

RELATIONSHIPS BETWEEN VECTOR VARIATIONAL-LIKE INEQUALITIES AND MULTI-OBJECTIVE PROGRAMMING INVOLVING GENERALIZED ARCWISE CONNECTED FUNCTIONS

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Abstract: This paper is devoted to the study of relationships between vector variational-like inequalities and multi-objective programming under the assumption of generalized arcwise connected convexities. By employing the methods in convex analysis and nonsmooth analysis, the notion of (ρ, b) -right differential arcwise connected functions are introduced, and then some examples are presented to illustrate their existences. It discloses the close relationships between the efficient solutions or weakly efficient solutions of (ρ, b) -right differential arcwise connected multi-objective programming and the solutions of vector variational-like inequalities, which generalize those conclusions in the case of convexity in literatures, enrich and deepen the theory of vector optimization.

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1 Introduction

Convexity and its extension are of great importance in mathematical programming, which serves as an efficient concept to investigate optimality conditions. There was several extensions and generalizations for classical convexity. Among them, one meaningful notion is called arcwise connected convexity, which was first introduced by Avrie and Zang [5]. Since then, several scholars have been made efforts to establish optimality conditions and duality results in mathematical programming under the new defined generalized arcwise connected convexities. For instance: sub-arcwise connected functions [13], arcwise cone connected type-I functions [18], arcwise connected cone-convex functions [9], arcwise connected cone-quasiconvex set-valued mappings [19] and B -arcwise connected functions [22], and so on.

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In this paper, we shall introduce a kind of generalized arcwise connected convexity, named (ρ, b) -right differential arcwise connected functions, which are proposed by combining arcwise connected convexity together with B -invexity given in [6].

It is well-known that vector variational-like inequalities play an important role in the field of applied mathematics. In the investigation of vector variational-like inequalities, scholars are focused on the following two topics: one is the relationship between vector variational-like inequalities and vector optimization; the other one is the existence results of vector variational-like inequalities. Since Giannessi [11] first discussed the relationship between Minty vector variational inequalities and vector optimality problems in the case of differentiable functions, many results were obtained by scholars related to this topic. For example: Homidan and Ansari [2] discussed the relationship between a kind of generalized Minty vector variational-like inequalities and vector optimality problems in the case of non-differentiable functions; Ansari and Lee [4] introduced the Minty vector variational inequalities and the Stampacchia vector variational inequalities by defining upper Dini derivative, and presented an existence result for the solutions of these two kinds of variational inequalities; Yu [19], Yu and Liu [20] disclosed the relationships between proper efficiency of generalized cone-preinvex set-valued optimization problem and a kind of vector variational-like inequalities; Kim et al. [21] discussed solution existence, stability and global error bound for strongly pseudomonotone variational inequalities.

This paper is a further study of Stampacchia and Minty vector variational-like inequalities involving the generalized arcwise connected convex functions. Under the assumption of (ρ, b) -right differential arcwise connected convexity, we show the close relationships between two kinds of vector variational-like inequalities and multi-objective programming. This paper is arranged as follows: Section 2 is concerned with some definitions, which will be applicable in proving our results. In Section 3, some relationships between vector variational-like inequalities and multi-objective programming are derived.

2 Notations and Preliminaries

Let \mathbb{R}^n , \mathbb{R} , \mathbb{R}_+ and \mathbb{R}_{++} be n -dimensional Euclidean space, the set of real numbers, the set of nonnegative real numbers and the set of positive real numbers. For arbitrary $x, y \in \mathbb{R}^n$, the inner product between x and y is denoted by $x^T y$. The following convention for vectors in \mathbb{R}^n will be followed

$$x \leq y \Leftrightarrow x_i \leq y_i, i = 1, \dots, n, \text{ with strict inequality holding for at least one } i;$$

$$x \leq y \Leftrightarrow x_i \leq y_i, i = 1, \dots, n;$$

$$x = y \Leftrightarrow x_i = y_i, i = 1, \dots, n;$$

$$x < y \Leftrightarrow x_i < y_i, i = 1, \dots, n.$$

Let us recall the notions of arcwise connected convexity for a set and a function.

