中多变量压缩映射的公共耦合不动点定理

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摘要: 本文提出了多变量映射的一致点的概念. 本文的结果是模糊偏序度量空间中不动点定理的一些主要结论从低维到高维的推广.

关键词: 不动点定理; 度量空间; 模糊度量空间; 偏序集; 相容映射

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