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VALUE-AT-RISK AND CONTINUOUS COHERENT RISK MEASURES ON L^P-SPACE

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Abstract: In this paper, we study the relation between value-at-risk and continuous coherent risk measures on L^p-space. By using the separation theorem for convex sets and the truncated approximation method, we obtain that VaR can be represented by continuous coherent risk measures on L^p-space. Meanwhile, we get a new method to prove the representation result for the continuous coherent risk measures on L^p-space, which extend the results in [2] from L^∞-space to L^p-space, and do some complements of that of Inoue [4], respectively.

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1 Introduction

Artzner et al. [1–2] first proposed the concept of coherent risk measures. Further, Delbaen [3] studied coherent risk measures on general probability spaces. Value-at-risk (VaR) is a popular risk measure, especially in practice. However, VaR is not of subadditivity. Artzner et al. [2] and Delbaen [3] provided expressions for VaR in terms of kinds of coherent risk measures on L^∞-space.

Coherent risk measures on L^p-space, $1 \le p < \infty$, were also studied in the literature, for example [4–9]. A natural and interesting issue is how about the relation between VaR and coherent risk measures on L^p-space. So far, we have not found any report on this issue. In this paper, we will provide an expression for VaR in terms of kinds of continuous coherent risk measures on L^p-space. Meanwhile, we also give a proof of the representation result for continuous coherent risk measures on L^p-space. It should be mentioned that Inoue [4] stated the representation for continuous coherent risk measures on L^p-space, $1 \le p \le \infty$, but only the proof in L^∞-space case was provided. It is well-known that the dual spaces of L^∞ and L^p $(1 \le p < \infty)$ are quite distinct. The proof of the representation result for continuous coherent risk measures on L^p-space $(1 \le p < \infty)$ deserves to be provided. The proof provided in this paper is different from that of Inoue [4].

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The rest of the paper is organized as follows. In Section 2, we will introduce preliminaries. The main results will be stated in Section 3. Finally, in Section 4, the proofs of the main results are given.

2 Preliminaries

In this section, we will briefly introduce the preliminaries. Let $(\Omega, \mathcal{F}, \mathbb{P})$ be a fixed probability space. We denote $\mathcal{H} = L^p(\Omega, \mathcal{F}, \mathbb{P})$ for $1 \leq p \leq +\infty$. When $1 \leq p < +\infty$, $\mathcal{H}$ is the space of random variables with finite p-order moment. $L^\infty(\Omega, \mathcal{F}, \mathbb{P})$ stands for the space of all essentially bounded random variables on $(\Omega, \mathcal{F}, \mathbb{P})$. The space $\mathcal{H}$ represents financial risk positions. Positive values of $X \in \mathcal{H}$ correspond to losses, while negative values correspond to gains. For $X \in \mathcal{H}$, define $\|X\|_p := \text{essup}|X|$, if $p = +\infty$; $[E_{\mathbb{P}}(|X|^p)]^{\frac{1}{p}}$, if $1 \leq p < \infty$, where $E_{\mathbb{P}}(|X|^p)$ means the integral of $|X|^p$ with respect to the probability $\mathbb{P}$, then $(\mathcal{H}, \|.\|_p)$ is a Banach space. For $1 \leq p < \infty$, q means the conjugate index of p, $\frac{1}{p} + \frac{1}{q} = 1$ ($q = \infty$ if $p = 1$).

We introduce more notations. Denote by $\mathfrak{M}_1(\mathbb{P})$ the set of $\mathbb{P}$-absolutely continuous probability measures on $(\Omega, \mathcal{F}, \mathbb{P})$. For $X \in \mathcal{H}$, we set $X \wedge n := \min(X, n)$, $X^+ := \max(X, 0)$ and $X^- := \max(-X, 0)$. Given a set A, $\mathbf{1}_A$ means the indicator function of A.

In general, a risk measure is defined as any function ρ from $\mathcal{H}$ to the real numbers $\mathbf{R}$. Given a position $X \in \mathcal{H}$, the quantity $\rho(X)$ is interpreted as the amount of risk capital that the holder of position X has to safely invest, in order to satisfy the regulator.

