

TO CONSTRUCT SOLUTIONS OF THE DIMENSIONALLY REDUCED VARIABLE-COEFFICIENT B-TYPE KADOMTSEV-PETVIASHVILI EQUATION

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Abstract: In this paper, we investigate a generalized (3+1)-dimensional variable-coefficient B-type Kadomtsev-Petviashvili equation in fluid dynamics. Based on bilinear forms, the solutions to dimensionally reduced generalized variable coefficient B-type Kadomtsev-Petviashvili equation in (3+1)-dimensions are computed through symbolic computation. The property of solutions is investigated and exhibited vividly by three dimensional plots and contour plots.

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1 Introduction

The study of nonlinear evolution equations which are regarded as the models to describe nonlinear phenomena became more attractive topic in physical science [1]. Many methods were proposed over years to address the nonlinear evolution equations, such as inverse scattering transformation [2, 3], Darboux transformation [4], Hirota method [5], Bäcklund transformation [6], Bell polynomials [7]. Through these methods, many kinds of solutions are presented. As a kind of solutions, the rational solution to some nonlinear partial differential equations are studied. The aim of this study is to use the Hirota bilinear equations to generate the generalized (3+1) dimensional variable coefficient B-type Kadomtsev-Petviashvili equation and then study the solutions to its two dimensionally reduced cases in (2+1)-dimensions.

The Kadomtsev-Petviashvili equation [8],

$$P_{KP}(u) = u_t + 6uu_t + u_{xxx} + \alpha u_{yy} = 0 \quad (1.1)$$

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is a nonlinear partial differential equation. The rational solutions were exhibited by symbolic computation [9]. Furthermore, the bilinear formulation plays a key role in the study of rational solutions, which is defined as [10]:

$$\begin{aligned} & D_{p,x}^m D_{p,t}^n f \cdot f \\ &= \left(\frac{\partial}{\partial x} + \alpha_p \frac{\partial}{\partial x'} \right)^m \left(\frac{\partial}{\partial t} + \alpha_p \frac{\partial}{\partial t'} \right)^n f(x, t) f(x', t') \Big|_{x'=x, t'=t} \\ &= \sum_{i=0}^m \sum_{j=0}^n \binom{m}{i} \binom{n}{j} \alpha_p^i \alpha_p^j \frac{\partial^{m-i}}{\partial x^{m-i}} \frac{\partial^i}{\partial x'^i} \frac{\partial^{n-j}}{\partial t^{n-j}} \frac{\partial^j}{\partial t'^j} f(x, t) f(x', t') \Big|_{x'=x, t'=t} \\ &= \sum_{i=0}^m \sum_{j=0}^n \binom{m}{i} \binom{n}{j} \alpha_p^i \alpha_p^j \frac{\partial^{m+n-i-j} f(x, t)}{\partial x^{m-i} t^{n-j}} \frac{\partial^{i+j} f(x, t)}{\partial x^i t^j}, \end{aligned}$$

where $m, n \geq 0$, $\alpha_p^s = (-1)^{r_p(s)}$, $s = r_p(s) \bmod p$.

Extended from (1.1), a generalized (3+1) dimensional variable-coefficient B-type Kadomtsev-Petviashvili equation [11],

$$P_{BKP}(u) = a(t)u_{xxxxy} + \rho a(t)(u_x u_y)_x + (u_x + u_y + u_z)_t + b(t)(u_{xx} + u_{zz}) = 0, \quad (1.2)$$

where $a(t)$ and $b(t)$ are the real function of t and ρ is a real non-zero constant.

In this study, we would like to present the bilinear form for (1.2) generated from a generalized bilinear differential equation. The solutions will be generated from polynomial solutions. In Section 2, the solutions to the first dimensionally reduced case, the second dimensionally reduced case, for $z = x$ and $z = y$, are constructed. Finally, some conclusions are given in Section 3.

2 Bilinear Forms and Solutions

Take $u = \frac{6}{\rho}(\ln f)_x$, which is suggested by bell polynomial theories [12]. A Hirota equation is proposed as follows

$$\begin{aligned} B_{BKP}(f) &= [a(t)D_x^3 D_y + D_x D_t + D_y D_t + D_z D_t + b(t)(D_x^2 + D_z^2)]f \cdot f \\ &= 2a(t)[f f_{xxxxy} - f_{xxx} f_y + 3f_{xx} f_{xy} - 3f_{xxy} f_x] \\ &\quad + 2[f_{xt} f - f_x f_t] + 2[f_{yt} f - f_y f_t] + 2[f_{zt} f - f_z f_t] \\ &\quad + 2b(t)[f_{xx} f - f_x^2 + f_{xx} f - f_x^2 + f_{zz} f - f_z^2] = 0, \end{aligned} \quad (2.1)$$

where f as a function of x, y, z , and t , the bilinear differential operators $D_x^3 D_y, D_x D_t, D_y D_t, D_x^2, D_z^2$ are the Hirota bilinear operators.

