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SELF-ADAPTIVE SLIDING MODE SYNCHRONIZATION OF A CLASS OF UNCERTAIN FRACTIONAL-ORDER VICTOR-CARMEN SYSTEMS

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Abstract: In this paper, we investigate the sliding mode synchronization problem of fractional-order uncertain Victor-Carmen systems. By using self-adaptive sliding mode control approach, sufficient conditions on sliding mode synchronization are provided for the fractional-order systems, which verifies that the master-slave systems of fractional-order Victor-Carmen systems are sliding mode synchronization by choosing proper sliding mode surface and controllers.

Keywords: uncertain fractional-order; Victor-Carmen systems; siding mode; self-adaptive
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1 Introduction

Recently, the chaos synchronization of fractional-order systems gained a lot of attention, such as [1–11]. Sun in [12] addressed the sliding mode synchronization problem of fractional-order uncertainty systems, which the master-slave systems can realize project synchronization. The authors in [13] studied the problem of self-adaptive sliding mode synchronization of a class of fractional-order chaos systems, which the drive-response systems achieved chaos synchronization. Chaos synchronization control problem was investigated for fractional-order systems in [14]. Zhang in [15] considered the self-adaptive trace project synchronization problem of the fractional-order Rayleigh-Duffing-like systems. Since the Victor-Carmen chaos systems involving lots of secreted key parameters and getting extensive use in communications, some results on this topic were investigated. For example, a novel chaotic systems was studied for random pulse generation in [16], and in [17], the terminal sliding mode chaos control of fractional-order systems was studied. In this paper, the problem of sliding mode synchronization of a class of fractional-order uncertain Victor-Carmen systems is tackled using self-adaptive sliding mode control approach, and sufficient conditions on sliding mode synchronization are derived for the fractional-order systems.

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Definition 1.1 (see [18]) The fractional derivative of Caputo is given as follows

$$ {}_cD^{\alpha}_{t_0,t}x(t) = D^{-(n-\alpha)}_{t_0,t}\frac{d^n}{dt^n}x(t) = \frac{1}{\Gamma(n-\alpha)}\int_{t_0}^{t}(t-\tau)^{n-\alpha-1}x^{(n)}(\tau)d\tau, n-1<\alpha<n\in Z^+. $$

2 Main Results

Consider the following integer-order Victor-Carmen systems

$$ \begin{cases} \dot{x_1} = -x_1 - \alpha x_2x_3, \\ \dot{x_2} = -x_2 + ax_3 - \beta x_1x_3, \\ \dot{x_3} = -bx_1 - ax_2 + x_3 + \gamma x_1x_2, \end{cases} \tag{2.1} $$

where $x_1, x_2, x_3 \in R^3$ are system states, $a, b, \alpha, \beta, \gamma$ are constant parameters.

The responsive systems are as follows

$$ \begin{cases} \dot{y_1} = -y_1 - \alpha y_2y_3 + \Delta f_1(y) + d_1(t) + u_1(t), \\ \dot{y_2} = -y_2 + ay_3 - \beta y_1y_3 + \Delta f_2(y) + d_2(t) + u_2(t), \\ \dot{y_3} = -by_1 - ay_2 + y_3 + \gamma y_1y_2 + \Delta f_3(y) + d_3(t) + u_3(t), \end{cases} \tag{2.2} $$

where $\Delta f_i(y)$ is uncertain, $d_i(t)$ is bounded disturbance, u_i is controller, subtracting (2.2) to (2.1), we get

$$ \begin{cases} \dot{e_1} = -e_1 - \alpha y_2y_3 + \alpha x_2x_3 + \Delta f_1(y) + d_1(t) + u_1(t), \\ \dot{e_2} = -e_2 + ae_3 - \beta y_1y_3 + \beta x_1x_3 + \Delta f_2(y) + d_2(t) + u_2(t), \\ \dot{e_3} = -be_1 - ae_2 + e_3 + \gamma y_1y_2 - \gamma x_1x_2 + \Delta f_3(y) + d_3(t) + u_3(t). \end{cases} \tag{2.3} $$

Assumption 2.1 $\Delta f_i(y)$ and $d_i(t)$ are bounded, $m_i, n_i > 0, |\Delta f_i(y)| < m_i, |d_i(t)| < n_i$.

Assumption 2.2 m_i and n_i are unknown for all $i = 1, 2, 3$.

