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EXTINCTION AND PERMANENCE OF A DIFFUSIVE PREDATOR-PREY MODEL WITH MODIFIED LESLIE-GOWER FUNCTIONAL RESPONSE

YANG Wen-sheng
(*School of Mathematics and Computer Science, Fujian Normal University, Fuzhou 350007, China*)

Abstract: The diffusive predator-prey system with modified Leslie-Gower functional response is studied in this paper. By using comparison principle, sufficient conditions for extinction of the prey are obtained. Furthermore, sufficient and necessary conditions which guarantee the predator and the prey species to be permanent are derived by using some lemmas, and our results complement and supplement earlier ones.

Keywords: diffusive system; modified Leslie-Gower functional response; permanence; extinction

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1 Introduction

One of dominant themes in both ecology and mathematical ecology is the dynamic relationship between predators and their prey due to its universal existence and importance in population dynamics. The investigations on predator-prey models were developed during these thirty years, and more realistic models were derived in view of laboratory experiments and observations (see [1–21] and the references therein). In [22, 23], Nindjin et al. considered a predator-prey model incorporating a modified version of Leslie-Gower functional response as well as the Holling-type II functional response

$$\begin{cases} \frac{dx}{dt} = x(a_1 - bx - \frac{c_1 y}{x+k_1}), \\ \frac{dy}{dt} = y(a_2 - \frac{c_2 y}{x+k_2}), \end{cases} \tag{1.1}$$

where (1.1) is considered associated with initial conditions $x(0) > 0, y(0) > 0$.

Model (1.1) describes a prey population x which serves as food for a predator with population y. The parameters $a_1, a_2, b, c_1, c_2, k_1, k_2$ are assumed to be only positive values: a_1 and $a_2$are the growth rate of prey x and predator y respectively, b measures the strength

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of competition among individuals of species x, c_1 is the maximum value of the per capita reduction rate of x due to y, k_1 and k_2 measure the extent to which environment provides protection to prey x and to predator y respectively, and c_2 has a similar meaning as c_1.

Recently, Tian and Weng [24] studied the following diffusive predator-prey system with modified Leslie-Gower functional response

$$\begin{cases} \frac{\partial u}{\partial t} = D\triangle u + u(1 - u - \frac{\beta_1 w^2}{u^2+k_1}), & x \in \Omega, t > 0, \\ \frac{\partial w}{\partial t} = \triangle w + \alpha w(1 - \frac{\beta_2 w}{u+k_2}), & x \in \Omega, t > 0, \\ \frac{\partial u}{\partial \mathbf{n}} = \frac{\partial w}{\partial \mathbf{n}} = 0, & x \in \partial\Omega, t > 0, \\ u(x,0) = u_0(x) \geq 0, w(x,0) = w_0(x) \geq 0, & x \in \Omega, \end{cases} \tag{1.2}$$

where $\mathbf{n}$ is the outward unit normal vector of the boundary $\partial\Omega$, u_0, w_0 are continuous functions of x. The intactness of the second equation of non-spatial system (1.2) contains a modified Leslie-Gower term $\frac{\alpha\beta_2 w}{u}$. If the growth of the predator population is of logistic form, then

$$\frac{dw}{dt} = w(\alpha - \frac{w}{C}),$$

here C measures the carry capacity set by the environmental resources. Let $C = \frac{u}{\alpha\beta_2}$, where $\frac{1}{\alpha\beta_2}$ is the conversion factor of prey into predators, then the above equation becomes

$$\frac{dw}{dt} = w(\alpha - \frac{\alpha\beta_2 w}{u}),$$

and the term $\frac{\alpha\beta_2 w}{u}$ measures the loss in the predator population due to the rarity (per capita w/u) of its favorite food. If this favorite food is lacking severely, the predator w will switch to other population, but its growth will be limited. By adding a positive constant to the denominator, the equation becomes

$$\frac{dw}{dt} = w(\alpha - \frac{\alpha\beta_2 w}{u + k_2}).$$

For more biological background of system (1.2), one could refer to [24] and the references cited therein.

Obviously, $E_1 = (0,0), E_2 = (1,0), E_3 = (0, \frac{k_2}{\beta_2})$ are the three trivial equilibrium of (1.2). Moreover, we can easily obtain that system (1.2) has a unique interior equilibrium $E_4 = (u^*, w^*)$ (i.e., $u^* > 0, w^* > 0$) if

$$(1 - \frac{\beta_1}{\beta_2^2})^2 < 3(k_1 + \frac{2\beta_1 k_2}{\beta_2^2}) \tag{1.3}$$

and

$$k_1 - \frac{\beta_1 k_2^2}{\beta_2^2} > 0 \tag{1.4}$$

hold.