If f solves generalized equation (2.1), then $u = \frac{6}{\rho}(\ln f)_x$ present the solutions to (1.2). In this study, we construct positive quadratic function solution to the dimensionally reduced Hirota bilinear equation (2.1) for two cases: $z = x$ and $z = y$, and begin with

$$\begin{aligned} f &= g^2 + h^2 + a_9, \\ g &= a_1 x + a_2 y + a_3 t + a_4, \\ h &= a_5 x + a_6 y + a_7 t + a_8, \end{aligned} \quad (2.2)$$

where a_i ($1 \leq i \leq 9$) are all real parameters to be determined.

2.1 With $z = x$

The dimensionally reduced Hirota bilinear equation (2.1) in (2+1) dimensions

$$2a(t)[ff_{xxxxy} - f_{xxx}f_y + 3f_{xx}f_{xy} - 3f_{xxy}f_x] + 4[f_{xt}f - f_xf_t] + 2[f_{yt}f - f_yf_t] + 4b(t)[f_{xx}f - f_x^2] = 0, \quad (2.3)$$

which is transformed into

$$a(t)[u_{xxxxy} - 3(u_xu_y)_x] + 2u_{xt} + u_{yt} + 2b(t)u_{xx} = 0. \quad (2.4)$$

Through the link between f and u , substituting (2.2) into (2.3) to the following set of constraining equations for the parameters

$$\left\{ \begin{array}{l} a_1 = a_1, a_2 = a_2, \\ a_3 = -\frac{2(2a_1^3 + a_1^2a_2 - a_2a_5^2 + 2a_1a_5(a_5 + a_6))b(t)}{(2a_1 + a_2)^2 + (2a_5 + a_6)^2}, \\ a_4 = a_4, a_5 = a_5, a_6 = a_6, \\ a_7 = -\frac{2(2a_1a_2a_5 + a_1^2(2a_5 - a_6) + a_5^2(2a_5 + a_6))b(t)}{(2a_1 + a_2)^2 + (2a_5 + a_6)^2}, \\ a_8 = a_8, a_9 = -\frac{3(a_1^2 + a_5^2)(a_1a_2 + a_5a_6)((2a_1 + a_2)^2 + (2a_5 + a_6)^2)a(t)}{2(a_2a_5 - a_1a_6)^2b(t)}, \end{array} \right. \quad (2.5)$$

which satisfies the conditions

$$(a_2a_5 - a_1a_6)b(t) \neq 0, \quad (2.6)$$

$$(a_1a_2 + a_5a_6)a(t) < 0 \quad (2.7)$$

to guarantee the rational localization of the solutions. The parameters in set (2.5) yield a class of positive quadratic function solution to (2.2) as

$$f = (a_1x + a_2y - \frac{[2(2a_1^3 + a_1^2a_2 - a_2a_5^2 + 2a_1a_5(a_5 + a_6))b(t)]t}{(2a_1 + a_2)^2 + (2a_5 + a_6)^2} + a_4)^2 + (a_5x + a_6y - \frac{[2(2a_1a_2a_5 + a_1^2(2a_5 - a_6) + a_5^2(2a_5 + a_6))b(t)]t}{(2a_1 + a_2)^2 + (2a_5 + a_6)^2} + a_8)^2 - \frac{3(a_1^2 + a_5^2)(a_1a_2 + a_5a_6)((2a_1 + a_2)^2 + (2a_5 + a_6)^2)a(t)}{2(a_2a_5 - a_1a_6)^2b(t)} \quad (2.8)$$

through transformation as

$$u^{(1)} = \frac{12(a_1g + a_5h)}{\rho f} \quad (2.9)$$

and the function g , h are given as follows

$$g = a_1x + a_2y - \frac{[2(2a_1^3 + a_1^2a_2 - a_2a_5^2 + 2a_1a_5(a_5 + a_6))b(t)]t}{(2a_1 + a_2)^2 + (2a_5 + a_6)^2} + a_4,$$

