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THE GELFAND-KIRILLOV DIMENSION OF QUANTIZED ENVELOPING ALGEBRA OF TYPE D_4

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Abstract: In this paper, we research the problem of computing the Gelfand-Kirillov dimension of quantized enveloping algebra of type D_4 by using the method of computing the Gelfand-Kirillov dimension given in [1] and the Gröbner-Shirshov basis for quantized enveloping algebra of type D_4 given in [2]. The main result we get is that the Gelfand-Kirillov dimension of quantized enveloping algebra of type D_4 is 28. We hope this result will provide some ideas to compute the Gelfand-Kirillov dimension of quantized enveloping algebra of type D_n.

Keywords: Gröbner-Shirshov basis; Poincaré-Birkhoff-Witt algebra; weight vector; Gelfand-Kirillov dimension

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1 Introduction

In contrast with the commutative case, for non-commutative algebras the classical Krull dimension is usually not a very useful tool, because it is defined by using chains of prime ideals. For finitely generated k-algebras R, the Gelfand-Kirillov dimension is far better invariant and coincides with the Krull dimension in the commutative case. The Gelfand-Kirillov dimension measures the asymptotic rate of growth of algebras and provides important structural information, so this invariant has become one of the standard tools in the study of finitely generated infinite dimensional algebras. But in general, the Gelfand-Kirillov dimension is extremely hard to compute.

In [1], the authors gave a detailed discussion of the Gelfand-Kirillov dimension of finitely generated k-algebras and modules over them, and also introduced an algorithm to compute the Gelfand-Kirillov dimension of several classical and non-classical examples (in the context of enveloping algebras and quantum groups).

In this paper by using the method in [1] and the Gröbner-Shirshov basis given in [2], we compute the Gelfand-Kirillov dimension $\mathrm{GKdim}(U_q(D_4))$ of the quantized enveloping algebra $U_q(D_4)$. We hope that this work might become a first step of computing the Gelfand-Kirillov dimension of quantized enveloping algebra of type D_n.

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2 Some Preliminaries

In this section, we recall the notion of the Gelfand-Kirillov dimension of an algebra from [3].

Let k be a field and A a finitely generated k-algebra. A finite dimensional k-vector space V contained in A and containing 1 is said to be a generating subspace of A if it generates A as a k-algebra. For any positive integer n, denote by V^n the set of all elements of A of the form $\sum v_1 \cdots v_n$, where $v_1,\cdots,v_n \in V$. In particular, $V^0 = k$ and $V^1 = V$. Obviously, $\{V^n\}_{n\geq 0}$ determines a filtration on A.

Definition 2.1 The growth function or Hilbert function HF_V of A relative to V is defined on $\mathbb{N}$ by putting

$$HF_V(n) = \dim_k(V^n)$$

for all positive integer n.

A function $f : \mathbb{N} \to \mathbb{R}$ is said to be positive if it only takes positive values. We say a positive function f is eventually monotone increasing if there exists a positive integer n_0 such that $f(n) \leq f(n+1)$ for all $n \geq n_0$. It is clear that the growth function HF_V above is eventually monotone increasing.

Lemma 2.2 Let $f : \mathbb{N} \to \mathbb{R}$ be a monotone increasing function and denote by $D(f)$ the set of all $x \in \mathbb{R}$ for which there exists some positive integer n_0 and some $c \in \mathbb{R}$ (depending on x) such that $f(n) \leq cn^x$ for all $n \geq n_0$. Then

$$\text{inf}D(f) = \limsup \log_n f(n),$$

where $\log_n$ denote the logarithm with base n and if $D(f) = \emptyset$, then we put $\text{inf}D(f) = \infty$.

Definition 2.3 If $f : \mathbb{N} \to \mathbb{R}$ is an eventually increasing function, then we put

$$d(f) = \text{inf}D(f) = \limsup \log_n f(n) \in [0, \infty].$$

We will call $d(f)$ the degree of growth of f. The following proposition tells us that the degree of growth of Hilbert function HF_V does not depend on the choices of the generating subspace V.

Proposition 2.4 Let A be a finitely generated k-algebra. Assume V and V' to be generating subspaces of A. Then $d(HF_V) = d(HF_{V'})$.

Now the following definition makes sense:

Definition 2.5 Let A be a finitely generated k-algebra, say with finite dimensional generating subspace V. The Gelfand-Kirillov dimension of A is then defined as

$$\text{GKdim}(A) = d(HF_V).$$

Now we recall the definition of the Gelfand-Kirillov dimension of a left A-module.

Let A be a finitely generated k-algebra and M a finitely generated left A-module. A generating subspace of M is just a finite dimensional k-subspace U of M such that $RU = M$.

Definition 2.6 Let A be a finitely generated k-algebra with generating subspace V and M a finitely generated left A-module with generating subspaces U. Then the growth function or Hilbert function $HF_{V,U}$ of M relative to V and U is defined by

$$HF_{V,U}(n) = \dim_k(V^n U)$$

for all positive integer n.