Assumption 2.3 Definite $\Delta f_i(y) + d_i(t) = g_i(t), i = 1, 2, 3$.

Assumption 2.4 $g_i(t)$ satisfies the condition $|g_i(t)| \le \varepsilon|e_i(t)|$, where $0 < \varepsilon < 1$.

Assumption 2.5 If $e_i(t) = 0$, then $g_i(t) = 0$ and if $e_i(t) \ne 0$, then $g_i(t) \ne 0$.

Lemma 2.6 (Barbalat's lemma, see [19]) If $f(t)$ is uniform continuity in $[0, +\infty)$, and $\int_0^{+\infty} f(t)dt$ is exist, then $\lim_{t\to\infty} f(t) = 0$.

Lemma 2.7 (see [19]) If there exists a symmetric and positive-definite matrix $\mathbf{P}$ such that $\mathbf{J}(\mathbf{x}(t)) = \mathbf{x}^T(t)\mathbf{P}D^{\alpha}_t\mathbf{x}(t) < 0$, where the systems order number $0 < \alpha \le 1$, then general fractional-order autonomous systems $D^{\alpha}_t\mathbf{x}(\mathbf{t}) = \mathbf{f}(\mathbf{x}(\mathbf{t}))$ is asymptotic stable.

Theorem 2.8 Under Assumptions 2.1–2.5, choosing sliding mode function $s(t) = e_1 + e_2 + e_3$, and the following controllers

$$ \begin{aligned} u_1 &= \alpha y_2y_3 - \alpha x_2x_3 - (\hat{m}_1 + \hat{n}_1)\mathrm{sgn}(s), \\ u_2 &= -ae_3 + \beta y_1y_3 - \beta x_1x_3 - (\hat{m}_2 + \hat{n}_2)\mathrm{sgn}(s), \\ u_3 &= (b+1)e_1 + (a+1)e_2 - e_3 + \gamma(x_1x_2 - y_1y_2) - (\hat{m}_3 + \hat{n}_3 - \eta)\mathrm{sgn}(s), \end{aligned} \tag{2.4} $$

where $\eta > 0$, $\hat{m}_i$ and $\hat{n}_i$ are the estimate values of m_i and n_i, and for all $i = 1, 2, 3$, designing self-adaptive laws

$$\begin{cases} \dot{\hat{m}}_i = |s|, \hat{m}_i(0) = \hat{m}_{i0}, \\ \dot{\hat{n}}_i = |s|, \hat{n}_i(0) = \hat{n}_{i0}, \end{cases} \quad i = 1, 2, 3.$$

Then the master-slave systems (2.1) and (2.2) of integer-order Victor-Carmen systems are self-adaptive sliding mode synchronization.

Proof When the systems state moving on the sliding mode surface, then we can get $s(t) = 0, \dot{s}(t) = 0$, because

$$s(t) = e_1 + e_2 + e_3 = 0. \tag{2.5}$$

If we substitute (2.4) to (2.3), then $\dot{e_i} = -e_i + g_i(t) - (\hat{m}_i + \hat{n}_i)\text{sgn}(s), i = 1, 2$ for $s(t) = 0$, it is easy to get $\dot{e_i} = -e_i + g_i(t), i = 1, 2$. On the other hand, for $\dot{e_3} = e_1 + e_2 + g_3(t) - (\hat{m}_3 + \hat{n}_3)\text{sgn}(s) - \eta\text{sgn}(s)$, from (2.5), it is easy to get $e_1 + e_2 = -e_3$, so we get $\dot{e_3} = -e_3 + g_3(t)$, so $\dot{e_i} = -e_i + g_i(t), i = 1, 2, 3$. According to Lyapunov stability theory, when $e_i(t) \neq 0$, found Lyapunov function $V(t) = \frac{1}{2}e^2(t)$, we get

$$\dot{V}(t) = e^T(t)\dot{e}(t) = \sum_{i=1}^{3} e_i\dot{e}_i = \sum_{i=1}^{3} e_i(-e_i + g_i(t)) \leq -(1-\varepsilon)\sum_{i=1}^{3} |e_i(t)|^2 < 0.$$

So the solution of $\dot{e_i} = -e_i + g_i(t)$ convergence to zero, which is $e_i(t) \to 0, i = 1, 2, 3$. For the systems state moving on the sliding mode surface, so the solution of errors equation (2.3) is asymptotic stable, then $e_i(t) \to 0, i = 1, 2, 3$.