Definition 2.1 [10] A set $X \subset \mathbb{R}^n$ is said to be arcwise connected, if for any $x, u \in X$,

there exists a continuous vector-valued function $H_{x,u} : [0, 1] \rightarrow X$, named an arc, such that

$$H_{x,u}(0) = x, H_{x,u}(1) = u.$$

Definition 2.2 [5] Let φ be a real valued function defined on an arcwise connected set $X \subset \mathbb{R}^n$. Let $x, u \in X$ and $H_{x,u}$ be the arc connecting x and u in X . The function φ is said to possess a right derivative or right differential with respect to the arc $H_{x,u}$ at $t = 0$ if

$$\lim_{t \rightarrow 0^+} \frac{\varphi(H_{x,u}(t)) - \varphi(x)}{t}$$

exists. This limit is denoted by $\varphi^+(H_{x,u}(0))$.

Definition 2.3 [20] Let $X \subset \mathbb{R}^n$ be arcwise connected set. A function $\varphi : X \rightarrow \mathbb{R}$ is called to be (ρ, b) -right differential arcwise connected with respect to $H_{x,u}$ at $x \in X$, if there exist real valued functions $b : X \times X \rightarrow \mathbb{R}_+, \rho : X \times X \rightarrow \mathbb{R}$ such that

$$b(x, u)[\varphi(u) - \varphi(x)] \geq \varphi^+(H_{x,u}(0)) + \rho(x, u), \quad \forall u \in X. \tag{2.1}$$

If φ is (ρ, b) -right differential arcwise connected at for all $x \in X$, then φ is called (ρ, b) -right differential arcwise connected on X ; φ is called to be strictly (ρ, b) -right differential arcwise connected with respect to $H_{x,u}$ at $x \in X$, if equation (2.1) takes strict inequality, that is

$$b(x, u)[\varphi(u) - \varphi(x)] > \varphi^+(H_{x,u}(0)) + \rho(x, u), \quad u \neq x, \quad \forall u \in X.$$

If φ is strictly (ρ, b) -right differential arcwise connected at for all $x \in X$, then φ is called strictly (ρ, b) -right differential arcwise connected on X .

Following example enables us to illustrate the above definition.

Example 1 Considering the functions $\varphi : [0, 1] \rightarrow \mathbb{R}, b : [0, 1] \times [0, 1] \rightarrow \mathbb{R}_+$ and $\rho : [0, 1] \times [0, 1] \rightarrow \mathbb{R}$, defined by $\varphi(x) = x^2, b(x, u) = x^4, \rho(x, u) = 0$. Further, for any $u, x \in [0, 1]$, define $H_{x,u}(t) = \sqrt{(1-t^2)x^2 + t^2u^2}, \forall t \in [0, 1]$. Then by definition of right derivative of φ , we obtain $\varphi^+(H_{x,u}(0)) = 0$. Taking $x = 0$, we have $\varphi(u) - \varphi(x) = u^2$. Therefore we get

$$b(x, u)[\varphi(u) - \varphi(x)] = x^4u^2, \quad \varphi^+(H_{x,u}(0)) + \rho(x, u) = 0,$$

it is obvious that for $x \in [0, 1]$,

$$b(x, u)[\varphi(u) - \varphi(x)] \geq \varphi^+(H_{x,u}(0)) + \rho(x, u), \quad \forall u \in [0, 1].$$

Thus φ is (ρ, b) -right differential arcwise connected with respect to $H_{x,u}$ at $x = 0$.

Definition 2.4 Let $X \subset \mathbb{R}^n$ be arcwise connected set, a function $\varphi : X \rightarrow \mathbb{R}$ is called to be pseudo (ρ, b) -right differential arcwise connected with respect to $H_{x,u}$ at $x \in X$, if there exist real valued functions $b : X \times X \rightarrow \mathbb{R}_+, \rho : X \times X \rightarrow \mathbb{R}$ such that

$$\varphi^+(H_{x,u}(0)) + \rho(x, u) \geq 0 \Rightarrow b(x, u)[\varphi(u) - \varphi(x)] \geq 0, \quad \forall u \in X.$$

Equivalently,

$$b(x, u)[\varphi(u) - \varphi(x)] < 0 \Rightarrow \varphi^+(H_{x,u}(0)) + \rho(x, u) < 0, \forall u \in X.$$

