

STATE FEEDBACK CONTROL FOR STOCHASTIC NONLINEAR SYSTEMS WITH HIGH-ORDER TERMS AND TIME-VARYING DELAY

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Abstract: In this paper, we study the state feedback control problem for a class of high-order stochastic nonlinear systems with time-varying delay. Using the Lyapunov method, a suitable Lyapunov functional is constructed and combined with the stability theory of stochastic systems, we obtain a state feedback controller which can make the global asymptotical stability (GAS) in probability of the closed-loop system. System in this paper has more general high-order terms, which generalizes the previous result of global asymptotic stability in probability of a single high-order term system. The efficiency of the state-feedback controller is demonstrated by a simulation example.

Keywords: high-order terms; time-varying delay; state feedback; Lyapunov-Krasovskii theory; stochastic nonlinear.

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1 Introduction

As we all know, the stochastic systems received more and more attention in physical applications and science. Ever since the stochastic stability theory was established and improved by Kushner, Mao, Deng and Has'minskii [1–4], the design and analysis of a backstepping controller for nonlinear stochastic systems achieved remarkable development in recent years [5–10]. The delay was also widely existed in practical system. The presence of time delay greatly complicates the stochastic control designs and makes them more difficult [11–12]. Therefore, the control design of stochastic nonlinear time-delay systems was received much attention in recent years [13–16].

In this paper, we consider the following stochastic nonlinear system with high-order terms and time-varying delay in the form

$$\begin{aligned} dx_i &= \sum_{j=1}^s h_{ij}(t)x_{i+1}^{p_j}(t)dt + f_i(t, \bar{x}_i(t), \bar{x}_i(t-d(t)))dt + g_i^T(t, \bar{x}_i(t), \bar{x}_i(t-d(t)))d\omega, \\ dx_n &= h_n(t)u(t)dt + f_n(t, \bar{x}_n(t), \bar{x}_n(t-d(t)))dt + g_n^T(t, \bar{x}_n(t), \bar{x}_n(t-d(t)))d\omega, \end{aligned} \quad (1.1)$$

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where $i = 1, 2, \dots, n-1$, $x = (x_1, x_2, \dots, x_n)^T \in R^n$ and $u \in R$ are the system state and input, respectively. $\bar{x}_i = (x_1, x_2, \dots, x_i)^T$, $\bar{x}_n(t) = x(t)$, $\bar{x}_i(t-d(t)) = (x_1(t-d(t)), x_2(t-d(t)), \dots, x_i(t-d(t)))^T$, $\bar{x}_n(t-d(t)) = x(t-d(t))$; $h_{ij}(t) : R_+ \rightarrow R_+$, $i = 1, 2, \dots, n-1$; $j = 1, 2, \dots, s$, are continuous functions; $p_j \in R_{\text{odd}}^{\geq 1} = \{q \in R: q \geq 1 \text{ and } q \text{ is a ratio of odd integers}\}$ is said to be the high-order of the system and $h_{ij}x_{i+1}^{p_j}$ is high-order term of the system. $d(t) : R_+ \rightarrow [0, d]$ is time-varying delay which satisfies $0 \leq \dot{d}(t) \leq h < 1$; ω is an m -dimensional standard Wiener process defined on a probability space (Ω, \mathcal{F}, P) with Ω being a sample space, \mathcal{F} being a filtration, and P being a probability measure; the drift terms $f_i : R_+ \times R^i \times R^i \rightarrow R$ and the diffusion terms $g_i : R_+ \times R^i \times R^i \rightarrow R^m$, $i = 1, 2, \dots, n$, are assumed to be locally Lipschitz in $(\bar{x}_i(t), \bar{x}_i(t-d(t)))$ and piecewise continuous in t with $f_i(t, 0, 0) = 0$ and $g_i(t, 0, 0) = 0$.

When $s = 1$ (i.e., single high-order term), for this kind of stochastic nonlinear time-varying delay system (1.1), by using backstepping methods, the design of global stabilization controller achieved remarkable development [17–20]. When $s > 1$, system (1.1) has more general terms $\sum_{j=1}^s h_{ij}(t)x_{i+1}^{p_j}$, it is more interesting since it includes the single high-order term case as a special case. When the diffusion term is zero in the stochastic nonlinear system (1.1), Wang and Zheng [21] studied the stability of this type of system for the first time and gave the state feedback controller. To the best of our knowledge, there are not any results on GAS in probability for system (1.1) at present, this paper is a promotion of previous literature.

In this paper, under the appropriate conditions on nonlinearities in drift and diffusion vector fields, the method of backstepping design and the method of power integration are combined to solve the state feedback stabilization problem for stochastic nonlinear system (1.1) with high-order terms and time-varying delay.