When the systems aren't moving on the sliding mode surface, we found Lyapunov function as $V(t) = \frac{1}{2}s^2(t) + \frac{1}{2}\sum_{i=1}^{3}\left((\hat{m}_i - m_i)^2 + (\hat{n}_i - n_i)^2\right)$, so it has

$$\begin{aligned} \dot{V} =& s\dot{s} + \sum_{i=1}^{3}(\hat{m}_i - m_i)|s| + \sum_{i=1}^{3}(\hat{n}_i - n_i)|s| \\ =& s[-e_1 + \Delta f_1(y) + d_1(t) - (\hat{m}_1 + \hat{n}_1)\text{sgn}(s) - e_2 + \Delta f_2(y) + d_2(t) - (\hat{m}_2 + \hat{n}_2)\text{sgn}(s) \\ &+ e_1 + e_2 + \Delta f_3(y) + d_3(t) - (\hat{m}_3 + \hat{n}_3)\text{sgn}(s) - \eta\text{sgn}(s)] \\ \leq& \sum_{i=1}^{3}(m_i + n_i)|s| - \sum_{i=1}^{3}(\hat{m}_i + \hat{n}_i)|s| + \sum_{i=1}^{3}(\hat{m}_i - m_i)|s| + \sum_{i=1}^{3}(\hat{n}_i - n_i)|s| - \eta|s| \\ =& -\eta|s| < 0. \end{aligned}$$

For $\dot{V} \leq -\eta|s|$, integral on the both sides

$$\int_0^t |s(\tau)|d\tau \leq \frac{-1}{\eta}\int_0^t \dot{V}(\tau)d\tau \leq \frac{V(0) - V(\infty)}{\eta} \leq \frac{V(0)}{\eta} < \infty,$$

so $s(t)$ is bounded and integrable. From Lemma 2.6, we get $s(t) \to 0 \Rightarrow e_i(t) \to 0$, so the errors converge to zero.

Consider the master systems of fractional-order Victor-Carmen systems

$$\begin{cases} D_t^q x_1 = -x_1 - \alpha x_2 x_3, \\ D_t^q x_2 = -x_2 + a x_3 - \beta x_1 x_3, \\ D_t^q x_3 = -b x_1 - a x_2 + x_3 + \gamma x_1 x_2. \end{cases} \tag{2.6}$$

Design the slave systems as following

$$\begin{cases} D_t^q y_1 = -y_1 - \alpha y_2 y_3 + \Delta f_1(y) + d_1(t) + u_1(t), \\ D_t^q y_2 = -y_2 + a y_3 - \beta y_1 y_3 + \Delta f_2(y) + d_2(t) + u_2(t), \\ D_t^q y_3 = -b y_1 - a y_2 + y_3 + \gamma y_1 y_2 + \Delta f_3(y) + d_3(t) + u_3(t), \end{cases} \tag{2.7}$$

where $\Delta f_i(y)$ is uncertainty, $y = [y_1 \;\; y_2 \;\; y_3], d_i(t)$ is bounded disturbance, u_i is controller, subtract (2.7) to (2.6), we get the following errors equation

$$\begin{cases} D_t^q e_1 = -e_1 - \alpha y_2 y_3 + \alpha x_2 x_3 + \Delta f_1(y) + d_1(t) + u_1(t), \\ D_t^q e_2 = -e_2 + a e_3 - \beta y_1 y_3 + \beta x_1 x_3 + \Delta f_2(y) + d_2(t) + u_2(t), \\ D_t^q e_3 = -b e_1 - a e_2 + e_3 + \gamma y_1 y_2 - \gamma x_1 x_2 + \Delta f_3(y) + d_3(t) + u_3(t). \end{cases} \tag{2.8}$$