Example 2 Consider the functions $\varphi : [0, 1] \mapsto \mathbb{R}$, $b : [0, 1] \times [0, 1] \mapsto \mathbb{R}_+$ and $\rho : [0, 1] \times [0, 1] \rightarrow \mathbb{R}$, defined by $\varphi(x) = x^2$, $b(x, u) = 1$, $\rho(x, u) = -2x^2$. Further, for any $u, x \in [0, 1]$, define $H_{x,u}(t) = \sqrt{(\sin(1-t)\frac{\pi}{2})x^2 + u^2}$, $\forall t \in [0, 1]$. Then we obtain

$$\varphi^+(H_{x,u}(0)) = u^2.$$

Assuming that $\varphi^+(H_{x,u}(0)) + \rho(x, u) \geq 0$, that is

$$\varphi^+(H_{x,u}(0)) + \rho(x, u) = u^2 - 2x^2 \geq 0, \forall u \in [0, 1],$$

we derive $f(u) - f(x) = u^2 - x^2 \geq 0$. This shows that φ is pseudo (ρ, b) -right differential arcwise connected with respect to $H_{x,u}$ at all $x \in [0, 1]$.

Definition 2.5 The right derivative of function $\varphi : X \subset \mathbb{R}^n \rightarrow \mathbb{R}$ is called monotone on X , if there exists real valued function $\rho : X \times X \rightarrow \mathbb{R}$ such that

$$\rho(u, x) + \varphi^+(H_{u,x}(0)) + \rho(x, u) + \varphi^+(H_{x,u}(0)) \leq 0, \forall x, u \in X.$$

Now, we give an example, which illustrate the monotonicity of right derivative for a real valued function.

Example 3 Consider the function $\varphi : (0, 1] \rightarrow \mathbb{R}$, $\rho : (0, 1] \times (0, 1] \rightarrow \mathbb{R}$ defined by

$$\varphi(x) = -\sin x^2, \quad \rho(u, x) = -(u - x)^2.$$

Further, for any $u, x \in X$, defining $H_{u,x}(t) = \sqrt{tx^{2t} + (1-t)u^2}$, $\forall t \in [0, 1]$. Then

$$\varphi^+(H_{u,x}(0)) = \lim_{t \rightarrow 0^+} \frac{\varphi(H_{u,x}(t)) - \varphi(u)}{t} = \lim_{t \rightarrow 0^+} \frac{-\sin(tx^{2t} + (1-t)u^2) + \sin u^2}{t} = (u^2 - 1) \cos u^2,$$

and $\varphi^+(H_{x,u}(0)) = (x^2 - 1) \cos x^2$. Thus for any $x, u \in (0, 1]$, one has

$$\rho(u, x) + \varphi^+(H_{u,x}(0)) + \rho(x, u) + \varphi^+(H_{x,u}(0)) = (u^2 - 1) \cos u^2 + (x^2 - 1) \cos x^2 - 2(u - x)^2.$$

It is clear that $\rho(u, x) + \varphi^+(H_{u,x}(0)) + \rho(x, u) + \varphi^+(H_{x,u}(0)) \leq 0$. Therefore the right derivative of function φ is monotone on $(0, 1]$.

From now on, unless otherwise specified we always assume that $X \subset \mathbb{R}^n$ is an arcwise connected subset of \mathbb{R}^n , $f(x) = (f_1(x), f_2(x), \dots, f_p(x))^T$, $f_i : X \rightarrow \mathbb{R}$ and f_i is (ρ, b) -right differential arcwise connected function, $i \in P = \{1, 2, \dots, p\}$.

Consider the following multi-objective programming problem

$$(MOP) \quad \begin{cases} \text{minimize} & f(x) = (f_1(x), f_2(x), \dots, f_p(x))^T, \\ \text{subject to} & x \in X. \end{cases}$$

Definition 2.6 A point $x \in X$ is said to be an efficient solution of (MOP), if there exists no $u \in X$ such that $f_i(u) - f_i(x) \leq 0, \forall i \in P$ with strict inequality for at least one i .

Definition 2.7 A point $x \in X$ is said to be a weak efficient solution of (MOP), if there exists no $u \in X$ such that $f_i(u) - f_i(x) < 0, \forall i \in P$.

Next, we are about to introduce the following Stampacchia and Minty arcwise connected vector variational-like inequalities, respectively, with also their weak formulations.