2 Preliminary Results

The following notations are used throughout the paper. R_+ denotes the set of all nonnegative real numbers, and R^n denotes the real n -dimensional space. For a given vector or matrix X , X^T denotes its transpose, $\text{Tr}\{X\}$ denotes its trace when X is square, and $|X|$ is the Euclidean norm of a vector X . $\mathcal{C}_{\mathcal{F}_0}^b([-d, 0]; R^n)$ denotes the family of all \mathcal{F}_0 -measurable bounded $\mathcal{C}([-d, 0]; R^n)$ -valued random variables $\xi = \{\xi(\theta) : -d \leq \theta \leq 0\}$. \mathcal{C}^i denotes the set of all functions with continuous i th partial derivatives. $\mathcal{C}^{2,1}(R^n \times [-d, \infty); R_+)$ denotes the family of all nonnegative functions $V(x, t)$ on $R^n \times [-d, \infty)$ which are \mathcal{C}^2 in x and \mathcal{C}^1 in t . \mathcal{K} denotes the set of all functions: $R_+ \rightarrow R_+$, which are continuous, strictly increasing and vanishing at zero; \mathcal{K}_∞ denotes the set of all functions which are of class \mathcal{K} and unbounded.

The following definitions and lemmas are used throughout the paper.

Definition 2.1 (see [22]) The equilibrium $x(t) = 0$ of the stochastic system (1.1) is said to be

- (i) globally stable in probability if for any $\varepsilon > 0$, there exists a function $\gamma(\cdot) \in \mathcal{K}$ such

that

$$P\{\|x(t, \phi)\| < \gamma(\sup_{-d \leq \theta \leq 0} |\phi|)\} \geq 1 - \varepsilon, \forall t \geq 0, \phi \in \mathcal{C}_{\mathcal{F}_0}^b([-d, 0]; R^n) \setminus \{0\};$$

(ii) globally asymptotically stable (GAS) in probability, one has

$$P\{\lim_{t \rightarrow \infty} \|x(t, \phi)\| = 0\} = 1, \forall t \geq 0, \phi \in \mathcal{C}_{\mathcal{F}_0}^b([-d, 0]; R^n) \setminus \{0\}.$$

Definition 2.2 For any given $V(x(t), t) \in \mathcal{C}^{2,1}$ associated with stochastic system (1.1), the differential operator \mathcal{L} is defined as

$$\mathcal{L}V = \frac{\partial V}{\partial t} + \frac{\partial V}{\partial x}f + \frac{1}{2}\text{Tr}\{g\frac{\partial^2 V}{\partial x^2}g^T\},$$

where $\frac{1}{2}\text{Tr}\{g\frac{\partial^2 V}{\partial x^2}g^T\}$ is called as the Hessian term of \mathcal{L} .

Lemma 2.3 (see [22]) For stochastic system (1.1), if there exist a function $V(x(t), t) \in \mathcal{C}^{2,1}(R^n \times [-d, \infty); R_+)$, two class- \mathcal{K}_∞ functions α_1, α_2 , and a class- \mathcal{K} function α_3 such that

$$\begin{aligned} \alpha_1(|x(t)|) &\leq V(x(t), t) \leq \alpha_2(\sup_{-d \leq s \leq 0} |x(t+s)|), \\ \mathcal{L}V(x(t), t) &\leq -\alpha_3(|x(t)|), \end{aligned}$$

then there exists a unique solution on $[-d, \infty)$ for (1.1), and the equilibrium $x(t) = 0$ is GAS in probability.

Lemma 2.4 (see [23]) Let $r \in R_{\text{odd}}^{\geq 1}$, and x, y be real-valued functions. One has

$$|x^r - y^r| \leq r|x - y|(x^{r-1} + y^{r-1}) \leq c|x - y|(|x - y|)^{r-1} + |y|^{r-1},$$

where, if $1 < r < 2$, then $c = r$; if $r > 2$, then $c = r2^{r-1}$.

Lemma 2.5 (see [24]) For any $x, y \in R$ and a constant $p \geq 1$, then

$$|x + y|^p \leq 2^{p-1}|x^p + y^p|, \quad (|x| + |y|)^{\frac{1}{p}} \leq |x|^{\frac{1}{p}} + |y|^{\frac{1}{p}}.$$

Moreover, if p is an odd integer, then

$$|x - y|^p \leq 2^{p-1}|x^p - y^p|, \quad |x^{\frac{1}{p}} - y^{\frac{1}{p}}| \leq 2^{1-\frac{1}{p}}|x - y|^{\frac{1}{p}}.$$

Lemma 2.6 (see [25]) For x_1, \dots, x_n being real variables and $p > 0$ being a constant, then

$$(x_1 + \dots + x_n)^p \leq \max\{n^{p-1}, 1\}(x_1^p + \dots + x_n^p).$$

Lemma 2.7 (see [23]) Let x, y be real variables, for any positive real numbers a, b, m and n , the following inequality holds

$$ax^m y^n \leq b|x|^{m+n} + \frac{n}{m+n} \left(\frac{m+n}{m}\right)^{-\frac{m}{n}} a^{\frac{m+n}{n}} b^{-\frac{m}{n}} |y|^{m+n}.$$