Definition 2.1 (see [4]) A risk measure ρ: $\mathcal{H} \to \mathbf{R}$ is called a coherent risk measure, if it satisfies the following four axioms

(A1) Monotonicity: $X \geq Y$ implies $\rho(X) \leq \rho(Y)$ for any $X, Y \in \mathcal{H}$.

(A2) Translation invariance: $\rho(X + a) = \rho(X) - a$ for any $X \in \mathcal{H}$ and $a \in \mathbf{R}$.

(A3) Positive homogeneity: $\rho(\lambda X) = \lambda \rho(X)$ for any $X \in \mathcal{H}$ and $\lambda \geq 0$.

(A4) Subadditivity: $\rho(X_1 + X_2) \leq \rho(X_1) + \rho(X_2)$ for any X_1, $X_2 \in \mathcal{H}$.

We call that a coherent risk measure ρ: $\mathcal{H} \to \mathbf{R}$ is continuous, if

$$\lim_{n\to\infty} \|X_n - X\|_p = 0 \text{ implies } \lim_{n\to\infty} \rho(X_n) = \rho(X)$$

for any $X_n, X \in \mathcal{H}$.

Remark 2.1 Given a coherent risk measure ρ: $\mathcal{H} \to \mathbf{R}$, ρ is continuous if and only if there exists $C \in (0, \infty)$ such that $|\rho(X)| \leq C\|X\|_p$ (see [4]).

Definition 2.2 (see [3]) If X is a real-valued random variable and $\alpha \in (0, 1)$, then we say that $q_\alpha(X)$ is an α-quantile of X if $\mathbb{P}[X < q_\alpha(X)] \leq \alpha \leq \mathbb{P}[X \leq q_\alpha(X)]$.

It is easy to see that the set of quantiles forms a closed interval with endpoints q_α^+ and q_α^-. These endpoints can be defined as

$$\begin{aligned} q_\alpha^+ &:= q_\alpha^+(X) := \inf\{x \in \mathbf{R} : \mathbb{P}[X \leq x] > \alpha\}, \\ q_\alpha^- &:= q_\alpha^-(X) := \inf\{x \in \mathbf{R} : \mathbb{P}[X \leq x] \geq \alpha\}. \end{aligned}$$

Definition 2.3 (see [10]) The quantity $VaR_\alpha(X) := -q_\alpha^+(X)$ is called the value at risk at level α for the random variable X.

Definition 2.4 (see [11]) Two random variables X and Y are called comonotonic, if there is no pair (ω_1, ω_2) in some Ω_0 with $\mathbb{P}(\Omega_0) = 1$ such that $X(\omega_1) < X(\omega_2)$ and $Y(\omega_1) > Y(\omega_2)$.

Definition 2.5 (see [11]) A risk measure ρ: $\mathcal{H} \to \mathbf{R}$ is called to be of comonotonic additivity, if for all comonotonic X_1, $X_2 \in \mathcal{H}$, we have $\rho(X_1 + X_2) = \rho(X_1) + \rho(X_2)$.

3 Main Results

In this section, we will state the representation result for continuous coherent risk measures on $\mathcal{H}$, and the relation between VaR and continuous coherent risk measures on $\mathcal{H}$.

Theorem 3.1 Let $1 \le p < \infty$. For a risk measure ρ: $\mathcal{H} \to \mathbf{R}$, the following conditions are equivalent :

(1) The risk measure ρ is a continuous coherent risk measure.

(2) There exists a set G of nonnegative random variables g with $E_{\mathbb{P}}[g] = 1$ such that

$$\sup_{g \in G} \|g\|_q < \infty, \tag{3.1}$$

$$\rho(X) = \sup_{g \in G} E_{\mathbb{P}}[(-X)g] \tag{3.2}$$

for any $X \in \mathcal{H}$.

(3) There exists a set Δ of $\mathfrak{M}_1(\mathbb{P})$ such that

$$\sup_{\mathbb{Q} \in \Delta} \|\frac{d\mathbb{Q}}{d\mathbb{P}}\|_q < \infty, \tag{3.3}$$

$$\rho(X) = \sup_{\mathbb{Q} \in \Delta} E_{\mathbb{Q}}[-X] \tag{3.4}$$

for any $X \in \mathcal{H}$.