(SVVI) For a given function $\rho_i, i \in P$, find $x \in X$ such that there is no $u \in X$, fulfilling

$$(\rho_1(x, u) + f_1^+(H_{x,u}(0)), \dots, \rho_p(x, u) + f_p^+(H_{x,u}(0)))^T \leq 0.$$

(MVVI) For a given function $\rho_i, i \in P$, find $x \in X$ such that there is no $u \in X$, fulfilling

$$(\rho_1(u, x) + f_1^+(H_{u,x}(0)), \dots, \rho_p(u, x) + f_p^+(H_{u,x}(0)))^T \geq 0.$$

(WSVVI) For a given function $\rho_i, i \in P$, find $x \in X$ such that there is no $u \in X$, fulfilling

$$(\rho_1(x, u) + f_1^+(H_{x,u}(0)), \dots, \rho_p(x, u) + f_p^+(H_{x,u}(0)))^T < 0.$$

(WMVVI) For a given function $\rho_i, i \in P$, find $x \in X$ such that there is no $u \in X$, fulfilling

$$(\rho_1(u, x) + f_1^+(H_{u,x}(0)), \dots, \rho_p(u, x) + f_p^+(H_{u,x}(0)))^T > 0.$$

Example 4 Consider the functions $f : [0, 1] \rightarrow \mathbb{R}^2, \rho_i : [0, 1] \times [0, 1] \rightarrow \mathbb{R}, i = 1, 2$, defined by $f(x) = (f_1(x), f_2(x))^T, f_1(x) = x^2, f_2(x) = x^4, \forall x \in [0, 1]$, and $\rho_1(u, x) = \rho_2(u, x) = 0$ for $u, x \in [0, 1]$. Let $H_{u,x}(t) = \sqrt{(1 - t^2)u^2 + t^2x^2}, \forall t \in [0, 1]$. Then

$$f_1^+(H_{u,x}(0)) = f_2^+(H_{u,x}(0)) = 0.$$

Thus

$$(\rho_1(u, x) + f_1^+(H_{u,x}(0)), \rho_2(u, x) + f_2^+(H_{u,x}(0)))^T = (0, 0)^T.$$

It is obvious that for $x \in [0, 1]$, there exists no $u \in [0, 1]$, satisfying

$$(\rho_1(u, x) + f_1^+(H_{u,x}(0)), \rho_2(u, x) + f_2^+(H_{u,x}(0)))^T > 0.$$

Therefore, each number in $[0, 1]$ is a solution of (WMVVI).

3 Relationship between Arcwise Connected Vector Variational-Like Inequalities and Multi-Objective Programing

In this section, we work on the relationships between Stampacchia and Minty vector variational-like inequalities and multi-objective programming, which are formulated in Section 2.

Theorem 3.1 Let u be arbitrary vector in $X, b_i : X \times X \rightarrow \mathbb{R}_{++}, \rho_i : X \times X \rightarrow \mathbb{R}, i \in P$ and for each $i \in P, f_i : X \rightarrow \mathbb{R}$ is (ρ_i, b_i) -right differential arcwise connected with respect to $H_{x,u}$ at $x \in X$. If x solves (SVVI), then x is an efficient solution of (MOP).

Proof Suppose that x is not an efficient solution of (MOP), then there exists $u \in X$ such that

$$f_i(u) - f_i(x) \leq 0, \forall i \in P \quad (3.1)$$

with strict inequality for at least one i . Because, for each $i \in P$, f_i is (ρ_i, b_i) -right differential arcwise connected with respect to $H_{x,u}$ at $x \in X$, it follows from Definition 2.3 that

$$b_i(x, u)[f_i(u) - f_i(x)] \geq f_i^+(H_{x,u}(0)) + \rho_i(x, u), \forall i \in P. \quad (3.2)$$

Noticing that $b_i(x, u) > 0$, we derive from (3.1) and (3.2) that $\rho_i(x, u) + f_i^+(H_{x,u}(0)) \leq 0$ with strict inequality for at least one i . This shows that there exists $u \in X$ such that

$$(\rho_1(x, u) + f_1^+(H_{x,u}(0)), \dots, \rho_p(x, u) + f_p^+(H_{x,u}(0)))^T \leq 0,$$

which is a contradiction to the fact that x solves (SVVI).

Theorem 3.2 Let u be arbitrary vector in X , $b_i : X \times X \rightarrow \mathbb{R}_{++}$, $\rho_i : X \times X \rightarrow \mathbb{R}_+$, $i \in P$ and for each $i \in P$, $-f_i : X \rightarrow \mathbb{R}$ is strictly (ρ_i, b_i) -right differential arcwise connected with respect to $H_{x,u}$ at $x \in X$. If x is a weak efficient solution of (MOP), then x solves (SVVI).