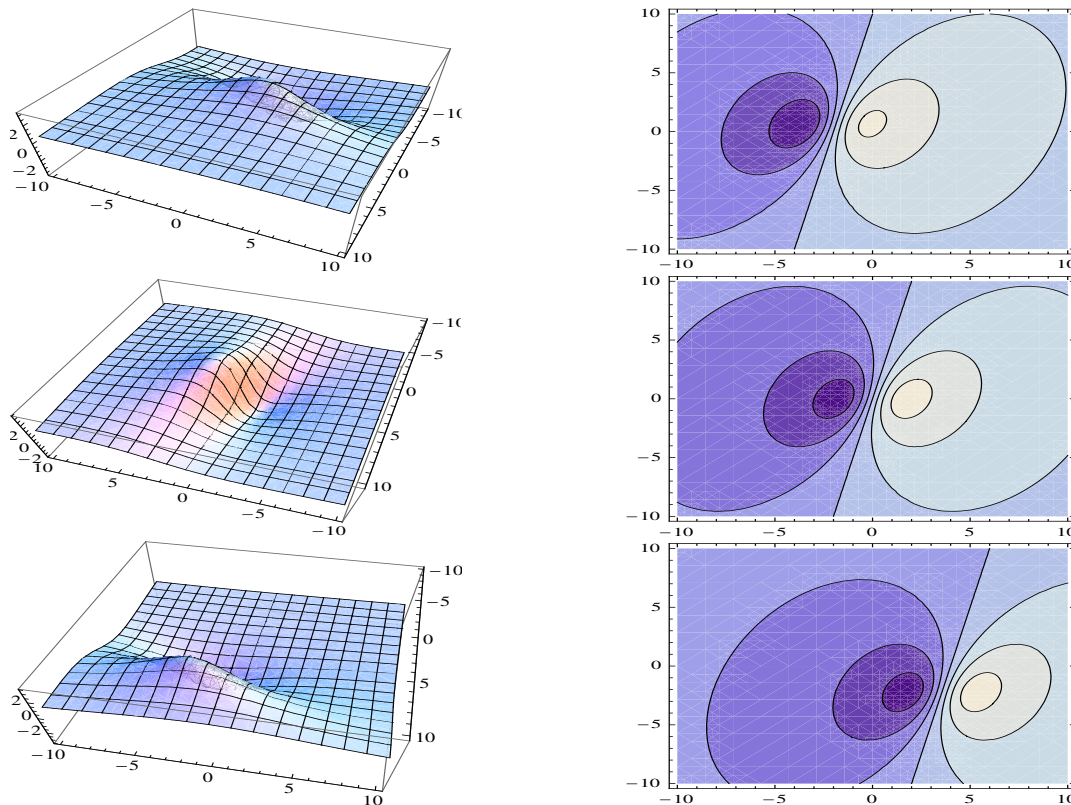


Figure 1: Profiles of (1.1) with $t = -2, 0, 4$: 3d plots (left) and contour plots (right)

$$h = a_5x + a_6y - \frac{[2(2a_1a_2a_5 + a_1^2(2a_5 - a_6) + a_5^2(2a_5 + a_6))b(t)]t}{(2a_1 + a_2)^2 + (2a_5 + a_6)^2} + a_8.$$

The solution u involves six parameters a_1, a_2, a_4, a_5, a_6 and a_8 . All six involved parameters are arbitrary and the rest are demanded to satisfy conditions (2.6) and (2.7). We choose the following special set of parameters

$$a_1 = 1, a_2 = 1, a_3 = -\frac{1}{3}, a_4 = 0, a_5 = 2,$$

$$a_6 = -1, a_7 = \frac{7}{3}, a_8 = 0, a_9 = 15.$$

2.2 With $z = y$

The dimensionally reduced Hirota bilinear equation (2.1) turn out to be

$$2a(t)[f f_{xxxy} - f_{xxx}f_y + 3f_{xx}f_{xy} - 3f_{xxy}f_x] + 2[f_{xt}f - f_x f_t] \\ + 4[f_{yt}f - f_y f_t] + 2b(t)[f_{xx}f - f_x^2 + f_{yy}f - f_y^2] = 0, \quad (2.10)$$

which is transformed into

$$a(t)[u_{xxxx} - 3(u_x u_y)_x] + u_{xt} + 2u_{yt} + b(t)(u_{xx} + u_{yy}) = 0. \quad (2.11)$$