Theorem 2.9 Under Assumptions 2.1–2.5, design sliding mode function $s(t) = D_t^{q-1}(e_1 + e_2 + e_3)$, choosing controller

$$\begin{aligned} u_1 &= \alpha y_2 y_3 - \alpha x_2 x_3 - (\hat{m}_1 + \hat{n}_1)\mathrm{sgn}(s), \\ u_2 &= -a e_3 + \beta y_1 y_3 - \beta x_1 x_3 - (\hat{m}_2 + \hat{n}_2)\mathrm{sgn}(s), \\ u_3 &= (b+1)e_1 + (a+1)e_2 - e_3 - \gamma y_1 y_2 + \gamma x_1 x_2 - (\hat{m}_3 + \hat{n}_3 - \eta)\mathrm{sgn}(s), \end{aligned} \tag{2.9}$$

where $\eta > 0$, $\hat{m}_i, \hat{n}_i$ are the estimate values of m_i, n_i, design self-adaptive laws

$$\begin{cases} \dot{\hat{m}}_i = |s|, \hat{m}_i(0) = \hat{m}_{i0}, \\ \dot{\hat{n}}_i = |s|, \hat{n}_i(0) = \hat{n}_{i0}, \end{cases} \quad i = 1, 2, 3.$$

Then the master-slave systems (2.6) and (2.7) of fractional-order Victor-Carmen systems are self-adaptive sliding mode synchronization.

Proof When the systems state moving on the sliding mode surface, $s(t) = 0, \dot{s}(t) = 0$, then $s(t) = D_t^{q-1}(e_1 + e_2 + e_3) = 0$, so we get $D_t^{1-q} D_t^{q-1}(e_1 + e_2 + e_3) = 0$, such that we have

$$e_1 + e_2 + e_3 = 0. \tag{2.10}$$

Substitute controller (2.9) to (2.8), we get $D_t^q e_i = -e_i + g_i(t) - (\hat{m}_i + \hat{n}_i)\mathrm{sgn}(s), i = 1, 2$, when the systems state moving on the sliding mode surface $s(t) = 0$, so it is easy to get

$$D_t^q e_i = -e_i + g_i(t), i = 1, 2.$$

On the other hand, $D_t^q e_3 = e_1 + e_2 + g_3(t) - (\hat{m}_3 + \hat{n}_3)\mathrm{sgn}(s) - \eta\mathrm{sgn}(s)$, according to (2.10), such we get $e_1 + e_2 = -e_3$ and $D_t^q e_3 = -e_3 + g_3(t)$, so it has $D_t^q e_i = -e_i + g_i(t), i = 1, 2, 3$.

According to Lemma 2.7, if $e_i(t) \neq 0$,

$$J = e^T(t)D_t^q e(t) = \sum_{i=1}^{3} e_i D_t^q e_i = \sum_{i=1}^{3} e_i(-e_i + g_i(t)) \leq -(1-\varepsilon)\sum_{i=1}^{3} |e_i(t)|^2 < 0.$$

According to Lemma 2.7, the solution of following equation $D_t^q e_i = -e_i + g_i(t)$, so $e_i(t) \to 0, i = 1,2,3$. When the systems state moving on the sliding mode surface $s(t) = 0$, then the solution of errors equation (2.8) is asymptotic stable such that we get $e_i(t) \to 0, i = 1,2,3$.

When the systems aren't moving on the sliding mode surface, we found Lyapunov function $V(t) = \frac{1}{2}s^2(t) + \frac{1}{2}\sum_{i=1}^{3}\left((\hat{m}_i - m_i)^2 + (\hat{n}_i - n_i)^2\right)$ such that we get

$$\dot{V} = s\dot{s} + \sum_{i=1}^{3}(\hat{m}_i - m_i)|s| + \sum_{i=1}^{3}(\hat{n}_i - n_i)|s| \leq -\eta|s| < 0.$$

According to Lemma 2.6, $s(t) \to 0$, so we get $e_i(t) \to 0$.

3 Numerical Simulation

In this section, the example is provided to verify the effectiveness of the proposed method. The systems appears chaos attractors, when

$$\alpha = 50, \beta = 20, \gamma = 4.1, a = 5, b = 9, q = 0.873,$$
$$\Delta f_1(y) = \cos(2\pi y_2), \Delta f_2(y) = 0.5\cos(2\pi y_3), \Delta f_3(y) = 0.3\cos(2\pi y_2),$$

the disturbance is bounded

$$d_1(t) = 0.2\cos t, d_2(t) = 0.6\sin t, d_3(t) = \cos 3t,$$
$$(\hat{m}_1, \hat{m}_2, \hat{m}_3) = (0.3, 0.5, 1), (\hat{n}_1, \hat{n}_2, \hat{n}_3) = (0.8, 0.6, 0.3).$$

From Figure 1, we see that the systems aren't getting synchronization without controller. From Figure 2, we see the systems getting rapidly synchronization with controller. From Figure 3, we see that the errors approaching zero,which verifies the systems getting chaos synchronization rapidly.