Remark 1.1 We should point out that there is a print error in [24]. $(1 - \frac{\beta_1}{\beta_2^2})^2 > 3(k_1 + \frac{2\beta_1 k_2}{\beta_2^2})$ (see (1.6) in [24]) should be $(1 - \frac{\beta_1}{\beta_2^2})^2 < 3(k_1 + \frac{2\beta_1 k_2}{\beta_2^2})$.

Remark 1.2 If we only prove the existence of the positive nontrivial equilibrium, an immediate consequence of the proof of the former result is that system (1.2) has a interior equilibrium $E_4 = (u^*, w^*)$ if (1.4) holds.

As well known, the permanence is an important topics in the study of dynamics for differential equations and population models (see [10, 12]). Tian and Weng [24] obtained sufficient conditions under which system (1.2) is permanent by using comparison techniques. To the best of the authors' knowledge, for the predator-prey system (1.2), whether one could obtain the sufficient and necessary conditions which insure the permanence of system (1.2) or not is still an open problem.

The aim of this paper is, by further developing the analysis technique of Hale and Waltman [25], to obtain sufficient and necessary conditions which ensure the permanence of system (1.2). It is shown that our result supplements and complements one of the main results of Tian and Weng in [24].

2 The Model without Diffusion

For the reaction-diffusion predator-prey system (1.2), the reduced system is an ordinary differential equation of the form

$$\begin{cases} \frac{du}{dt} = u(1 - u - \frac{\beta_1 w^2}{u^2+k_1}), \\ \frac{dw}{dt} = \alpha w(1 - \frac{\beta_2 w}{u+k_2}). \end{cases} \tag{2.1}$$

Obviously, the equilibria of (2.1) consist of three trivial critical points $E_1 = (0,0), E_2 = (1,0), E_3 = (0, \frac{k_2}{\beta_2})$ on the boundary of $\Omega = \{(N,P) : N \geq 0, P \geq 0\}$. Moreover, system (2.1) has a nontrivial critical point $E_4 = (u^*, w^*)$ if (1.4) holds.

The Jacobian matrix of system (2.1) at $E_1 = (0,0)$ is

$$J_{E_1} = \begin{pmatrix} 1 & 0 \\ 0 & \alpha \end{pmatrix},$$

then it is clear that the Jacobian matrix J_{E_1} has two eigenvalues $\lambda_1 = 1, \lambda_2 = \alpha > 0$. Hence the equilibria $E_1 = (0,0)$ is unstable.

The Jacobian matrix of system (2.1) at $E_2 = (1,0)$ is

$$J_{E_2} = \begin{pmatrix} -1 & 0 \\ 0 & \alpha \end{pmatrix},$$

then it is clear that the Jacobian matrix J_{E_2} has two eigenvalues $\lambda_1 = -1, \lambda_2 = \alpha$. Hence the equilibria $E_2 = (1,0)$ is a saddle point, the unstable manifolds lie on the w-axis.

The Jacobian matrix of system (2.1) at $E_3 = (0, \frac{k_2}{\beta_2})$ is

$$J_{E_3} = \begin{pmatrix} 1 - \frac{\beta_1}{k_1}(\frac{k_2}{\beta_2})^2 & 0 \\ \frac{\alpha}{\beta_2} & -\alpha \end{pmatrix}.$$

Hence the equilibria $E_3=(0,\frac{k_2}{\beta_2})$ is a saddle point if (1.4) holds, and it is locally asymptotically stable when $1-\frac{\beta_1}{k_1}(\frac{k_2}{\beta_2})^2\le 0$.

3 Extinction of the Predator

We first show a well known conclusion on the Logistic equation.

Lemma 3.1 (see [15]) Assume that $u(x,t)$ is defined by

$$\begin{cases} \frac{\partial u}{\partial t}=d_1\triangle u+ru(1-\frac{u}{K}), & x\in\Omega,\ t>0,\\ \frac{\partial u}{\partial v}=0, & x\in\partial\Omega,\ t>0,\\ u(x,0)=u_0(x)>0, & x\in\Omega, \end{cases} \tag{3.1}$$

then $\lim\limits_{t\to\infty} u(x,t)=K$.

