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INFINITELY MANY SOLUTIONS FOR A SUPERLINEAR FRACTIONAL SCHRÖDINGER EQUATIONS

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Abstract: In this paper, we study the existence of infinitely many large energy solutions for the superlinear fractional Schrödinger equations via the Fountain Theorem in the critical point theory. In particular, we do not use the classical Ambrosetti-Rabinowitz condition, which extends the result in [12].

Keywords: fractional Schrödinger equations; superlinear; fountain theorem

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1 Introduction and Main Results

In this article, we consider the following fractional Schrödinger equations

$$(-\Delta)^s u + V(x)u = f(x,u), \quad x \in \mathbb{R}^N, \tag{1.1}$$

where $s \in (0\,,1)$, $(-\Delta)^s$ stands for the fractional Laplacian, and $f \in C(\mathbb{R}^N, \mathbb{R})$, $N > 2s$. Here the fractional Laplacian $(-\Delta)^s$ of a function $u \in \mathcal{S}$ is defined by

$$\mathcal{F}((-\Delta)^s u)(\xi) = |\xi|^{2s}\mathcal{F}(u)(\xi), \quad \forall\, s \in (0\,,1),$$

where $\mathcal{S}$ denotes the Schwartz space of rapidly decreasing C^∞ functions in $\mathbb{R}^N$, $\mathcal{F}$ is the Fourier transform, i.e.,

$$\mathcal{F}(u)(\xi) = \frac{1}{(2\pi)^{\frac{N}{2}}}\int_{\mathbb{R}^N} e^{-2\pi i\,\xi\cdot x}u(x)dx.$$

If $u \in C^\infty(\mathbb{R}^N)$, it can be computed by the following singular integral

$$(-\Delta)^s u(x) = c_{N,s}\text{P.V.}\int_{\mathbb{R}^N}\frac{u(x)-u(y)}{|x-y|^{N+2s}}dy,$$

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here P.V. is the principle value and $c_{N,s}$ is a normalization constant.

In recent years, equation (1.1) was widely studied under various conditions on f, for example [4, 5, 8–14]. Specially, in [9], Felmer, Quaas, and Tan studied the existence and regularity of positive solution of (1.1) with $V(x) = 1$ when f has subcritical growth and satisfies Ambrosetti-Rabinowitz condition, i.e., there exists $\theta > 2$ such that

$$0 < \theta F(x,u) \leq tf(x,t) \quad \forall t > 0 \text{ a.e. } x \in \mathbb{R}^N, \tag{AR}$$

where $F(x,u) = \int_0^u f(x,t)dt$. As is well-known, (AR) condition implies that the nonlinearity f is super-quadric at infinity. In [12], Secchi obtained the existence of ground state solutions of (1.1) when $V(x) \to +\infty$ as $|x| \to +\infty$ and (AR) condition holds, and Chang [4] investigated the existence of ground state solutions of (1.1) when $f(x,t)$ is asymptotically linear with respect to t at infinity.

In this article, we use the Fountain Theorem to find infinitely many large energy solutions to eq. (1.1) when the nonlinearity f does not satisfies (AR) condition. We give the following assumptions.

(V) The function $V \in C(\mathbb{R}^N, \mathbb{R})$ satisfies $\inf\limits_{x\in\mathbb{R}^N} V(x) \geq a > 0$, where $a > 0$ is a constant. Moreover, for every $M > 0$, $\text{meas}(\{x \in \mathbb{R}^N : V(x) \leq M\}) < \infty$, where meas denote the Lebesgue measure in $\mathbb{R}^N$;

(f_1) $f(x,-t) = -f(x,t)$ for any $x \in \mathbb{R}^N$;

(f_2) $f \in C(\mathbb{R}^N \times \mathbb{R}, \mathbb{R})$ and for some $2 < p < 2_s^* := \frac{2N}{N-2s}$, $c_1 > 0$,

$$|f(x,t)| \leq c_1(|t| + |t|^{p-1}), \text{ for a.e. } x \in \mathbb{R}^N$$

and $\lim\limits_{t\to 0} \frac{f(x,t)}{t} = 0$ uniformly in $x \in \mathbb{R}^N$;

(f_3) $\lim\limits_{|t|\to\infty} \frac{F(x,t)}{|t|^2} = +\infty$, uniformly in $x \in \mathbb{R}^{\mathbb{N}}$ and $F(x,0) \equiv 0$, $F(x,t) \geq 0$ for all $(x,t) \in \mathbb{R}^N \times \mathbb{R}$.