In Theorem 2.8, $g_1(t) = \cos(2\pi y_2) + 0.2\cos t, g_2(t) = 0.5\cos(2\pi y_3) + 0.6\sin t, g_3(t) = 0.3\cos(2\pi y_2) + \cos 3t, \eta = 2.5$. The uncertainty and outer disturbance as Theorem 2.9, $\eta = 3, q = 0.873$, the systems errors as Figure 4.

4 Conclusion

In this paper, we study the self-adaptive sliding mode synchronization problem of a class of fractional-order Victor-Carmen systems based on fractional-order calculus. The conclusion indicates that the systems are self-adaptive synchronization if designing appropriate

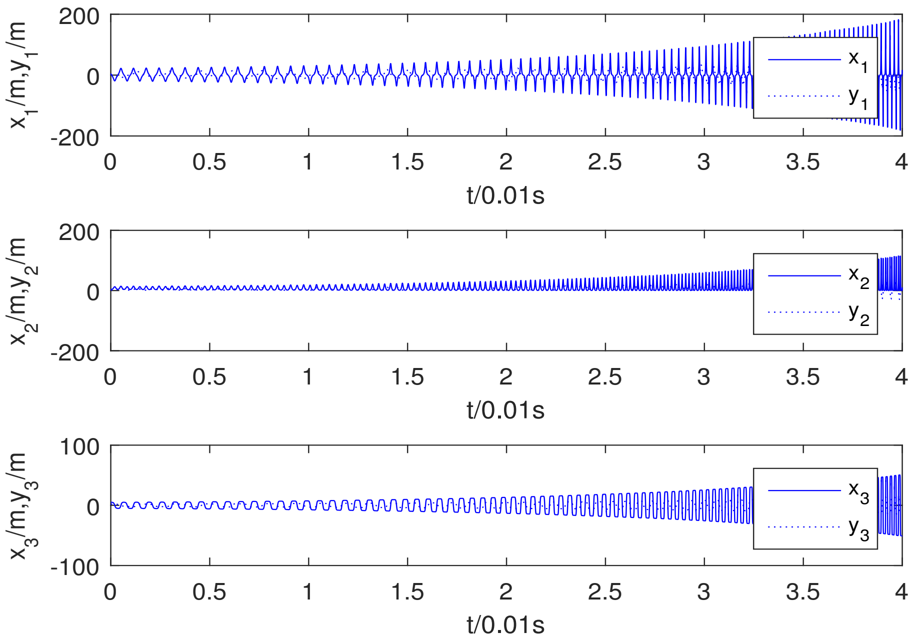

Figure 1: State of master-slave with no control

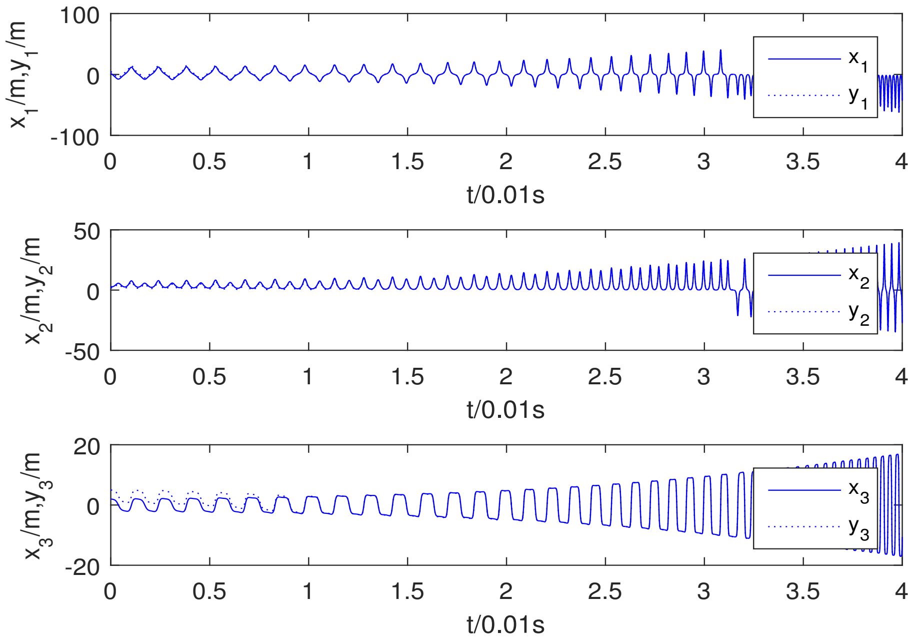

Figure 2: State of master-slave with control

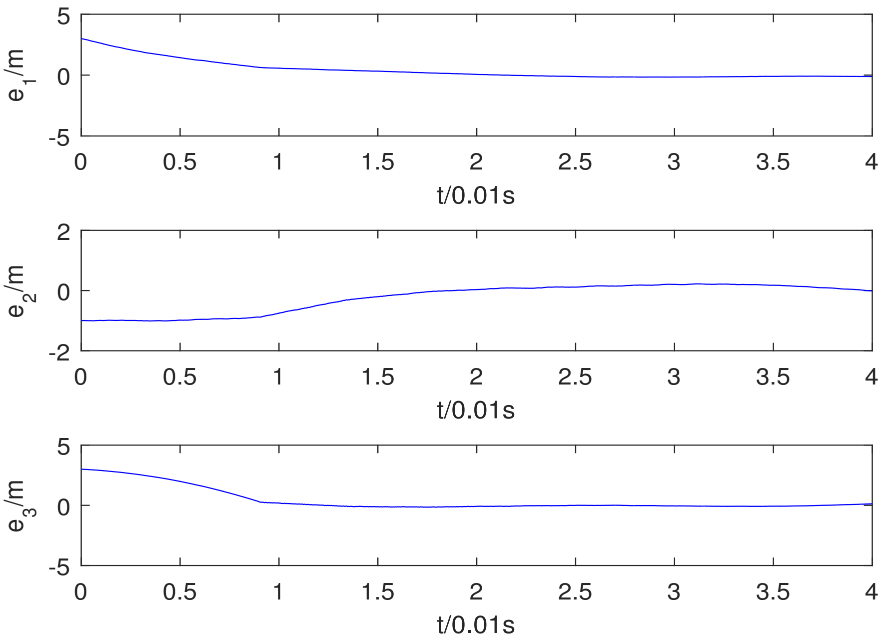

Figure 3: The system errors of Theorem 2.8

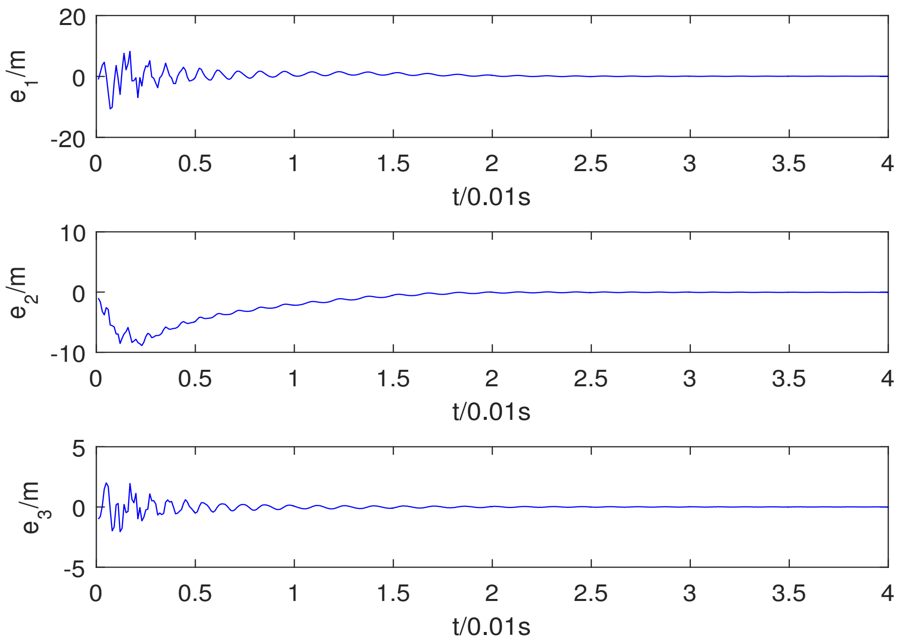

Figure 4: The system errors of Theorem 2.9

controller and sliding mode function. We give out the strict proof in mathematics, and the numerical simulation demonstrates the effectiveness of the proposed method.