Proof We proceed by contradiction. Assume that x is a weak efficient solution of (MOP), but does not solve (SVVI). Then there exists $u \in X$ such that

$$(\rho_1(x, u) + f_1^+(H_{x,u}(0)), \dots, \rho_p(x, u) + f_p^+(H_{x,u}(0)))^T \leq 0,$$

that is, there exists $u \in X$ such that

$$\rho_i(x, u) + f_i^+(H_{x,u}(0)) \leq 0, \forall i \in P \quad (3.3)$$

with strict inequality for at least one i . Because, for each $i \in P$, $-f_i$ is strictly (ρ_i, b_i) -right differential arcwise connected with respect to $H_{x,u}$ at $x \in X$, it follows from Definition 2.3 that

$$\rho_i(x, u) + (-f_i)^+(H_{x,u}(0)) < b_i(x, u)[-f_i(u) - (-f_i(x))], x \neq u, \forall u \in X \text{ and } i \in P. \quad (3.4)$$

By (3.3) and (3.4), we obtain that there exists $u \in X$ such that $f_i(u) - f_i(x) < 0, \forall i \in P$, which contributes to a contradiction. Hence x is a solution of problem (SVVI).

Theorem 3.3 Let u be arbitrary vector in X , $b_i : X \times X \rightarrow \mathbb{R}_{++}$, $\rho_i : X \times X \rightarrow \mathbb{R}$, $i \in P$ and for each $i \in P$, $f_i : X \rightarrow \mathbb{R}$ is pseudo (ρ_i, b_i) -right differential arcwise connected with respect to $H_{x,u}$ at $x \in X$. If x solves (WSVVI), then x is a weak efficient solution of (MOP).

Proof We proceed by contradiction. Assume that x is not a weak efficient solution of (MOP), namely, there exists $u \in X$ such that $f_i(u) - f_i(x) < 0, \forall i \in P$. Since, for each $i \in P$, f_i is pseudo (ρ_i, b_i) -right differential arcwise connected with respect to $H_{x,u}$ at $x \in X$, hence, for $u \in X$, we can derive

$$\rho_i(x, u) + f_i^+(H_{x,u}(0)) < 0, \forall i \in P.$$

That is, there exists $u \in X$ such that

$$(\rho_1(x, u) + f_1^+(H_{x,u}(0)), \dots, \rho_p(x, u) + f_p^+(H_{x,u}(0)))^T < 0,$$

which leads to a contradiction.

Theorem 3.4 Let $u, x \in X$ be arbitrary vectors, $b_i : X \times X \rightarrow \mathbb{R}_{++}$, $\rho_i : X \times X \rightarrow \mathbb{R}$, $i \in P$ and for each $i \in P$, $f_i : X \rightarrow \mathbb{R}$ is (ρ_i, b_i) -right differential arcwise connected with respect to $H_{u,x}$ on X . If x is an efficient solution of (MOP), then x solves (MVVI).

Proof Suppose, contrary to the result, that x does not solve (MVVI), then there exists $u \in X$, such that

$$(\rho_1(u, x) + f_1^+(H_{u,x}(0)), \dots, \rho_p(u, x) + f_p^+(H_{u,x}(0)))^T \geq 0.$$

That is, there exists $u \in X$ such that

$$\rho_i(u, x) + f_i^+(H_{u,x}(0)) \geq 0, \forall i \in P \tag{3.5}$$

with strict inequality for at least one i . Because, for each $i \in P$, f_i is (ρ_i, b_i) -right differential arcwise connected with respect to $H_{u,x}$ on X , so we have

$$b_i(u, x)[f_i(x) - f_i(u)] \geq f_i^+(H_{u,x}(0)) + \rho_i(u, x), \forall i \in P.$$

By condition (3.5) and the above inequality, it follows that there exists $u \in X$, satisfying

$$f_i(u) - f_i(x) \leq 0, \forall i \in P$$

with strict inequality for at least one i , which contradicts the hypothesis that x is an efficient solution of (MOP).

Theorem 3.5 Let $u, x \in X$ be arbitrary vectors, $\rho_i : X \times X \rightarrow \mathbb{R}$, $i \in P$ and for each $i \in P$, the right derivative of $f_i : X \rightarrow \mathbb{R}$ is monotone on X . If $x \in X$ solves (WSVVI), then x solves (WMVVI).