From [24], we can obtain that

Theorem 3.1 (see [24]) All the solutions of (1.2) are nonnegative and defined for all $t>0$. Furthermore, the nonnegative solution $(u(x,t),w(x,t))$ of (1.2) satisfies

$$\limsup_{t\to+\infty}\max_{x\in\bar\Omega}u(x,t)\le 1,\limsup_{t\to+\infty}\max_{x\in\bar\Omega}w(x,t)\le\frac{1+k_2}{\beta_2}. \tag{3.2}$$

Remark 3.1 An immediate consequence of the proof of the former result is that for a given $\varepsilon>0$,

$$\Lambda=\Big\{(u,w)\in R^2: u\in[0,1+\varepsilon], w\in[0,\frac{1+k_2}{\beta_2}+\varepsilon]\Big\}$$

is an absorbing set for system (1.2). Hence system (1.2) is dissipative.

Now, let us identify an interesting situation where the prey cannot survive.

Theorem 3.2 If $\frac14<k_1<\beta_1(\frac{k_2}{\beta_2})^2$, then $E_3=(0,\frac{k_2}{\beta_2})$ is global asymptotically stable with respect to nonnegative initial functions.

Proof Let us assume that $k_1-\beta_1(\frac{k_2}{\beta_2})^2<0$, we can easily obtain that for small enough $\varepsilon>0$,

$$k_1-\beta_1(\frac{k_2}{\beta_2}-\varepsilon)^2<0. \tag{3.3}$$

From the second equation of the system, there is a $t_1>0$ such that

$$\begin{array}{ll} \frac{\partial w}{\partial t}\ge\triangle w+\alpha w(1-\frac{\beta_2 w}{k_2}), & x\in\Omega,\ t>t_1,\\ \frac{\partial w}{\partial \mathbf{n}}=0, & x\in\partial\Omega,\ t>t_1,\\ w(x,t_1)>0, & x\in\Omega. \end{array} \tag{3.4}$$

Consider the corresponding initial value problem

$$\begin{array}{l} z'(t)=\alpha z(1-\frac{\beta_2 z}{k_2}), t>t_1,\\ z(t_1)=\min\limits_{x\in\bar\Omega}w(t_1,x)>0. \end{array} \tag{3.5}$$

An application of the comparison principle gives

$$\liminf_{t\to+\infty}\min_{x\in\bar\Omega}w(x,t)\ge\frac{k_2}{\beta_2}. \tag{3.6}$$

From (3.6), it follows that for the above $\varepsilon > 0$, there exists a $T > 0$, such that $w(x,t) > \frac{k_2}{\beta_2} - \varepsilon$ for all $t \geq T, x \in \Omega$.

Therefore from the first equation of system (1.2) and $\frac{1}{4} < k_1 < \beta_1(\frac{k_2}{\beta_2})^2$, we obtain that

$$\begin{aligned} \frac{\partial u}{\partial t} - D\triangle u &= u(1 - u - \frac{\beta_1 w^2}{u^2+k_1}) \leq u(1 - u - \frac{\beta_1(\frac{k_2}{\beta_2}-\varepsilon)^2}{u^2+k_1}) \\ &= u\Big(\frac{k_1-\beta_1(\frac{k_2}{\beta_2}-\varepsilon)^2-u(u^2-u+k_1)}{u^2+k_1}\Big) \\ &= u\Big(\frac{k_1-\beta_1(\frac{k_2}{\beta_2}-\varepsilon)^2-u[(u-\frac{1}{2})^2+k_1-\frac{1}{4}])}{u^2+k_1}\Big) < 0. \end{aligned} \tag{3.7}$$

Invoking again the comparison principle, we get that $u(x,t) \to 0$ as $t \to \infty$. Hence there is $T_1 > 0$ such that $u(x,t) < \varepsilon$ for all $t \geq T_1$.

Now, from the second equation of system (1.2), we obtain

$$\frac{\partial w}{\partial t} - \triangle w \leq \alpha w(1 - \frac{\beta_2 w}{k_2+\varepsilon}). \tag{3.8}$$

By the comparison principle, we get that $w(x,t) \leq z(t)$ for any $t \geq T_1$, where $z(t)$ is the solution of the equation $z'(t) = \alpha z(1 - \frac{\beta_2 z}{k_2+\varepsilon})$. Finally our assertion follows from the arbitrariness of ε and the fact that the solutions with positive initial data of the equation $z'(t) = \alpha z(1 - \frac{\beta_2 z}{k_2})$ tends exponentially to $z = \frac{k_2}{\beta_2}$.