Through the link between f and u , substituting (2.2) into (2.10), the following set of constraining equations for the parameters

$$\left\{ \begin{array}{l} a_1 = a_1, a_2 = a_2, \\ a_3 = \frac{-[a_1((a_1 + a_2)^2 + a_5^2 + 4a_5a_6 - a_6^2) + 2a_2(a_2^2 - a_5^2 + a_5a_6 + a_6^2)]b(t)}{(a_1^2 + 2a_2^2) + (a_5^2 + 2a_6^2)}, \\ a_4 = a_4, a_5 = a_5, a_6 = a_6, \\ a_7 = \frac{-[(a_1^2 - a_2^2)(a_5 - 2a_6) + 2a_1a_2(2a_5 + a_6) + (a_5 + 2a_6)(a_5^2 + a_6^2)]b(t)}{(a_1^2 + 2a_2^2) + (a_5^2 + 2a_6^2)}, \\ a_8 = a_8, a_9 = \frac{3(a_1^2 + a_5^2)(a_1a_2 + a_5a_6)[(a_1 + 2a_2)^2 + (a_5 + 2a_6)^2]a(t)}{5(a_2a_5 - a_1a_6)^2b(t)}, \end{array} \right. \quad (2.12)$$

which needs to satisfy the conditions

$$(a_2a_5 - a_1a_6)b(t) \neq 0, \quad (2.13)$$

$$(a_1a_2 + a_5a_6)a(t) < 0 \quad (2.14)$$

to guarantee the analyticity and rational localization of the solutions. The parameters in set (2.12) yield a class of positive quadratic function solution to (2.10) as

$$\begin{aligned} f = & [a_1x + a_2y - \frac{[a_1^3 + 2a_1^2a_2 + a_1(a_2^2 + a_5^2 + 4a_5a_6 - a_6^2) + 2a_2(a_2^2 - a_5^2 + a_5a_6 + a_6^2)]b(t)}{(a_1^2 + 2a_2^2) + (a_5^2 + 2a_6^2)}]_t + a_4]^2 \\ & + [a_5x + a_6y - \frac{[a_1^2(a_5 - 2a_6) - a_2^2(a_5 - 2a_6) + 2a_1a_2(2a_5 + a_6) + (a_5 + 2a_6)(a_5^2 + a_6^2)]b(t)}{(a_1^2 + 2a_2^2) + (a_5^2 + 2a_6^2)}]_t + a_8]^2 \\ & - \frac{3(a_1^2 + a_5^2)(a_1a_2 + a_5a_6)[(a_1 + 2a_2)^2 + (a_5 + 2a_6)^2]a(t)}{5(a_2a_5 - a_1a_6)^2b(t)} \end{aligned} \quad (2.15)$$

through transformation as

$$u^{(2)} = \frac{12(a_1g + a_5h)}{\rho f}, \quad (2.16)$$

and the functions g, h are given as follows

$$\begin{aligned} g = & a_1x + a_2y + a_4 \\ & - \frac{[a_1^3 + 2a_1^2a_2 + a_1(a_2^2 + a_5^2 + 4a_5a_6 - a_6^2) + 2a_2(a_2^2 - a_5^2 + a_5a_6 + a_6^2)]b(t)}{(a_1^2 + 2a_2^2) + (a_5^2 + 2a_6^2)}]_t, \\ h = & a_5x + a_6y + a_8 \\ & - \frac{[a_1^2(a_5 - 2a_6) - a_2^2(a_5 - 2a_6) + 2a_1a_2(2a_5 + a_6) + (a_5 + 2a_6)(a_5^2 + a_6^2)]b(t)}{(a_1^2 + 2a_2^2) + (a_5^2 + 2a_6^2)}]_t. \end{aligned}$$

The solution u involves six parameters a_1, a_2, a_4, a_5, a_6 and a_8 . All six involved parameters are arbitrary and the rest are demanded to satisfy (2.13) and (2.14). To get the special solutions of the equation, let us choose the following special set of the parameters

$$\begin{aligned} a_1 = -2, a_2 = 1, a_3 = \frac{1}{4}, a_4 = 0, a_5 = \frac{2}{3}, \\ a_6 = 1, a_7 = \frac{4}{3}, a_8 = 0, a_9 = \frac{32}{9}. \end{aligned}$$