Proof Suppose x solves (WSVVI), then there exists no $u \in X$, such that

$$(\rho_1(x, u) + f_1^+(H_{x,u}(0)), \dots, \rho_p(x, u) + f_p^+(H_{x,u}(0)))^T < 0,$$

i.e., there exists $u \in X$ such that

$$\rho_i(x, u) + f_i^+(H_{x,u}(0)) < 0, \forall i \in P. \tag{3.6}$$

Because, for each $i \in P$, the right derivative of f_i is monotone on X , we have

$$\rho_i(u, x) + f_i^+(H_{u,x}(0)) + \rho_i(x, u) + f_i^+(H_{x,u}(0)) \leq 0, \forall i \in P.$$

The above inequality can be rewritten as, for all $x, u \in X$ and $i \in P$, we get

$$\rho_i(u, x) + f_i^+(H_{u,x}(0)) \leq -[\rho_i(x, u) + f_i^+(H_{x,u}(0))]. \tag{3.7}$$

By inequalities (3.6) and (3.7), it follows that, there exists no $u \in X$ such that

$$\rho_i(u, x) + f_i^+(H_{u,x}(0)) > 0, \forall i \in P.$$

That is, there exists no $u \in X$, such that

$$(\rho_1(u, x) + f_1^+(H_{u,x}(0)), \dots, \rho_p(u, x) + f_p^+(H_{u,x}(0)))^T > 0,$$

which implies that, x solves (WMVVI).

Theorem 3.6 Let $u, x \in X$ be arbitrary vectors, $b_i : X \times X \rightarrow \mathbb{R}_{++}$, $\rho_i : X \times X \rightarrow \mathbb{R}$, $i \in P$ and for each $i \in P$, $f_i : X \rightarrow \mathbb{R}$ is strictly (ρ_i, b_i) -right differential arcwise connected with respect to $H_{u,x}$ on X . If x is a weak efficient solution of (MOP), then x solves (MVVI).

Proof We proceed by contradiction. Suppose that x is a weak efficient solution of (MOP) but does not solve (MVVI), then there exists $u \in X$, satisfying

$$(\rho_1(u, x) + f_1^+(H_{u,x}(0)), \dots, \rho_p(u, x) + f_p^+(H_{u,x}(0)))^T \geq 0,$$

that is, there exists $u \in X$ such that

$$\rho_i(u, x) + f_i^+(H_{u,x}(0)) \geq 0, \forall i \in P \quad (3.8)$$

with strict inequality for at least one i . Because, for each $i \in P$, f_i is strictly (ρ_i, b_i) -right differential arcwise connected with respect to $H_{u,x}$ on X , so we have

$$b_i(u, x)[f_i(x) - f_i(u)] > f_i^+(H_{u,x}(0)) + \rho_i(u, x), \quad x \neq u, \forall x, u \in X \text{ and } i \in P. \quad (3.9)$$

From (3.8) and (3.9), it follows, there exists $u \in X$ such that $f_i(u) - f_i(x) < 0$, $\forall i \in P$, which contradicts the fact that x is a weak efficient solution of (MOP).

Theorem 3.7 Let $u, x \in X$ be arbitrary vectors, $b_i : X \times X \rightarrow \mathbb{R}_{++}$, $\rho_i : X \times X \rightarrow \mathbb{R}$, $i \in P$ and for each $i \in P$, $f_i : X \rightarrow \mathbb{R}$ is (ρ_i, b_i) -right differential arcwise connected with respect to $H_{u,x}$ on X . If x is a weak efficient solution of (MOP), then x solves (WMVVI).

Proof The proof is similar to that of Theorem 3.4 and therefore being omitted.

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广义弧连通凸向量变分不等式与多目标规划解之间的关系

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摘要: 本文研究了广义弧连通凸性条件下向量变分不等式与多目标规划解之间关系的问题. 利用凸分析和非光滑分析的方法, 引入了一类 (ρ, b) -右可微弧连通函数的概念, 并举例说明了这类广义凸函数的存在性. 获得了 (ρ, b) -右可微弧连通凸多目标规划的有效解或弱有效解与向量变分不等式解之间存在紧密关系的结果, 推广了文献中凸性假设下的相应结论. 本文所得成果是向量优化理论研究内容的丰富和深化.

关键词: 弧连通凸性; 向量变分不等式; 单调性; 多目标规划

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