Remark 3.1 The statement of Theorem 3.1 can also be found in Inoue [4]. However, Inoue [4] gave only the proof for $X \in L^\infty$. In this paper, we will complement the proof for the case of $X \in L^p(\Omega, \mathcal{F}, \mathbb{P})$, $1 \le p < \infty$.

Now, we are in a position to state another main result of the present paper, which is a generalization of Proposition 5.2 in Artzner et al. [2] and Theorem 6.8 in Delbaen [3].

Theorem 3.2 Let $1 \le p < \infty$. For every $X \in \mathcal{H}$ and any α with $0 < \alpha < 1$, we have

$$\mathrm{VaR}_\alpha(X) = \inf\{\rho(X) : \rho \text{ is a continuous coherent risk measure on } \mathcal{H} \text{ and } \rho \ge \mathrm{VaR}_\alpha \text{ on } \mathcal{H}\}.$$

4 Proofs of Main Results

In this section, we will provide proofs of Theorems 3.1 and 3.2.

First, we will adopt the dual method to prove Theorem 3.1, which can also be seen in Yan [11].

Proof of Theorem 3.1 (2)$\Leftrightarrow$(3) follows form the Randon-Nikodym theorem.

(2)⇒(1) is also obvious. Hence, it suffices to prove (1)⇒(2). To do this we only need to show that for any $X \in \mathcal{H}$, there exists $g_X \in L^q(\Omega, \mathcal{F}, \mathbb{P})$, such that

$$\begin{aligned} &\sup_{g \in \{g_X : X \in \mathcal{H}\}} \|g\|_q < \infty, \\ &\rho(X) = E[-X g_X] \end{aligned} \tag{4.1}$$

and

$$E[-Y g_X] \le \rho(Y) \tag{4.2}$$

for all $Y \in \mathcal{H}$ hold.

In fact, by (4.1) and (4.2), we have

$$\rho(X) = \sup_{g \in G} E[-Xg] \tag{4.3}$$

for all $X \in \mathcal{H}$, where $G := \{g_X : X \in \mathcal{H}\}$.

By the translation invariance of ρ, with no loss of generality, we can assume that $\rho(X) = 1$. Since ρ is a continuous coherent risk measure, by Remark 2.1, there exists $C \in (0, \infty)$ such that $\rho(X) \le C\|X\|_p$ for all $X \in \mathcal{H}$.

Let

$$\begin{aligned} &B_1 := \{Y \in \mathcal{H} : \|Y\|_p < \frac{1}{C}\}, \\ &B := \{Y \in \mathcal{H}\colon\ \rho(Y) < 1\}. \end{aligned}$$

Then $B_1 \subset B$, $X \notin B$. Since B is a convex set, by the Hahn-Banach theorem, there exists a nontrivial $h \in L^q(\Omega, \mathcal{F}, \mathbb{P})$, such that

$$\sup_{Y \in B} h(-Y) \le h(-X),$$

where $h(-X) := E_{\mathbb{P}}[(-X)h]$. Obviously, $B_1 \subset B$, $h(-X) > 0$. As a result, we can choose h such that $h(-X) = 1$. We further claim that h has the following three properties

(1) $h(Y) \ge 0$ for any $Y \ge 0$, $Y \in \mathcal{H}$.

(2) $h(1) = 1$.

(3) $h(-Y) \le \rho(Y)$ for any $Y \in \mathcal{H}$.

First, we prove (1). For any $Y \ge 0$, $Y \in \mathcal{H}$ and any $s > 0$, $sY \in B$. Hence $h(-sY) \le h(-X) = 1$. By arbitrariness of $s > 0$, we conclude that $h(Y) \ge 0$. From property (1) we obtain that h is a nonnegative random variable.

Second, we prove (2). On one hand, for any $-1 < s < 0$, $s \in B$. Then $h(-s) \le h(-X) = 1$, which implies $h(1) \le 1$, because $-1 < s < 0$ is arbitrary.