(f_4) There exists a constant $\theta \geq 1$ such that

$$\theta H(x,t) \geq H(x,rt)$$

for all $x \in \mathbb{R}^N$, $t \in \mathbb{R}$ and $r \in [0,1]$, where $H(x,t) = tf(x,t) - 2F(x,t)$.

Remark 1.1 Obviously, (f_3) can be derived from (AR). Under (AR), any (PS) sequence of the corresponding energy functional is bounded, which plays an important role of the application of variational methods. Indeed, there are many superlinear functions which do not satisfy (AR) condition. For instance, the function $f(x,t) = t\ln(1+|t|)$ does not satisfy (AR) condition.

The main result is as follows.

Theorem 1.1 Assume that (V), (f_1)–(f_4) hold. Then eq. (1.1) has infinitely many solutions $\{u_n\} \subset H^s(\mathbb{R}^N)$ satisfying

$$\frac{1}{2}\int\limits_{\mathbb{R}^N} |\xi|^{2s}|\hat{u}_n(\xi)|^2 d\xi + \frac{1}{2}\int\limits_{\mathbb{R}^N} V(x)u_n^2 dx - \int\limits_{\mathbb{R}^N} F(x,u_n)dx \to +\infty,$$

where $\hat{u} := \mathcal{F}(u)$ denotes the Fourier transform of u.

The paper is organized as follows. In Section 2, we introduce a variational setting of the problem and present some preliminary results. In Section 3, we apply Fountain Theorem to prove the existence of infinitely many weak solutions of eq.(1.1).

2 Variational Settings and Preliminary Results

In this section, we collect our basic assumptions and recall some known results for future reference. In this paper, the symbols C, C_i will be used to denote various positive constants. $B_R(0)$ denotes the ball centered at the origin with radius R.

For any $s \in (0\,,\,1)$, the fractional Sobolev space $H^s(\mathbb{R}^N)$ is defined by

$$H^s(\mathbb{R}^N) := \Big\{u \in L^2(\mathbb{R}^N) : \int\limits_{\mathbb{R}^{2N}} \frac{|u(x)-u(y)|^2}{|x-y|^{N+2s}} dxdy < +\infty\Big\},$$

endowed with the natural norm

$$\|u\|_{H^s(\mathbb{R}^N)} = \Big(\int\limits_{\mathbb{R}^N} |u|^2 dx + \int\limits_{\mathbb{R}^{2N}} \frac{|u(x)-u(y)|^2}{|x-y|^{N+2s}} dxdy\Big)^{\frac{1}{2}},$$

where the term

$$[u]_{H^s(\mathbb{R}^N)} = \Big(\int\limits_{\mathbb{R}^{2N}} \frac{|u(x)-u(y)|^2}{|x-y|^{N+2s}} dxdy\Big)^{\frac{1}{2}}$$

is the so-called Gagliardo semi-norm of u.

Moreover, one can see that an alternative definition of the fractional Sobolev space $H^s(\mathbb{R}^N)$ via the Fourier transform is as follows

$$H^s(\mathbb{R}^N) := \Big\{u \in L^2(\mathbb{R}^N) : \int\limits_{\mathbb{R}^N} (1+|\xi|^{2s})|\hat{u}(\xi)|^2 d\xi < +\infty\Big\},$$

where $\hat{u} := \mathcal{F}(u)$ denotes the Fourier transform of u. The following identity yields the relation between the fractional operator $(-\Delta)^s$ and the fractional Sobolev space $H^s(\mathbb{R}^N)$,

$$[u]_{H^s(\mathbb{R}^N)} = C\Big(\int\limits_{\mathbb{R}^N} |\xi|^{2s}|\hat{u}(\xi)|^2 d\xi\Big)^{\frac{1}{2}} = C\|(-\Delta)^{\frac{s}{2}} u\|_{L^2(\mathbb{R}^N)}$$

for a suitable positive constant C depending only on s and N.