3 Control Design and Stability Analysis

where

$$c_{i-1,k} = \begin{cases} \alpha_{i-1} \cdots \alpha_k, & k = 1, 2, \dots, i-1, \\ 1, & k = i. \end{cases}$$

Consider the following Lyapunov-Krasovskii functional

$$V = \frac{1}{4} \sum_{i=1}^n \xi_i^4 + \int_{t-d(t)}^t W(\xi(\sigma)) d\sigma, \quad (3.5)$$

where $W(\xi) = \sum_{i=1}^n \sum_{j=1}^s \frac{n-i+1}{2(1-h)} \xi_i^{3+p_j}$. By (3.4), one gets

$$\begin{aligned} \mathcal{L}V &\leq \xi_n^3 h_n(t)u + \sum_{i=1}^{n-1} \sum_{j=1}^s \xi_i^3 h_{ij}(t)x_{i+1}^{p_j} + \sum_{i=2}^n \xi_i^3 \sum_{k=1}^{i-1} c_{i-1,k} \sum_{j=1}^s h_{kj} x_{k+1}^{p_j} \\ &\quad + \sum_{i=1}^n \xi_i^3 \sum_{k=1}^i c_{i-1,k} f_k(t, \bar{x}_k, \bar{x}_k(t-d(t))) + \frac{3}{2} \sum_{i=1}^n \xi_i^2 \left(\sum_{k=1}^i c_{i-1,k} g_k(t, \bar{x}_k, \bar{x}_k(t-d(t))) \right)^2 \\ &\quad + W(\xi) - (1-h)W(\xi(t-d(t))) \\ &= \xi_n^3 h_n(t)u + \sum_{i=1}^{n-1} \sum_{j=1}^s \xi_i^3 \underline{h} x_{i+1}^{*p_j} + \sum_{i=1}^{n-1} \sum_{j=1}^s \xi_i^3 (h_{ij}(t)x_{i+1}^{p_j} - \underline{h} x_{i+1}^{*p_j}) + \sum_{i=2}^n \xi_i^3 \sum_{k=1}^{i-1} c_{i-1,k} \sum_{j=1}^s h_{kj} x_{k+1}^{p_j} \\ &\quad + \sum_{i=1}^n \xi_i^3 \sum_{k=1}^i c_{i-1,k} f_k(t, \bar{x}_k, \bar{x}_k(t-d(t))) + \frac{3}{2} \sum_{i=1}^n \xi_i^2 \left(\sum_{k=1}^i c_{i-1,k} g_k(t, \bar{x}_k, \bar{x}_k(t-d(t))) \right)^2 \\ &\quad + W(\xi) - (1-h)W(\xi(t-d(t))). \end{aligned} \quad (3.6)$$

Next, we estimate the right side of (3.6). The following propositions supply these estimates, with their respective proofs located in Appendix.

Proposition 1 There exists a positive constant β_i such that

$$\sum_{j=1}^s \xi_i^3 (h_{ij} x_{i+1}^{p_j} - \underline{h} x_{i+1}^{*p_j}) \leq \frac{1}{4(1-h)} \sum_{j=1}^s \xi_i^{3+p_j} + \beta_{i+1} \sum_{j=1}^s \xi_{i+1}^{3+p_j}, \quad i = 1, 2, \dots, n-1. \quad (3.7)$$

Proposition 2 There exists a positive constant η_i such that

$$\xi_i^3 \sum_{k=1}^{i-1} c_{i-1,k} \sum_{j=1}^s h_{kj} x_{k+1}^{p_j} \leq \frac{1}{4(1-h)} \sum_{k=1}^{i-1} \sum_{j=1}^s \xi_k^{3+p_j} + \eta_i \sum_{j=1}^s \xi_i^{3+p_j}, \quad i = 2, 3, \dots, n. \quad (3.8)$$

Proposition 3 There exists a positive constant γ_i such that

$$\begin{aligned} \xi_i^3 \sum_{k=1}^i c_{i-1,k} f_k(t, \bar{x}_k, \bar{x}_k(t-d(t))) &\leq \frac{1}{4(1-h)} \sum_{k=1}^i \sum_{j=1}^s \xi_k^{3+p_j} + \frac{1}{4} \sum_{k=1}^i \sum_{j=1}^s \xi_k^{3+p_j} (t-d(t)) \\ &\quad + \gamma_i \sum_{j=1}^s \xi_i^{3+p_j}, \quad i = 1, 2, \dots, n. \end{aligned} \quad (3.9)$$

Proposition 4 There exists a positive constant μ_i such that

$$\begin{aligned} \frac{3}{2}\xi_i^2 \left(\sum_{k=1}^i c_{i-1,k} g_k(t, \bar{x}_k, \bar{x}_k(t-d(t)))\right)^2 &\leq \frac{1}{4(1-h)} \sum_{k=1}^i \sum_{j=1}^s \xi_k^{3+p_j} + \frac{1}{4} \sum_{k=1}^i \sum_{j=1}^s \xi_k^{3+p_j} (t-d(t)) \\ &+ \mu_i \sum_{j=1}^s \xi_i^{3+p_j}, i = 1, 2, \dots, n. \end{aligned} \tag{3.10}$$