From the above discussion and (3.6), we obtain that $E_3 = (0, \frac{k_2}{\beta_2})$ is global asymptotically stable.

Remark 3.2 In [24], Tian and Weng obtained the global asymptotical stability of the equilibria $E_3 = (0, \frac{k_2}{\beta_2})$ under the assumption that

$$\frac{1}{3} < k_1 < \beta_1(\frac{k_2}{\beta_2})^2. \tag{3.9}$$

Inequality (3.9) implies $\frac{1}{4} < k_1 < \beta_1(\frac{k_2}{\beta_2})^2$, but not conversely, for

$$\frac{1}{4} < \frac{1}{3} < k_1 < \beta_1(\frac{k_2}{\beta_2})^2.$$

Therefore, we have improved the global asymptotical stability condition of [24] for system (1.2). It is shown that these conditions are weaker than those of Tian and Weng [24].

4 Permanence

In this section, we will show that any nonnegative solution $(u(x,t), w(x,t))$ of (1.2) lies in a certain bounded region as $t \to \infty$ for all $x \in \Omega$.

Now, we will summarize some facts contained in Hale and Waltman, see [25], about the permanence for abstract dynamical systems.

Suppose that Ω is a complete metric space with $\Omega = \Omega_0 \cup \partial\Omega_0$ for an open set Ω_0, where $\partial\Omega_0$ is the boundary of the set Ω_0. We will typically choose Ω_0 to be the positive cone in an ordered Banach space. A flow or semiflow on Ω under which Ω_0 and $\partial\Omega_0$ are forward invariant is said to be permanent if it is dissipative and if there is a number $\eta > 0$ such that any trajectory starting in Ω_0 will be at least a distance η from $\partial\Omega_0$ for all sufficiently large t. To state a theorem implying permanence we need a few definitions. An invariant set M for the flow or semiflow is said to be isolated if it has a neighborhood U such that M is the maximal invariant subset of U. Let $\omega(\partial\Omega_0) \subset \partial\Omega_0$ denote the union of the sets $\omega(u)$ over $u \in \partial\Omega_0$ (this differs from the standard definition of the ω-limit set of a set but it is more convenient for our purposes; see [26] for a discussion). The set $\omega(\Omega_0)$ is said to be isolated if it has a covering $M = \cup_{k=1}^{n} M_k$ of pairwise disjoint, both sets M_k which are isolated and invariant with respect to the flow or semiflow both on $\partial\Omega_0$ and on $\Omega = \Omega_0 \cup \partial\Omega_0$. The covering M is then called an isolated covering. Suppose N_1 and N_2 are isolated invariant sets (not necessarily distinct). The set N_1 is said to be chained to N_2 (denoted $N_1 \to N_2$) if there exists $u \notin N_1 \cup N_2$ with $u \in W^u(N_1) \cap W^s(N_2)$ (as usual, W^u and W^s denote the unstable and stable manifolds, respectively). A finite sequence $N_1, N_2, \cdots, N_k$ of isolated invariant sets is a chain if $N_1 \to N_2 \to N_3 \to \cdots \to N_k$ (this is possible for $k = 1$ if $N_1 \to N_1$). The chain is called a cycle if $N_k = N_1$. The set $\omega(\partial\Omega_0)$ is said to be acyclic if there exists an isolated covering $\cup_{k=1}^{n} M_k$ such that no subset of $\{M_k\}$ is a cycle. We now state a lemma that can be used to establish permanence.

Lemma 4.1 (see [25]) Suppose that Ω is a complete metric space with $\Omega = \Omega_0 \cup \partial\Omega_0$, where Ω_0 is open. Suppose that a semiflow on Ω leaves both Ω_0 and $\partial\Omega_0$ forward invariant, maps bounded sets in Ω to precompact sets for $t > 0$, and is dissipative. If in addition

(1) $\omega(\partial\Omega_0)$ is isolated and acyclic,

(2) $W^s(M_k) \cap \Omega_0 = \emptyset$ for all k, where $\cup_{k=1}^{n} M_k$ is the isolated covering used in the definition of acyclicity of $\omega(\partial\Omega_0)$;

then the semiflow is permanent, i.e., there exist $\eta > 0$ such that any trajectory with initial data in Ω_0 will be bounded away from $\partial\Omega_0$ by a distance greater than η for t sufficiently large.