On the other hand, for any $s > 1$, we have $2 - sh(1) = h(-2X - s) = h[-(2X + s)] \le h(-X) = 1$, which implies $h(1) \ge \frac{1}{s}$. Since $s > 1$ is arbitrary, we obtain $h(1) \ge 1$. Thus property (2) is proved, which implies $E_{\mathbb{P}}(g) = 1$.

Finally, we prove (3). For any Y, let $Y_1 := Y + \rho(Y) - 1$, then for any $s > 1$, $\frac{Y_1}{s} \in B$, which implies $h(-Y_1) \leq 1$, because $s > 1$ is arbitrary. Hence $h(-Y) \leq \rho(Y)$. From the choice of h and property (3), we obtain (3.2).

Next, we will prove (3.1). Note that ρ is continuous, for any $X \in \mathcal{H}$,

$$\rho(-|X|) = \sup_{g\in G} E[|X|g] \leq C\|X\|_p,$$

which yields for all $g \in G$,

$$E[|X|g] \leq \rho(-|X|) \leq C\|X\|_p. \tag{4.4}$$

When $p = 1$, for any $1 \leq t < \infty$, let $X := g^{t-1}$ in (4.4), then $\|g\|_t \leq C$ for any $g \in G$. Furthermore, $\|g\|_\infty = \lim\limits_{t\to\infty} \|g\|_t \leq C$ for any $g \in G$, which implies $\sup\limits_{g\in G} \|g\|_\infty \leq C < \infty$.

So with no loss of generality, we assume that $p \neq 1$. Taking $X := g^{q-1}$ in (4.4) yields

$$E[g^q] \leq C\|g\|_q^{\frac{q}{p}}, \tag{4.5}$$

from which and the fact $\|g\|_q < \infty$ it follows that $\|g\|_q \leq C$ for any $g \in G$, which implies $\sup\limits_{g\in G} \|g\|_q \leq C < \infty$, and the proof of Theorem 3.1 is completed.

Next, we will borrow the idea of the proof of Artzner et al. [2, Proposition 5.2] to prove Theorem 3.2. However, more lemmas and more delicate arguments will be needed. Let us begin with lemmas.

Lemma 4.1 Let X be a non-negative random variable. Denote $X_n := X \wedge n$, $n \geq 1$. Then $\lim\limits_{n\to\infty} q_\alpha^+(X_n) = q_\alpha^+(X)$ for any $\alpha \in (0, 1)$.

Proof Obviously, $X_n \uparrow X$ and $X_n \xrightarrow{\text{a.s.}} X$. So $\{q_\alpha^+(X_n) : n \geq 1\}$ is an increasing sequence and $q_\alpha^+(X)$ is an upper bound of $q_\alpha^+(X_n)$. That is,

$$q_\alpha^+(X_n) \leq q_\alpha^+(X). \tag{4.6}$$

Hence $\lim\limits_{n\to\infty} q_\alpha^+(X_n)$ exists.

Note that $q_\alpha^+(X) < +\infty$. Thus there exists a positive integer N such that $q_\alpha^+(X) \leq N-1$. For any $0 < \epsilon < 1$, there is a positive integer $M = M(\epsilon) > N$ such that

$$\{X_n \leq q_\alpha^+(X_n) + \epsilon\} = \{X \leq q_\alpha^+(X_n) + \epsilon\}$$

for any $n \geq M$. Hence

$$\mathbb{P}[X \leq q_\alpha^+(X_n) + \epsilon] = \mathbb{P}[X_n \leq q_\alpha^+(X_n) + \epsilon] > \alpha$$

for any $n \geq M$. This shows that $q_\alpha^+(X) \leq q_\alpha^+(X_n) + \epsilon$ for any $n \geq M$, from which, the arbitrariness of ϵ and (4.6) it follows that $\lim\limits_{n\to\infty} q_\alpha^+(X_n) = q_\alpha^+(X)$. Lemma 4.1 is proved.

Similarly, one can steady show the following lemma.

Lemma 4.2 Let X be a non-negative random variable. Denote $X_n := X \wedge n$, $n \geq 1$, then $\lim\limits_{n\to\infty} q_\alpha^-(X_n) = q_\alpha^-(X)$ for any $\alpha \in (0, 1)$.