Now, from Propositions 3.4 and 3.6 of [7], we have the norms on $H^s(\mathbb{R}^N)$,

$$\begin{aligned} &u \mapsto \|u\|_{H^s(\mathbb{R}^N)}, \\ &u \mapsto \Big(\|u\|^2_{L^2(\mathbb{R}^N)} + \|(-\Delta)^{\frac{s}{2}} u\|^2_{L^2(\mathbb{R}^N)}\Big)^{\frac{1}{2}}, \\ &u \mapsto \Big(\|u\|^2_{L^2(\mathbb{R}^N)} + \int\limits_{\mathbb{R}^N} |\xi|^{2s}|\hat{u}(\xi)|^2 d\xi\Big)^{\frac{1}{2}} \end{aligned} \tag{2.1}$$

are all equivalent. Moreover, it follows from the results of [7, 11, 12] that we have the following results for the fractional Sobolev space.

Lemma 2.1 The space $H^s(\mathbb{R}^N)$ is continuously embedded into $L^p(\mathbb{R}^N)$ for $p \in [2\,,\,2_s^*]$ and compactly embedded into $L^p_{loc}(\mathbb{R}^N)$ for $p \in [2\,,\,2_s^*)$.

In this paper, we consider a Hilbert space

$$E = \Big\{ u \in H^s(\mathbb{R}^N) : \int\limits_{\mathbb{R}^N} V(x)|u|^2 dx < +\infty \Big\}$$

with the norm by

$$\|u\| := \Big(\int\limits_{\mathbb{R}^N} (1 + |\xi|^{2s}) |\hat{u}(\xi)|^2 d\xi + \int\limits_{\mathbb{R}^N} |u|^2 dx \Big)^{\frac{1}{2}}.$$

Together with (V), it follows from (2.1) that the norm $\|\cdot\|$ is equivalent to the norm

$$\|u\|_E := \Big(\int\limits_{\mathbb{R}^N} |\xi|^{2s} |\hat{u}(\xi)|^2 d\xi + \int\limits_{\mathbb{R}^N} V(x)|u|^2 dx \Big)^{\frac{1}{2}}.$$

Then throughout out this paper, we use the norm $\|\cdot\|_E$ in E. Obviously, the embedding $E \hookrightarrow L^p(\mathbb{R}^N)$ is continuous for any $p \in [2, 2_s^*]$, and is compact for any $p \in [2, 2_s^*)$.

The energy functional $I : E \to \mathbb{R}$ of eq. (1.1) is defined by

$$I(u) = \frac{1}{2}\|u\|_E^2 - \int_{\mathbb{R}^N} F(x,u)\,\mathrm{d}x.$$

Evidently, it is well-known that I is a C^1 functional with the derivative given by

$$\langle I'(u), v\rangle = \int\limits_{\mathbb{R}^N} |\xi|^{2s} \hat{u}(\xi)\hat{v}(\xi) d\xi + \int\limits_{\mathbb{R}^N} V(x)uvdx - \int\limits_{\mathbb{R}^N} f(x,u)vdx, \tag{2.2}$$

and its critical points are the solutions of (1.1). It is easy to know that I exhibits a strong indefiniteness, namely, it is unbounded both from below and from above on infinitely dimensional subspaces. This indefiniteness can be removed using the reduction method described in [2, 15], by which we are led to study a one-variable functional that does not present such a strongly indefinite nature.

For reader's convenience, we introduce the Cerami ((C) for short) condition, which was established by Cerami [3].

Definition 2.1 Assume that the functional Φ is C^1. For $c \in \mathbb{R}$, if any sequence $\{u_n\}$ in E satisfying $\Phi(u_n) \to c$ and $(1 + \|u_n\|)\|\Phi'(u_n)\| \to 0$ has a convergence subsequence, we say Φ satisfies Cerami condition at the level c.

To complete the proof of our theorem, we need the following Fountain Theorem.

Theorem 2.1 Let X be a Banach space with the norm $\|\cdot\|$ and let X_j be a sequence of subspace of X with $\dim X_j < \infty$ for each $j \in \mathbb{N}$. Further, $X = \overline{\bigoplus_{j\in\mathbb{N}} X_j}$, the closure of the direct sum of all X_j. Set $W_k = \bigoplus_{j=0}^{k} X_j$, $Z_k = \overline{\bigoplus_{j=k}^{\infty} X_j}$. Consider an even functional $\Phi \in C^1(X, \mathbb{R})$. If, for every $k \in \mathbb{N}$, there exist $\rho_k > r_k > 0$ such that

(Φ_1) $a_k := \max_{u\in W_k, \|u\|=\rho_k} \Phi(u) \le 0$,

(Φ_2) $b_k := \inf_{u\in Z_k, \|u\|=r_k} \Phi(u) \to +\infty$, as $k\to\infty$,

(Φ_3) the Cerami condition holds at any level $c>0$. Then Φ has an unbounded sequence of critical values.