We can obtain the following formula by combing (3.6) with (3.7)–(3.10)

$$\begin{aligned} \mathcal{L}V &\leq \xi_n^3 h_n(t)u + \sum_{i=1}^{n-1} \sum_{j=1}^s \xi_i^3 \underline{h} x_{i+1}^{*p_j} + \sum_{i=1}^{n-1} \left(\frac{1}{4(1-h)} \sum_{j=1}^s \xi_i^{3+p_j} + \beta_{i+1} \sum_{j=1}^s \xi_{i+1}^{3+p_j}\right) \\ &+ \sum_{i=2}^n \left(\frac{1}{4(1-h)} \sum_{k=1}^{i-1} \sum_{j=1}^s \xi_k^{3+p_j} + \eta_i \sum_{j=1}^s \xi_i^{3+p_j}\right) + \sum_{i=1}^n \left(\frac{1}{2(1-h)} \sum_{k=1}^i \sum_{j=1}^s \xi_k^{3+p_j}\right) \\ &+ \frac{1}{2} \sum_{k=1}^i \sum_{j=1}^s \xi_k^{3+p_j} (t-d(t)) + (\gamma_i + \mu_i) \sum_{j=1}^s \xi_i^{3+p_j} + W(\xi) - (1-h)W(\xi(t-d(t))) \\ &= \xi_n^3 h_n(t)u + \sum_{i=1}^{n-1} \sum_{j=1}^s \xi_i^3 \underline{h} x_{i+1}^{*p_j} + \sum_{i=1}^{n-1} \frac{1}{4(1-h)} \sum_{j=1}^s \xi_i^{3+p_j} \\ &+ \sum_{i=2}^n \beta_i \sum_{j=1}^s \xi_i^{3+p_j} + \sum_{i=1}^{n-1} \frac{n-i}{4(1-h)} \sum_{j=1}^s \xi_i^{3+p_j} + \sum_{i=2}^n \eta_i \sum_{j=1}^s \xi_i^{3+p_j} + \sum_{i=1}^n \frac{n-i+1}{2(1-h)} \sum_{j=1}^s \xi_i^{3+p_j} \\ &+ \sum_{i=1}^n \frac{n-i+1}{2} \sum_{j=1}^s \xi_i^{3+p_j} (t-d(t)) + \sum_{i=1}^n (\gamma_i + \mu_i) \sum_{j=1}^s \xi_i^{3+p_j} \\ &+ \sum_{i=1}^n \frac{n-i+1}{2(1-h)} \sum_{j=1}^s \xi_i^{3+p_j} - (1-h) \cdot \sum_{i=1}^n \frac{n-i+1}{2(1-h)} \sum_{j=1}^s \xi_i^{3+p_j} (t-d(t)) \\ &= [h_n(t) - \underline{h}] \xi_n^3 u + \xi_n^3 [\underline{h}u + \left(\frac{1}{1-h} + \beta_n + \eta_n + \gamma_n + \mu_n\right) \sum_{j=1}^s \xi_n^{p_j}] \\ &+ \xi_1^3 \sum_{j=1}^s [\underline{h}x_2^{*p_j} + \left(\frac{5n}{4(1-h)} + \gamma_1 + \mu_1\right) \xi_1^{p_j}] \\ &+ \sum_{i=2}^{n-1} \xi_i^3 \sum_{j=1}^s [\underline{h}x_{i+1}^{*p_j} + \left(\frac{5(n-i+1)}{4(1-h)} + \beta_i + \eta_i + \gamma_i + \mu_i\right) \xi_i^{p_j}]. \end{aligned} \tag{3.11}$$

According to the above inequalities, one obtains the virtual controllers

$$x_2^* = -\left(\frac{1}{\underline{h}}\left(\lambda_1 + \frac{5n}{4(1-h)} + \gamma_1 + \mu_1\right)\right)^{\frac{1}{p_*}} \xi_1 = -\alpha_1 \xi_1, \tag{3.12}$$

where $\alpha_1 = \left(\frac{1}{\underline{h}}\left(\lambda_1 + \frac{5n}{4(1-h)} + \gamma_1 + \mu_1\right)\right)^{\frac{1}{p_*}}$, $p_* = \min_{1 \leq j \leq s} p_j$,

$$x_{i+1}^* = -\left(\frac{1}{\underline{h}}\left(\lambda_i + \frac{5(n-i+1)}{4(1-h)} + \beta_i + \eta_i + \gamma_i + \mu_i\right)\right)^{\frac{1}{p_*}} \xi_i, 2 \leq i \leq n-1, \tag{3.13}$$