Lemma 4.3 Let $1 \leq p < \infty$, ρ be a continuous coherent risk measure on $L^p(\Omega, \mathcal{F}, \mathbb{P})$ and the set Δ be as in (3.3) and (3.4), then $\rho \geq VaR_\alpha$ on $L^p(\Omega, \mathcal{F}, \mathbb{P})$ if and only if for every B with $\mathbb{P}(B) > \alpha$ and any $\varepsilon > 0$, there is a measure $\mu \in \Delta$ with $\mu(B) > 1 - \varepsilon$.

Proof (1) Necessity: For any $\varepsilon > 0$ and any B with $\mathbb{P}(B) > \alpha$, since

$$\mathrm{VaR}_\alpha(-\mathbf{1}_B) = -\inf\{x : \mathbb{P}[-\mathbf{1}_B \leq x] > \alpha\} = 1,$$

we have $\rho(-\mathbf{1}_B) \geq 1$. This implies that there exists a measure $\mu \in \Delta$ with $\mu(B) > 1 - \varepsilon$.

(2) Sufficiency: First, we consider the case where $X \in L^p(\Omega, \mathcal{F}, \mathbb{P})$ is bounded.

Given a bounded random variable X and any $\varepsilon > 0$, let $B := \{X \leq q_\alpha^+ + \varepsilon\}$, where $q_\alpha^+ := q_\alpha^+(X)$. Then $\mathbb{P}(B) > \alpha$. So there exists a measure $\mu \in \Delta$ such that $\mu(B) > 1 - \varepsilon$. Hence,

$$\begin{aligned} E_\mu[-X] &\geq (-q_\alpha^+ - \varepsilon)\mu(X \leq q_\alpha^+ + \varepsilon) - E_\mu[X\mathbf{1}_{B^c}] \\ &\geq (-q_\alpha^+ - \varepsilon)\mu(X \leq q_\alpha^+ + \varepsilon) - \|X\|_\infty \varepsilon. \end{aligned} \tag{4.7}$$

Taking into account the fact that

$$(1 - \varepsilon) \leq \mu(X \leq q_\alpha^+ + \varepsilon) \leq 1,$$

we claim that

$$\lim_{\varepsilon \downarrow 0} (-q_\alpha^+ - \varepsilon)\mu(X \leq q_\alpha^+ + \varepsilon) = -q_\alpha^+. \tag{4.8}$$

1) In fact, if $-q_\alpha^+ - \varepsilon > 0$, then

$$(-q_\alpha^+ - \varepsilon)(1 - \varepsilon) \leq (-q_\alpha^+ - \varepsilon)\mu(X \leq q_\alpha^+ + \varepsilon) \leq (-q_\alpha^+ - \varepsilon),$$

which yields (4.8) by letting $\varepsilon \to 0$.

2) On the other hand, if $-q_\alpha^+ - \varepsilon < 0$, then

$$(-q_\alpha^+ - \varepsilon)(1 - \varepsilon) \geq (-q_\alpha^+ - \varepsilon)\mu(X \leq q_\alpha^+ + \varepsilon) \geq (-q_\alpha^+ - \varepsilon),$$

which also yields (4.8) by letting $\varepsilon \to 0$.

Combining (4.7) and (4.8) gives rise to

$$\overline{\lim_{\varepsilon \downarrow 0}} E_\mu[-X] \geq \lim_{\varepsilon \downarrow 0} \left[(-q_\alpha^+ - \varepsilon)\mu(X \leq q_\alpha^+ + \varepsilon) - \|X\|_\infty \varepsilon\right] = -q_\alpha^+.$$

Therefore

$$\rho(X) = \sup_{\mathbb{Q} \in \Delta} E_{\mathbb{Q}}[-X] \geq \overline{\lim_{\varepsilon \downarrow 0}} E_\mu[-X] \geq -q_\alpha^+.$$

Next, let us consider the general case where $X \in L^p(\Omega, \mathcal{F}, \mathbb{P})$. Let $X_n^+ := X^+ \wedge n$, and $X_n^- := X^- \wedge n$. Then $X_n^+ \xrightarrow{\text{a.s.}} X^+$, $X_n^- \xrightarrow{\text{a.s.}} X^-$, $X_n^+ - X_n^- \xrightarrow{\text{a.s.}} X$ and $\lim\limits_{n \to \infty} \|X_n^+ - X_n^- - X\|_p = 0$. It is easy to see that $-X_n^-$ and X_n^+ are comonotonic, as well as X^+ and X^-.