Remark 2.1 Cerami condition is weaker than (PS) condition. However, it was shown in [1] that from Cerami condition a deformation lemma follows and, as a consequence, we can also get minimax theorems.

3 Proof of Theorem

To obtain critical points of the functional I, first, we need to show that I satisfies $(\mathrm{C})_c$ condition.

Lemma 3.1 Under assumptions (f_2)–(f_4), the functional I satisfies the $(\mathrm{C})_c$ condition at any $c\in\mathbb{R}$.

Proof Assume that $\{u_n\}$ is a $(\mathrm{C})_c$ sequence, that is, for some $c\in\mathbb{R}$,

$$I(u_n) = \frac{1}{2}\|u_n\|_E^2 - \int_{\mathbb{R}^N} F(x,u_n)dx \to c \quad \text{as } n\to\infty \tag{3.1}$$

and

$$(1+\|u_n\|_E)I'(u_n)\to 0 \quad \text{as } n\to\infty. \tag{3.2}$$

From (3.1) and (3.2), for n large enough, we have

$$\begin{aligned} 1+c &\ge I(u_n) - \frac{1}{2}\langle I'(u_n),u_n\rangle \\ &= \frac{1}{2}\int_{\mathbb{R}^N} f(x,u_n)u_n dx - \int_{\mathbb{R}^N} F(x,u_n)dx. \end{aligned} \tag{3.3}$$

We claim that $\{u_n\}_n$ is bounded. Otherwise there should exist a subsequence of $\{u_n\}_n$ satisfying $\|u_n\|_E\to\infty$ as $n\to\infty$. Denote $w_n = \frac{u_n}{\|u_n\|_E}$, then $\{w_n\}_n$ is bounded. Up to a subsequence, for some $w\in E$, we obtain

$$\begin{aligned} &w_n \rightharpoonup w \quad \text{in } E, \\ &w_n\to w \quad \text{in } L^p_{\mathrm{loc}}(\mathbb{R}^N),\ \forall 2\le p<2_s^*, \\ &w_n(x)\to w(x) \quad \text{a.e. in } \mathbb{R}^N. \end{aligned} \tag{3.4}$$

We claim that $w\neq 0$ in E. In fact, if not, we assume that $w\equiv 0$. Then for any $m>0$, we define

$$\overline{w}_n = \sqrt{2m}\frac{u_n}{\|u_n\|_E} = \sqrt{2m}\, w_n.$$

By (f_2), we obtain

$$F(x,t)\le \frac{c_1}{2}|t|^2 + \frac{c_1}{p}|t|^p,\ \ \forall\, x\in\mathbb{R}^N \text{ and } t\in\mathbb{R}. \tag{3.5}$$

Therefore, due to (3.4) and (3.5), we get

$$\lim_{n\to\infty}\int\limits_{\mathbb{R}^N} F(x,\overline{w}_n)\,\mathrm{d}x \le \lim_{n\to\infty}\left(\frac{a_1}{2}\int\limits_{\mathbb{R}^N}|\overline{w}_n|^2\,\mathrm{d}x+\frac{c_1}{p}\int\limits_{\mathbb{R}^N}|\overline{w}_n|^p\,\mathrm{d}x\right)=0. \tag{3.6}$$