References

[1] Shahiri M, Ghadri R, Ranjbar N, Hosseinnia S H, Momani S. Chaotic fractional-order coullet system: synchronization and control approach[J]. Commun. Nonl. Sci. Numer. Simul., 2010, 15(3): 665–674.

[2] Hamamci S E, Koksal M. Calculation of all stabilizing fractional-order PD controllers for integrating time delay systems[J]. Comput. Math. Appl., 2010, 59(5): 1621–1629.

[3] Matouk A E. Chaos feedback and synchronization of fractional-order modified autonomous Vanderpol-Duffing circuit[J]. Commun. Nonl. Sci. Numer. Simul., 2011, 16(2): 975–986.

[4] Ahmad W M, El-Khazali R. Fractional-order dynamical models of love[J]. Chaos Soliton Frac., 2007, 33(4): 1367–1375.

[5] Mao Beixing, Zhang Yuxia. Finite-time chaos synchronization of complex networks systems with nonlinear coupling[J]. J. Jilin Univ. (Sci. Ed.), 2015, 53(4): 757–761.

[6] Mohammad P A. Robust finite-time stabilization of fractional-order chaotic systems based on fractional Lyapunov stability theory[J]. J. Comput. Nonl. Dyn., 2012, 7(2): 1011–1015.

[7] Milad Mohadeszadeh, Hadi Delavari. Synchronization of fractional-order hyper-chaotic systems based on a new adaptive sliding mode control[J]. Int. J. Dynam. Control, 2015, 10(7): 435–446.

[8] Wang X Y, He Y J. Projective synchronization of fractional order chaotic system based on linear separation[J]. Phys. Lett. A, 2008, 372(4): 435–441.

[9] Bhat S P, Bernstein D S. Geometric homogeneity with applications to finite-time stability[J]. Math. Contr. Sig. Sys., 2005, 17(2): 101–127.

[10] Mohammad P A, Sohrab K, Ghassem A. Finite-time synchronization of two different chaotic systems with unknown parameters via sliding mode technique[J]. Appl. Math. Model., 2011, 35(6): 3080–3091.

[11] Chen Baoying, Zhang Jiajun, Yuan Zhanjiang. Sychronization of chaotic fractional-order Rucklidge systems[J]. J. Dyn. Contr., 2010, 8(3): 234–238.

[12] Sun Ning, Zhang Huaguang, Wang Zhiliang. Projective synchronization of uncertain fractional order chaotic system using sliding mode controller[J]. J. Zhejiang Univ. (Engin. Sci.), 2010, 44(7): 1288–1291.

[13] Yu M Z, Zhang Y A. Sliding mede adaptive synchronization for a class of fractional-order chaotic systems with uncertainties[J]. J. Beijing Univ. Aeron. Astro., 2014, 40(9): 1276–1280.

[14] Zhong Qilong, Shao Yonghui, Zheng Yongai. Synchronization of the fractional order chaotic systems based on TS models[J]. J. Yangzhou Univ. (Nat. Sci. Ed.), 2012, 17(2): 46–49.

[15] Zhang Yanlan. Adaptive tracking generalized projective synchronization of fractional Rayleigh-Duffling-like system[J]. J. Dyn. Contr., 2014, 12(4): 348–352.

[16] Grigoras V, Grigoras C. A novel chaotic systems for random pulse generation[J]. Adv. Electr. Comp. Engin., 2014, 14(2): 109–112.

[17] Xu Ruiping, Gao Mingmei. Synchronization of chaotic susyems with uncertainty using adaptive terminal sliding mode controller[J]. Contr. Engin. China, 2016, 23(5): 715–719.

[18] Mei Shengwei, Shen Tielong, Liu Zhikang. Modern robust control theory and application[M]. Beijing: Qinghua Univ. Pub., 2003.

[19] Hu Jianbing, Zhao Lingdong. Research of stability theory of fractional-order systems and control[J]. Acta. Phys. Sin., 2013, 62(24): 5041–5047.

一类不确定分数阶Victor-Carmen系统自适应滑模同步

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摘 要: 本文研究了分数阶不确定Victor-Carmen系统滑模同步问题. 利用自适应滑模方法, 得到了分数阶系统取得滑模同步的充分条件. 结论表明, 选择合适的滑模面和控制器, 分数阶Victor-Carmen系统的主从系统是滑模同步的.

关 键 词: 不确定分数阶; Victor-Carmen系统; 滑模; 自适应

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