From Lemmas 4.1, 4.2 and the comonotonic additivity of VaR it follows that

$$\begin{aligned}\rho(X) &= \lim_{n\to\infty} \rho(X_n^+ - X_n^-) \geq \overline{\lim_{n\to\infty}}\, \mathrm{VaR}_\alpha(X_n^+ - X_n^-)\\ &\geq \overline{\lim_{n\to\infty}} \left\{\mathrm{VaR}_\alpha(X_n^+) + \mathrm{VaR}_\alpha(-X_n^-)\right\} = \lim_{n\to\infty} \mathrm{VaR}_\alpha(X_n^+) + \lim_{n\to\infty} \mathrm{VaR}_\alpha(-X_n^-)\\ &= -\lim_{n\to\infty} q_\alpha^+(X_n^+) + \lim_{n\to\infty} q_{1-\alpha}^-(X_n^-) = -q_\alpha^+(X^+) + q_\alpha^-(X^-)\\ &= \mathrm{VaR}_\alpha(X^+) + \mathrm{VaR}_\alpha(-X^-) = \mathrm{VaR}_\alpha(X^+ - X^-)\\ &= \mathrm{VaR}_\alpha(X).\end{aligned}$$

The proof of Lemma 4.3 is completed.

Next, we will use Proposition 3.1 and Lemma 4.3 to prove Theorem 3.2.

Proof of Theorem 3.2 We only need to show that for given $X \in \mathcal{H}$, there is a continuous coherent risk measure ρ_X such that $\rho_X \geq \mathrm{VaR}_\alpha$ on $\mathcal{H}$ and with the property that $\rho_X(X) \leq \mathrm{VaR}_\alpha(X)$.

To this end, for any set $B \in \mathcal{F}$ with $\mathbb{P}(B) > \alpha$, we have $\mathbb{P}[B \cap \{X \geq q_\alpha^+(X)\}] > 0$ and we can define h_B as $\mathbf{1}_{B\cap\{X\geq q_\alpha^+(X)\}}/\mathbb{P}[B \cap \{X \geq q_\alpha^+(X)\}]$ and $\mathbb{Q}_B = h_B \cdot \mathbb{P}$. Then

$$\mathbb{Q}_B(B) = \int_B h_B dP = 1.$$

Let

$$\Delta := \{\mathbb{Q}_B : \mathbb{Q}_B = h_B \cdot \mathbb{P},\ \mathbb{P}(B) > \alpha\}$$

and define a risk measure ρ_X: $\mathcal{H} \to \mathbf{R}$ by

$$\rho_X(Y) = \sup_{\mathbb{Q}\in\Delta} E_{\mathbb{Q}}[-Y], \quad Y \in \mathcal{H}.$$

By Proposition 3.1 and Lemma 4.3, we know that ρ_X is a continuous coherent risk measure on $\mathcal{H}$, and $\rho_X \geq \mathrm{VaR}_\alpha$ on $\mathcal{H}$ with the property that $\rho_X(X) \leq -q_\alpha^+(X) = \mathrm{VaR}_\alpha(X)$. The proof of Theorem 3.2 is completed.

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在险值与L^p -空间上的连续一致风险度量

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摘 要: 本文研究了在险值和L^p-空间上的连续一致风险度量之间的关系. 利用凸集分离定理和截尾逼近方法, 获得了在险值可以用L^p-空间上的连续一致风险度量表示的结果, 并且得到了L^p-空间上的表示定理的一种新的证明方法. 它们分别是文献[2]的相关结论从L^∞-空间到L^p-空间上的推广和对Inoue [4] 做的一些补充证明.

关键词: L^p-空间; 连续一致风险度量; 在险值

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