Moreover, the function $I(tu_n)$ is continuous in t, and there exists $\{t_n\}\subset\mathbb{R}$ such that

$$I(t_nu_n)=\max_{t\in[0,1]} I(tu_n). \tag{3.7}$$

Then for n large enough, there exists m such that $\frac{\sqrt{m}}{\|u_n\|_E}\in[0,1]$, taking account of (3.6) and (3.7), and we have

$$I(t_nu_n)\ge I(\overline{w}_n)=m-\int\limits_{\mathbb{R}^N}F(x,\overline{w}_n)\,\mathrm{d}x\ge m, \tag{3.8}$$

which implies $\lim\limits_{n\to\infty} I(t_nu_n)=+\infty$. Thus by ($\mathrm{f}_4$) we obtain

$$\begin{aligned} I(u_n)-\frac{1}{2}\langle I'(u_n),u_n\rangle &= \int\limits_{\mathbb{R}^N}\left[\frac{1}{2}f(x,u_n)u_n-F(x,u_n)\right]dx \\ &= \frac{1}{2}\int\limits_{\mathbb{R}^N}H(x,u_n)\,dx \ge \frac{1}{2\theta}\int\limits_{\mathbb{R}^N}H(x,t_nu_n)\,dx \\ &= \frac{1}{\theta}\int\limits_{\mathbb{R}^N}\left[\frac{1}{2}f(x,t_nu_n)t_nu_n-F(x,t_nu_n)\right]dx \\ &= \frac{1}{\theta}\Big(I(t_nu_n)-\frac{1}{2}\langle I'(t_nu_n),t_nu_n\rangle\Big) \\ &\to +\infty. \end{aligned}$$

This contradicts (3.3). So $w\neq 0$. Let $\Omega=\{x\in\mathbb{R}^N:\ w(x)\neq 0\}$. Dividing by $\|u_n\|_E^2$ in both sides of (3.1), we obtain

$$\int\limits_{\mathbb{R}^N}\frac{F(x,u_n)}{\|u_n\|_E^2}dx=\frac{1}{2}+\frac{c}{\|u_n\|_E^2}=\frac{1}{2}+o_n(1). \tag{3.9}$$

Then by (f_3) for all $x\in\Omega$,

$$\frac{F(x,u_n)}{\|u_n\|_E^2}=\frac{F(x,u_n)}{|u_n|^2}w_n^2(x)\to+\infty \quad \text{as } n\to\infty.$$

Since $|\Omega|>0$, using Fatou's lemma, we obtain

$$\int\limits_{\mathbb{R}^N}\frac{F(x,u_n)}{\|u_n\|_E^2}\,\mathrm{d}x\to+\infty \quad \text{as } n\to\infty.$$

This contradicts (3.9). So $\{u_n\}_n$ is bounded. In view of Lemma 2.1, up to a subsequence, we can assume that $u_n \rightharpoonup u$ in E and $u_n \to u$ in $L^p_{\rm loc}(\mathbb{R}^N)$ for any $p \in [2, 2_s^*)$. By (2.2), we easily get

$$\|u_n - u\|_E^2 = \langle I'(u_n) - I'(u), u_n - u\rangle + \int_{\mathbb{R}^N} (f(x,u_n) - f(x,u))(u_n - u)\, dx.$$

It is clear that

$$\langle I'(u_n) - I'(u), u_n - u\rangle \to 0 \text{ as } n \to +\infty.$$

Moreover, according to assumption (f_2) and Hölder inequality, we obtain

$$\begin{aligned}&\int_{\mathbb{R}^N} (f(x,u_n) - f(x,u))(u_n - u)\, dx\\ \le &\int_{\mathbb{R}^N} \left[\frac{c_1}{2}(|u_n| + |u|) + \frac{c_1}{p}\left(|u_n|^{p-1} + |u|^{p-1}\right)\right] |u_n - u|\, \mathrm{d}x\\ \le &\frac{c_1}{2}\left(\|u_n\|_{L^2}^2 + \|u\|_{L^2}^2\right)\|u_n - u\|_{L^2}^2 + \frac{c_1}{p}\left(\|u_n\|_{L^p}^{p-1} + \|u\|_{L^p}^{p-1}\right)\|u_n - u\|_{L^p}.\end{aligned}$$

Since $u_n \to u$ in $L^p_{\rm loc}(\mathbb{R}^N)$ for any $p \in [2, 2_s^*)$, we have

$$\int_{\mathbb{R}^N} (f(x,u_n) - f(x,u))(u_n - u)\, \mathrm{d}x \to 0 \quad \text{as} \quad n \to \infty.$$

Then $\|u_n - u\|_E \to 0$ implies that I satisfies $(\mathrm{C})_c$ condition for any $c \in \mathbb{R}$.