Now,we show that system (1.2) is permanent. From the viewpoint of biology, this implies that the two species of prey and predator will always coexist at any time and any location of the inhabit domain, no matter what their diffusion coefficients are.

Theorem 4.1 System (1.2) is permanent if and only if (1.4) holds.

Proof Let us set $F = (F_1, F_2), U = (u, w)$ and $D = \text{diag}[D, 1]$, where

$$F_1(u, w) = u(1 - u - \frac{\beta_1 w^2}{u^2 + k_1}), F_2(u, w) = \alpha w(1 - \frac{\beta_2 w}{u + k_2}).$$

Henceforth, considering also an initial condition, system (1.2) can be rewritten as

$$\begin{cases} \frac{\partial U}{\partial t}(x, t) = D\triangle U(x, t) + F(U), & x \in \Omega,\ t > 0, \\ \frac{\partial U}{\partial \mathbf{n}}(x, t) = 0, & x \in \partial\Omega,\ t > 0, \\ U(x, 0) = \varphi(x) = (\varphi_1(x), \varphi_2(x)), & x \in \Omega. \end{cases} \tag{4.1}$$

Denoting by

$$\Omega_0 = \{\varphi \in X_\Lambda : \varphi(x) > 0, x \in \overline{\Omega}\},$$

where $X_\Lambda = \Omega_0 \cup \partial\Omega_0$. Assume that operator $S(t)$ is compact for $t > 0$, which is defined by $[S(t)\varphi](x) = U(x,t;\varphi)$, where $U(x,t;\varphi)$ is the classical solution of system (1.2). Moreover, from Theorem 3.1, it follows that the semiflow $\{S(t)\}_{t\geq 0}$ is pointwise dissipative.

A direct application of the maximum principle shows that $S(t)$ is positively invariant on $\partial\Omega_0$, which in turn implies that $S(t)$ is positively invariant on Ω_0.

Now let us show that $\omega(\partial\Omega_0) = \{E_1, E_2, E_3\}$.

First, we consider the equation

$$\begin{cases} \frac{\partial u}{\partial t} = D\triangle u + u(1-u), & x\in\Omega,\ t>0, \\ \frac{\partial u}{\partial \mathbf{n}} = 0, & x\in\partial\Omega,\ t>0, \\ u(x,0) = \varphi_1(x) > 0, & x\in\Omega. \end{cases} \tag{4.2}$$

There is no loss of generality assuming that $\varphi_1(x) > 0$ for all $x \in \overline{\Omega}$. Indeed, if $\varphi_1(x) \geq 0$ for all $x \in \overline{\Omega}$ with $\varphi_1(x) \not\equiv 0$ and applying the strong maximum principle, we obtain that $u(x,t) > 0$ for all $t > 0$ and $x \in \overline{\Omega}$. One could change the origin of time to a positive time t_1 and choose $\varphi_1(x) = u(x,t_1)$.

Let $p(t)$ and $q(t)$ be the solutions of the equation $z' = z(1-z)$ such that $p(0) = m = \min\limits_{x\in\overline{\Omega}} \varphi_1(x) > 0$ and $q(0) = M = \max\limits_{x\in\overline{\Omega}} \varphi_1(x) > 0$, respectively. From the comparison principle, we obtain

$$p(t) \leq u(x,t) \leq q(t), t > 0, x \in \overline{\Omega}.$$

Taking into account that $z(t) = 1$ attracts any positive solution, we conclude that $u(x,t) \to 1$ as $t \to \infty$.

If $\varphi_1(x) \equiv 0$, we can easily get that $u(x,t) \equiv 0$.

Let us now consider

$$\begin{cases} \frac{\partial w}{\partial t} = \triangle w + \alpha w(1 - \frac{\beta_2 w}{k_2}), & x\in\Omega,\ t>0, \\ \frac{\partial w}{\partial \mathbf{n}} = 0, & x\in\partial\Omega,\ t>0, \\ w(x,0) = \varphi_2(x) > 0, & x\in\Omega. \end{cases} \tag{4.3}$$

Similarly, by the comparison principle, we obtain that $w(x,t) \to \frac{k_2}{\beta_2}$ as $t \to \infty$.