where $\alpha_i = (\frac{1}{h}(\lambda_i + \frac{5(n-i+1)}{4(1-h)} + \beta_i + \eta_i + \gamma_i + \mu_i))^{\frac{1}{p^*}}$ ($2 \leq i \leq n-1$), and the controller

$$u = - \sum_{j=1}^s (\frac{1}{h}(\lambda_n + \frac{1}{1-h} + \beta_n + \eta_n + \gamma_n + \mu_n))^{\frac{p_j}{p^*}} \xi_n^{p_j} = - \sum_{j=1}^s \alpha_n^{p_j} \xi_n^{p_j}, \tag{3.14}$$

where $\alpha_n = (\frac{1}{h}(\lambda_n + \frac{1}{1-h} + \beta_n + \eta_n + \gamma_n + \mu_n))^{\frac{1}{p^*}}$, $\lambda_i \geq h$ ($i = 1, 2, \dots, n$) are positive constants. According to $[h_n(t) - h]\xi_n^3 u \leq 0$, we can obtain the following inequality by substituting (3.12), (3.13), (3.14) into (3.12)

$$\mathcal{L}V \leq - \sum_{i=1}^n \sum_{j=1}^s \lambda_i \xi_i^{3+p_j}. \tag{3.15}$$

Based on (3.5) and (3.15), we state the main result in this paper.

Theorem 3.1 Under Assumptions 1 and 2, there is a state feedback controller (3.14) such that the closed-loop stochastic system consisting of (1.1) and controller (3.14) has a global unique solution, and the equilibrium at the origin is GAS in probability.

Proof By $V = \frac{1}{4} \sum_{i=1}^n \xi_i^4 + \int_{t-d(t)}^t W(\xi(\sigma))d\sigma \geq \frac{1}{4} \sum_{i=1}^n \xi_i^4$ and Lemma 2.6, one has

$$\frac{1}{4} \sum_{i=1}^n \xi_i^4 \geq \frac{1}{4n} (\sqrt{\sum_{i=1}^n \xi_i^2})^4.$$

According to Lemma 2.3, there is a \mathcal{K}_∞ function $\alpha_1(\xi) = \frac{1}{4n}\xi^4$ such that $\alpha_1(|\xi|) \leq V(\xi)$. Furthermore, by mean value theorem, one obtains

$$\begin{aligned} V &= \frac{1}{4} \sum_{i=1}^n \xi_i^4 + \int_{t-d(t)}^t W(\xi(\sigma))d\sigma \\ &\leq \frac{1}{4} \sum_{i=1}^n \sup_{-d \leq s \leq 0} \xi_i^4(t+s) + \frac{nd}{2(1-h)} \sum_{i=1}^n \sum_{j=1}^s \sup_{-d \leq s \leq 0} \xi_i^{3+p_j}(t+s) \\ &\leq \frac{1}{4} (\sqrt{\sum_{i=1}^n \sup_{-d \leq s \leq 0} \xi_i^2(t+s)})^4 + \frac{nd}{2(1-h)} \sum_{j=1}^s (\sqrt{\sum_{i=1}^n \sup_{-d \leq s \leq 0} \xi_i^2(t+s)})^{3+p_j}. \end{aligned}$$

Define another \mathcal{K}_∞ function $\alpha_2(\xi) = \frac{1}{4}\xi^4 + \frac{nd}{2(1-h)} \sum_{j=1}^s \xi^{3+p_j}$, we have

$$\alpha_2(\sup_{-d \leq s \leq 0} |\xi(t+s)|) = \frac{1}{4} (\sup_{-d \leq s \leq 0} |\xi(t+s)|)^4 + \frac{nd}{2(1-h)} \sum_{j=1}^s (\sup_{-d \leq s \leq 0} |\xi(t+s)|)^{3+p_j},$$

then $V(\xi, t) \leq \alpha_2(\sup_{-d \leq s \leq 0} |\xi(t+s)|)$. By $\alpha_1(|\xi|) \leq V(\xi) \leq \alpha_2(\sup_{-d \leq s \leq 0} |\xi(t+s)|)$ and (3.15), we can know that the conditions of Lemma 2.3 are founded, so there is a global unique solution for the closed-loop system consisting of (1.1) and (3.14), meanwhile, the equilibrium $\xi = 0$ is GAS in probability.

Remark 3.2 In the procedure of design and analysis of controller, the main obstacles are the appearance of high-order terms, time-varying delay, nonlinear assumptions and Hessian terms, which will unavoidably produce many more nonlinear terms and inequalities, and how to deal with them is not an easy work.

4 A Simulation Example

In this section, an example is given to illustrate the effectiveness of the proposed design approach.