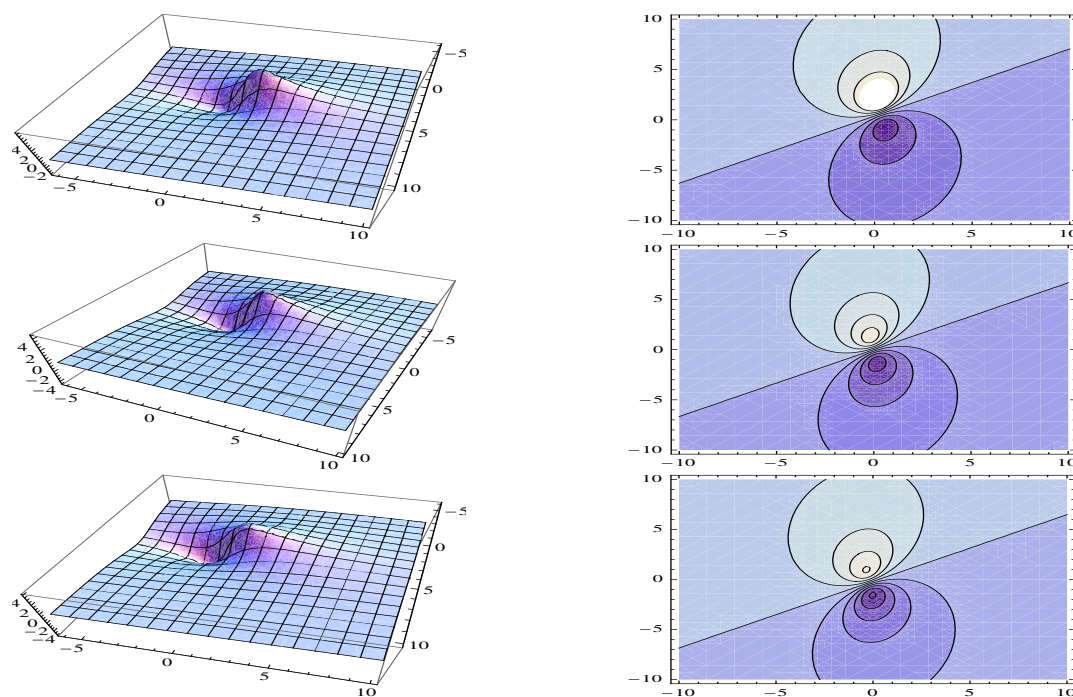


Figure 2: Profiles of (1.1) with $t = -1, 0, \frac{1}{2}$: 3d plots (left) and contour plots (right)

3 Conclusion

In this paper, via the Bell polynomials and Hirota method, we have derived bilinear form for (1.2). Then by searching for positive quadratic function solutions to (2.3) and (2.10), two classes of solutions to the dimensionally reduced equations are presented. These results provide some salutary information on the relevant fields in nonlinear science. Therefore, we expect that the results presented in this work will also be useful to study lump solutions in a variety of other high-dimensional nonlinear equations.

References

- [1] Zheng S. Nonlinear evolution equations[M]. Providence RI: Amer. Math. Soc., 1988.
- [2] Ablowitz M J, Clarkson P A. Solitons, nonlinear evolution equations and inverse scattering[M]. Cambridge: Cambridge Univ. Press, 1991.
- [3] Bekir A. Painlevé test for some (2+1)-dimensional nonlinear equations[J]. Chaos Soli. Frac., 2007, 32(2): 449–455.
- [4] Matveev V B, Salle M A. Darboux transformations and solitons[M]. Berlin: Springer, 1991.
- [5] Hirota R. The direct method in soliton theory[M]. Cambridge: Cambridge Univ. Press, 2004.
- [6] Xing L, Ma W X, Khaliq C M. A direct bilinear Bäcklund transformation of a (2+1)-dimensional Korteweg-de vries-like model[J]. Appl. Math. Lett., 2015, 50: 37–42.
- [7] Ma W X. Trilinear equations, Bell polynomials, and resonant solutions[J]. Front. Math. China, 2013, 8(5): 1139–1156.

- [8] Zhang H. A note on exact complex travelling wave solutions for (2+1)-dimensional B-type Kadomtsev-Petviashvili equation[J]. Appl. Math. Comput., 2010, 216(9): 2771–2777.
- [9] Ma W X. Lump solutions to the Kadomtsev-Petviashvili equation[J]. Phys. Lett. A, 2015, 379(36): 1975–1978.
- [10] Matveev V B. Generalized Wronskian formula for solutions of the KdV equations: first applications[J]. Phys. Lett. A, 1992, 166(3-4): 205–208.
- [11] Gao X Y. Bäcklund transformation and shock-wave-type solutions for a generalized (3+1)-dimensional variable-coefficient B-type Kadomtsev-Petviashvili equation in fluid mechanics[J]. Ocean Engin., 2015, 96: 245–247.
- [12] Bell E T. Exponential polynomials[J]. Ann. Math., 1934, 35(2): 258–277.

构造维数约化的变系数的B-type KP方程的解

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摘要: 本文研究了流体动力学中一个一般形式的(3+1)维的非线性变系数的B-type KP方程解的问题. 利用双线性化及符号计算的方法, 获得了维约化后的一般形式的(3+1)维的非线性变系数的B-type KP方程解, 并用三维立体图形和等值线图将所求解的性质形象地展示出来.

关键词: 有理函数解; 贝尔多项式理论; 双线性形式; 流体动力学

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