Henceforth, we conclude that $\omega(\partial\Omega_0) = \{E_1, E_2, E_3\}$ and it is isolated and acyclic. By choosing $M_1 = E_1$, $M_2 = E_2$ and $M_3 = E_3$, then $M = M_1 \cup M_2 \cup M_3$ is the covering required by Lemma 4.1.

Taking into account that $\omega(\partial\Omega_0)$ is positively invariant and the fact that the manifolds of the equilibrium E_1 lie on $\omega(\partial\Omega_0)$. Indeed, let us linearize (1.2) about E_1, obtaining the following system

$$\begin{cases} \frac{\partial u}{\partial t} = D\triangle u + u, \\ \frac{\partial w}{\partial t} = \triangle w + \alpha w, \end{cases} \tag{4.4}$$

we obtain that $W^s(E_1) \cap \Omega_0 = \emptyset$. In order to show that $W^s(E_2) \cap \Omega_0 = \emptyset$, let us linearize (1.2) about E_2, obtaining the following system

$$\begin{cases} \frac{\partial u}{\partial t} = D\triangle u - u, \\ \frac{\partial w}{\partial t} = \triangle w + \alpha w, \end{cases} \tag{4.5}$$

we get that the unstable manifold to E_2 point into the region Ω_0. Moreover, from the above performed computations we know that the stable manifold corresponding to the equilibrium E_2 lies on $\partial\Omega_0$. This implies that $W^s(E_2)\cap\Omega_0=\emptyset$.

Similarly, in order to show that $W^s(E_3)\cap\Omega_0=\emptyset$, let us linearize (1.2) about E_3, obtaining the following system

$$\begin{cases} \frac{\partial u}{\partial t}=D\triangle u+(1-\frac{\beta_1}{k_1}(\frac{k_2}{\beta_2})^2)u, \\ \frac{\partial w}{\partial t}=\triangle w+\frac{\alpha}{\beta_2}u-\alpha w, \end{cases} \tag{4.6}$$

we get that the unstable manifold to E_3 point into the region Ω_0. Moreover, from the above performed computations we know that the stable manifold corresponding to the equilibrium E_3 lies on $\partial\Omega_0$. This implies that $W^s(E_3)\cap\Omega_0=\emptyset$.

Since all hypothesis of Lemma 4.1 are fulfilled, we may conclude that system (1.2) is permanent.

Permanence implies that condition (1.4) is an immediate consequence of Section 2. Indeed, from Section 2 and Theorem 3.2 in Section 3, we can easily obtain that the trivial equilibrium E_3 of system (1.2) is locally asymptotically stable if $1-\frac{\beta_1}{k_1}(\frac{k_2}{\beta_2})^2\le 0$, which is a contradiction. Therefore, we can easily get that system (1.2) is permanent if and only if (1.4) holds.

This concludes the proof of our claim.

Remark 4.1 In [24], Tian and Weng obtained the permanence of system (1.2) under the assumption that

$$1-\frac{\beta_1}{k_1}(\frac{1+k_2}{\beta_2})^2>0. \tag{4.7}$$

Inequality (4.7) implies (1.4), but not conversely, for

$$\frac{\beta_1}{k_1}(\frac{1+k_2}{\beta_2})^2>\frac{\beta_1}{k_1}(\frac{k_2}{\beta_2})^2. \tag{4.8}$$

Therefore, we have improved the permanence condition of [24] for system (1.2). It is shown that these conditions are weaker than those of Tian and Weng [24].

Remark 4.2 Moreover, in this paper we obtain the sufficient and necessary conditions which insure the permanence of system (1.2) by using the method in [25], but Tian and Weng [24] only obtained sufficient conditions under which system (1.2) is permanent by using comparison techniques. It is shown that our result supplements and complements one of the main results of Tian and Weng in [24].

Remark 4.3 From Theorem 4.1, we can easily obtain that diffusion has no influence on the permanence of system (1.2).

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一类具有修正Leslie-Gower功能性反应的扩散捕食系统的持久性与绝灭性

杨文生

(福建师范大学数学与计算机科学学院, 福建 福州 350007)

摘 要: 本文研究了一类具有修正Leslie-Gower功能性反应的捕食者-食饵模型. 利用比较原理以及一些引理的方法, 获得了保证食饵绝灭的充分条件以及保证捕食者和食饵永久持续生存的充分必要条件, 所得结论完善和补充了前人的结果.

关键词: 扩散系统; 修正Leslie-Gower 功能性反应; 永久持续生存; 绝灭性

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