Consider the following stochastic nonlinear system

$$\begin{aligned} dx_1 &= ((1.1 + 0.1 \sin(t))x_2 + (1.05 + \frac{1}{20} \sin(t))x_2^{\frac{9}{2}} + \frac{1}{4}x_1 \sin x_1 + \frac{1}{4}x_1^{\frac{9}{7}}(t - d(t)))dt \\ &\quad + \frac{1}{6}x_1^{\frac{8}{7}}(t - d(t))d\omega, \\ dx_2 &= ((1.2 + 0.2 \cos(t))u + \frac{1}{6}(x_1^{\frac{9}{7}} + x_2^{\frac{9}{2}}) + \frac{1}{3}x_2^{\frac{9}{7}}(t - d(t)))dt + \frac{1}{7}x_2^{\frac{8}{7}}(t - d(t))d\omega, \end{aligned} \quad (4.1)$$

where $p_1 = 1, p_2 = \frac{9}{7}, d(t) = \frac{1 + \sin(t)}{10}$, $f_1(t, x_1, x_1(t - d(t))) = \frac{1}{4}x_1 \sin x_1 + \frac{1}{4}x_1^{\frac{9}{7}}(t - d(t))$, $f_2(t, \bar{x}_2, \bar{x}_2(t - d(t))) = \frac{1}{6}(x_1^{\frac{9}{7}} + x_2^{\frac{9}{2}}) + \frac{1}{3}x_2^{\frac{9}{7}}(t - d(t))$ and $g_1(t, x_1, x_1(t - d(t))) = \frac{1}{6}x_1^{\frac{8}{7}}(t - d(t))$, $g_2(t, \bar{x}_2, \bar{x}_2(t - d(t))) = \frac{1}{7}x_2^{\frac{8}{7}}(t - d(t))$. Since $s = 2$ in (4.1), obviously, the previous methods are not applicable to system (4.1). It is easy to verify that Assumptions 1 and 2 are satisfied.

That is,

$$\begin{aligned} |f_1(t, x_1, x_1(t - d(t)))| &\leq \frac{1}{4}|x_1|^{\frac{9}{7}} + \frac{1}{4}|x_1(t - d(t))|^{\frac{9}{7}}, \\ |f_2(t, \bar{x}_2, \bar{x}_2(t - d(t)))| &\leq \frac{1}{6}(|x_1|^{\frac{9}{7}} + |x_2|^{\frac{9}{2}}) + \frac{1}{3}|x_2(t - d(t))|^{\frac{9}{2}}, \\ |g_1(t, x_1, x_1(t - d(t)))| &= \frac{1}{6}|x_1(t - d(t))|^{\frac{8}{7}}, \\ |g_2(t, \bar{x}_2, \bar{x}_2(t - d(t)))| &= \frac{1}{7}|x_2(t - d(t))|^{\frac{8}{7}}. \end{aligned}$$

Choosing $h = 0.1$, by following the design procedure in ‘Control design and stability analysis’, we obtain the controller

$$u = -8.54(x_2 + 3.95x_1) - 48.27(x_2 + 3.95x_1)^{\frac{9}{2}}. \quad (4.2)$$

In the simulation, we choose $\lambda_1 = \lambda_2 = 1$ and the initial values $x_1(0) = -1, x_2(0) = -1$. Figure 1 and figure 2 give the system response of the closed-loop system consisting of (4.1) and (4.2), from which, the efficiency of the controller is demonstrated.

5 Conclusion

The main work of this paper is to give the state-feedback controller for the stochastic nonlinear systems with high-order terms and time-varying delay. This controller ensures that the equilibrium point of the whole system is globally asymptotically stable in probability. In the future work, there are still a lot of problems to be solved. One is to consider more

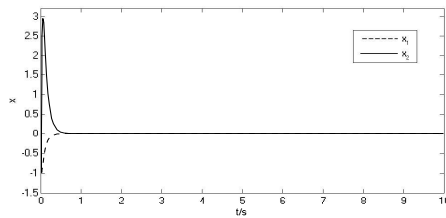


figure 1: system states

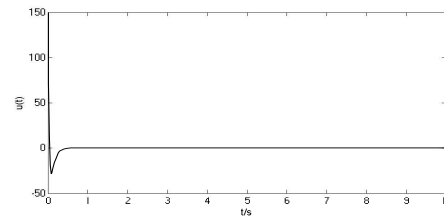


figure 2: control input

general systems with weaker assumptions. Another is to design an output feedback controller for system (1.1). The third is to deal with the stability of such a type of upper triangular system. These will be the focus of the next work.

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一类高阶随机非线性时变时滞系统的状态反馈控制

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摘要: 本文研究了一类高阶随机非线性时变时滞系统的状态反馈控制问题. 通过构造恰当的李雅普诺夫泛函, 并结合随机系统的稳定性理论, 获得了一个能使整个闭环系统为依概率全局渐近稳定的状态反馈控制器. 本文系统具有更一般的高阶项, 推广了以往单一高阶项系统依概率全局渐近稳定的结果. 数值仿真验证了所提状态反馈控制器方案的有效性.

关键词: 高阶项; 时变时滞; 状态反馈; Lyapunov-Krasovskii 理论; 随机非线性.