Due to Lemma 3.1, the functional I satisfies $(\mathrm{C})_c$ condition. Next, we verify that I satisfies the rest conditions of Theorem 2.1. To complete the proof of our theorem we choose an orthogonal basis $\{\varphi_j\}$ of E and define

$$W_k := \mathrm{span}\{\varphi_1, \cdots, \varphi_k\}, \quad Z_k := W_{k-1}^{\perp}.$$

Then from the Lemma 2.5 of [6], we have

Lemma 3.2 For any $2 < p < 2_s^*$, we have that

$$\beta_k := \sup_{u \in Z_k, \|u\|_E = 1} \|u\|_{L^p} \to 0 \quad \text{as} \quad k \to +\infty.$$

Proof of Theorem 1.1 First, we verify that I satisfies (Φ_1). It follows from (f_2) and (f_3) that there exist $C_1 > 0$ and $C_2 > 0$ such that

$$F(x,u) \ge C_1|u|^2 - C_2|u|$$

for any $x \in \mathbb{R}^N$ and all $u \in \mathbb{R}$. Hence we have

$$I(u) \le \frac{1}{2}\|u\|_E^2 - C_1\int_{\mathbb{R}^N} |u|^2 dx + C_2 \int_{\mathbb{R}^N} |u| dx.$$

Since, on the finitely dimensional space W_k all norms are equivalent, we have that

$$I(u) \le \Big(\frac{1}{2} - C_1 M_1\Big)\|u\|_E^2 + C_2 M_2 \|u\|_E, \quad \forall u \in W_k,$$

where M_1 and M_2 are positive constants. Now since $\frac{1}{2} - C_1 M_1 < 0$, when C_1 is large enough, it follows that

$$a_k := \max_{u \in W_k, \|u\|_E = \rho_k} I(u) \le 0$$

for some $\rho_k > 0$ large enough.

Second, we prove that I satisfies (Φ_2). By using (f_2) and (f_3), for any $\varepsilon > 0$, there is $C_\varepsilon > 0$ such that

$$|F(x,u)| \le \varepsilon |u|^2 + C_\varepsilon |u|^p, \quad \forall p \in (2, 2_s^*).$$

Then we have

$$\begin{aligned} I(u) &\ge \frac{1}{2}\|u\|_E^2 - \varepsilon \|u\|_{L^2}^2 - C_\varepsilon \|u\|_{L^p}^p \\ &\ge \Big(\frac{1}{2} - \frac{\varepsilon}{a}\Big)\|u\|_E^2 - C_\varepsilon {\beta_k}^p \|u\|_E^p, \end{aligned}$$

where $a > 0$ is a lower bound of $V(x)$ from (V) and β_k are defined in Lemma 3.2. Choosing $r_k := (C_\varepsilon\, p \beta_k^p)^{1/(2-p)}$, we obtain

$$\begin{aligned} b_k &= \inf_{u \in Z_k, \|u\|_E = r_k} I(u) \\ &\ge \inf_{u \in Z_k, \|u\|_E = r_k} \Big[\Big(\frac{1}{2} - \frac{\varepsilon}{a}\Big)\|u\|_E^2 - C_\varepsilon {\beta_k}^p \|u\|_E^p\Big] \\ &\ge \Big(\frac{1}{2} - \frac{\varepsilon}{a} - \frac{1}{p}\Big)(C_\varepsilon p \beta_k^p)^{\frac{2}{2-p}}. \end{aligned}$$

Because $\beta_k \to 0$ as $k \to 0$ and $p > 2$, we have

$$b_k \ge \Big(\frac{1}{2} - \frac{\varepsilon}{a} - \frac{1}{p}\Big)(C_\varepsilon\, p \beta_k^p)^{\frac{2}{2-p}} \to +\infty \ \text{ as } \ k \to +\infty$$

for enough small ε. This proves (Φ_2). Now, we apply Lemma 3.1 and Fountain Theorem 2.1 to complete the proof of Theorem 1.1.

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超线性分数次薛定谔方程无穷多解的存在性

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摘要: 本文研究了一类分数次薛定谔方程解的存在性问题. 利用喷泉定理, 得到了在超线性增长条件下方程存在无穷多非平凡解, 并且证明了相应解的能量是无界的. 本文中非线性项不满足Ambrosetti-Rabinowitz条件, 推广了文献[12]中的结果.

关键词: 分数次薛定谔方程; 超线性; 喷泉定理

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