MR(2010)主题分类号: 34D20; 35G50 中图分类号: O231.3

Appendix

In Appendix, we give the proofs of Propositions 1–4.

Proof of Proposition 1 With the help of (3.3), we can know that $-\sum_{j=1}^s \xi_i^3 x_{i+1}^{*p_j} \geq 0$.

It follows that

$$\begin{aligned} \sum_{j=1}^s \xi_i^3 (h_{ij} x_{i+1}^{p_j} - \underline{h} x_{i+1}^{*p_j}) &\leq \sum_{j=1}^s |\xi_i^3 h_{ij} x_{i+1}^{p_j} - \xi_i^3 \underline{h} x_{i+1}^{*p_j}| \leq \sum_{j=1}^s |\xi_i^3 h_{ij} x_{i+1}^{p_j} - \xi_i^3 h_{ij} x_{i+1}^{*p_j}| \\ &\leq \bar{h} \sum_{j=1}^s |\xi_i^3 (x_{i+1}^{p_j} - x_{i+1}^{*p_j})| = \bar{h} \sum_{j=1}^s |\xi_i|^3 |x_{i+1}^{p_j} - x_{i+1}^{*p_j}|. \end{aligned}$$

By $p_j \geq 1$, Lemmas 2.4, 2.7 and (3.3), one has

$$\begin{aligned} &\bar{h} \sum_{j=1}^s |\xi_i|^3 |x_{i+1}^{p_j} - x_{i+1}^{*p_j}| \leq \bar{h} c |\xi_i|^3 |\xi_{i+1}| \sum_{j=1}^s |\xi_{i+1}^{p_j-1} + x_{i+1}^{*p_j-1}| \\ &\leq \bar{h} c \sum_{j=1}^s |\xi_i|^3 |\xi_{i+1}| (|\xi_{i+1}|^{p_j-1} + \alpha_i^{p_j-1} |\xi_i|^{p_j-1}) = \sum_{j=1}^s \bar{h} c |\xi_i|^3 |\xi_{i+1}|^{p_j} + \sum_{j=1}^s \bar{h} c \alpha_i^{p_j-1} |\xi_i|^{2+p_j} |\xi_{i+1}| \\ &\leq \frac{1}{8(1-h)} \sum_{j=1}^s \xi_i^{3+p_j} + \beta_{i+1,1} \sum_{j=1}^s \xi_{i+1}^{3+p_j} + \frac{1}{8(1-h)} \sum_{j=1}^s \xi_i^{3+p_j} + \beta_{i+1,2} \sum_{j=1}^s \xi_{i+1}^{3+p_j} \\ &= \frac{1}{4(1-h)} \sum_{j=1}^s \xi_i^{3+p_j} + \beta_{i+1} \sum_{j=1}^s \xi_{i+1}^{3+p_j} \quad (\beta_{i+1} = \beta_{i+1,1} + \beta_{i+1,2}), \end{aligned}$$

where $\beta_{i+1,1}, \beta_{i+1,2}$ are positive constants.

Proof of Proposition 2 By Lemma 2.5 and (3.3), one has

$$\sum_{j=1}^s |x_{i+1}|^{p_j} = \sum_{j=1}^s |\xi_{i+1} - \alpha_i \xi_i|^{p_j} \leq \sum_{j=1}^s s_i (|\xi_i|^{p_j} + |\xi_{i+1}|^{p_j}), \quad (5.1)$$

where $s_i = 2^{p_j-1} \max\{1, |\alpha_i|^{p_j}\}$, and define $s_0 = 1$. According to (5.1), we deduce that

$$\begin{aligned} \xi_i^3 \sum_{k=1}^{i-1} c_{i-1,k} \sum_{j=1}^s h_{kj} x_{k+1}^{p_j} &\leq |\xi_i|^3 \sum_{k=1}^{i-1} \sum_{j=1}^s c_{i-1,k} \bar{h} s_k (|\xi_k|^{p_j} + |\xi_{k+1}|^{p_j}) \\ &= \sum_{k=1}^i \sum_{j=1}^s l_{i,k} |\xi_i|^3 |\xi_k|^{p_j}, \end{aligned}$$

where

$$l_{i,k} = \begin{cases} c_{i-1,k} \bar{h} s_k, & k = 1, \\ c_{i-1,k} \bar{h} s_k + c_{i-1,k-1} \bar{h} s_{k-1}, & k = 2, 3, \dots, i-1, \\ c_{i-1,k-1} \bar{h} s_{k-1}, & k = i. \end{cases}$$

By Lemma 2.7, one has

$$\sum_{j=1}^s l_{i,k} |\xi_i|^3 |\xi_k|^{p_j} \leq \frac{1}{4(1-h)} \sum_{j=1}^s \xi_k^{3+p_j} + \eta_{i,k} \sum_{j=1}^s \xi_i^{3+p_j}.$$

So we have

$$\xi_i^3 \sum_{k=1}^{i-1} c_{i-1,k} \sum_{j=1}^s h_{kj} x_{k+1}^{p_j} \leq \sum_{k=1}^{i-1} \sum_{j=1}^s \frac{1}{4(1-h)} \xi_k^{3+p_j} + \eta_i \sum_{j=1}^s \xi_i^{3+p_j},$$

where $\eta_i = l_{i,i} + \sum_{k=1}^{i-1} \eta_{i,k}$, and $\eta_{i,k}$ is associated with $l_{i,k}$.

Proof of Proposition 3 We prove (3.9). By (3.2) and (5.1), one can obtain

$$\begin{aligned} & \xi_i^3 \sum_{k=1}^i c_{i-1,k} f_k(t, \bar{x}_k, \bar{x}_k(t-d(t))) \\ & \leq |\xi_i|^3 \sum_{k=1}^i c_{i-1,k} \sum_{j=1}^k \sum_{m=1}^s (a_{k,j} |x_j|^{p_m} + \tilde{a}_{k,j} |x_j(t-d(t))|^{p_m}) \\ & = \sum_{j=1}^i [\sum_{k=j}^i a_{k,j} c_{i-1,k}] \sum_{m=1}^s |\xi_i|^3 |x_j|^{p_m} + \sum_{j=1}^i [\sum_{k=j}^i \tilde{a}_{k,j} c_{i-1,k}] \sum_{m=1}^s |\xi_i|^3 |x_j(t-d(t))|^{p_m} \\ & \leq \sum_{k=1}^i p_{i,k} \sum_{m=1}^s |\xi_i|^3 |\xi_k|^{p_m} + \sum_{k=1}^i q_{i,k} \sum_{m=1}^s |\xi_i|^3 |\xi_k(t-d(t))|^{p_m}, \end{aligned}$$

where

$$p_{i,k} = \begin{cases} s_{k-1} \sum_{j=k}^i a_{j,k} c_{i-1,j} + s_k \sum_{j=k+1}^i a_{j,k+1} c_{i-1,j}, & k = 1, 2, \dots, i-1, \\ s_{k-1} \sum_{j=k}^i a_{j,k} c_{i-1,j}, & k = i; \end{cases}$$

$$q_{i,k} = \begin{cases} s_{k-1} \sum_{j=k}^i \tilde{a}_{j,k} c_{i-1,j} + s_k \sum_{j=k+1}^i \tilde{a}_{j,k+1} c_{i-1,j}, & k = 1, 2, \dots, i-1, \\ s_{k-1} \sum_{j=k}^i \tilde{a}_{j,k} c_{i-1,j}, & k = i. \end{cases}$$

From Lemma 2.7, such that

$$p_{i,k} \sum_{m=1}^s |\xi_i|^3 |\xi_k|^{p_m} \leq \frac{1}{4(1-h)} \sum_{m=1}^s \xi_k^{3+p_m} + \tilde{p}_{i,k} \sum_{m=1}^s \xi_i^{3+p_m},$$

$$q_{i,k} \sum_{m=1}^s |\xi_i|^3 |\xi_k(t-d(t))|^{p_m} \leq \frac{1}{4} \sum_{m=1}^s \xi_k^{3+p_m}(t-d(t)) + \tilde{q}_{i,k} \sum_{m=1}^s \xi_i^{3+p_m}.$$

So we have

$$\begin{aligned} & \xi_i^3 \sum_{k=1}^i c_{i-1,k} f_k(t, \bar{x}_k, \bar{x}_k(t-d(t))) \\ & \leq \sum_{k=1}^i \sum_{m=1}^s \frac{1}{4(1-h)} \xi_k^{3+p_m} + \sum_{k=1}^i \sum_{m=1}^s \frac{1}{4} \xi_k^{3+p_m}(t-d(t)) + \gamma_i \sum_{m=1}^s \xi_i^{3+p_m}, \end{aligned}$$

where $\gamma_i = \sum_{k=1}^i (\tilde{p}_{i,k} + \tilde{q}_{i,k})$, $\tilde{p}_{i,k}$ is associated with $p_{i,k}$ and $\tilde{q}_{i,k}$ is associated with $q_{i,k}$.

Proof of Proposition 4 We prove (3.10). Similar to Proposition 3. According to (3.2), Lemmas 2.5, 2.6 and 2.7, we deduce that

$$\begin{aligned} & \frac{3}{2} \xi_i^2 \left(\sum_{k=1}^i c_{i-1,k} g_k(t, \bar{x}_k, \bar{x}_k(t-d(t))) \right)^2 \\ & \leq \sum_{k=1}^i \sum_{m=1}^s \frac{1}{4(1-h)} \xi_k^{3+p_m} + \sum_{k=1}^i \sum_{m=1}^s \frac{1}{4} \xi_k^{3+p_m} (t-d(t)) + \mu_i \sum_{m=1}^s \xi_i^{3+p_m}, \end{aligned}$$

where μ_i is a positive constant.