Proposition 2.7 Let A be a finitely generated k-algebra and M a finitely generated left A-module. Assume V and $V^{'}$ to be generating subspaces of A and U and $U^{'}$ to be generating subspaces of M. Then $d(HF_{V,U}) = d(HF_{V',U'})$.

So the following definition makes sense:

Definition 2.8 Let A be a finitely generated k-algebra and M a finitely generated left A-module. Assume V and U to be generating subspaces of A and M, respectively. The Gelfand-Kirillov dimension of M is then defined as

$$\mathrm{GKdim}(M) = d(HF_{V,U}).$$

Let $\mathbb{N}$ be the set of nonnegative integers and n a positive integer.

Definition 2.9 An admissible order on $(\mathbb{N}, +)$ is a total order $\preceq$ with following two properties:

(1) $0 \prec \alpha$ for every $0 \neq \alpha \in \mathbb{N}^n$;

(2) $\alpha + \gamma \prec \beta + \gamma$ for all $\alpha, \beta, \gamma \in \mathbb{N}^n$ with $\alpha \prec \beta$.

Definition 2.10 Let $\omega = (\omega_1, \cdots, \omega_n) \in \mathbb{N}^n$. The weighted total degree with respect to ω of the element $\alpha = (\alpha_1, \cdots, \alpha_n) \in \mathbb{N}^n$ is the dot product

$$|\alpha|_\omega = \langle \omega, \alpha \rangle = \sum_{i=1}^{n} \omega_i \alpha_i.$$

The ω-weighted degree lexicographical order $\preceq_\omega$ on $\mathbb{N}^n$ with $\varepsilon_1 \prec \varepsilon_2 \prec \cdots \prec \varepsilon_n$ is defined by letting

$$\alpha \preceq_\omega \beta \Leftrightarrow \begin{cases} |\alpha|_\omega < |\beta|_\omega \\ \text{or} \\ |\alpha|_\omega = |\beta|_\omega \text{ and } \alpha \preceq_{lex} \beta, \end{cases}$$

where $\varepsilon_1, \cdots, \varepsilon_n$ is the standard bases of $\mathbb{N}^n$ and $\preceq_{lex}$ is the lexicographical ordering.

Let A be an associative k-algebra generated by $x_1, \cdots, x_n$ and $\preceq$ an admissible order on $\mathbb{N}^n$ (see [1] for the definition). An element of the form $x_1^{a_1} \cdots x_n^{a_n}$ in A is called standard term and denoted by X^α, where $\alpha = (a_1, \cdots, a_n) \in \mathbb{N}^n$. If an element $f \in A$ can be expressed uniquely as

$$f = \sum_{\alpha \in \mathbb{N}} c_\alpha X^\alpha,$$

then we define

$$\exp(f) = \max\{\alpha \in N^n | c_\alpha \neq 0\}.$$

Definition 2.11 A PBW algebra A over a field k is an associated algebra generated by finitely many elements $x_1, \cdots, x_n$ subject to the relations

$$Q = \{x_j x_i = q_{ji} x_i x_j + p_{ji}\} \quad (1 \leq i < j \leq n),$$

where each p_{ji} is a finite k-linear combination of standard terms $X^\alpha = x_1^{a_1} \cdots x_n^{a_n}$, with $\alpha = (a_1, \cdots, a_n) \in \mathbb{N}^n$ and where each q_{ji} is a non-zero scalar in k. The algebra is required to satisfy the following two conditions:

(1) there is an admissible order $\preceq$ on N^n such that $\exp(p_{ji}) \prec \varepsilon_i + \varepsilon_j$ for every $1 \leq i < j \leq n$, where ε_i, ε_j are the standard bases vectors in $\mathbb{N}^n$;

(2) the standard terms X^α with $\alpha \in N^n$ forms a basis of A as a k-vector space.

This PBW k-algebra A is also denoted as $A = k\{x_1, \cdots x_n; Q, \preceq\}$. By Corollary 1.7 of Chapter 3 in [1], we also denote A as $A = k\{x_1, \cdots x_n; Q, \preceq_\omega\}$, for some vector ω with strictly positive components. For any subset $N \subseteq A$, we define

$$\mathrm{Exp}(N) = \{\exp(f) | f \in N\}.$$

Definition 2.12 Let $\alpha = (a_1, \cdots, a_n) \in \mathbb{N}^n$. The support of α is the set

$$\mathrm{supp}(\alpha) = \{i \in \{1, 2, \cdots, n\} | a_i \neq 0\},$$

then it is clear that $\mathrm{supp}(\alpha) = \emptyset$ if and only if $\alpha = 0$.

For any monoideal (see [1] for the definition) E of $\mathbb{N}^n$, we define

$$V(E) = \{\sigma \subseteq \{1, 2, \cdots, n\} | \text{ for any } \alpha \in E, \sigma \cap \mathrm{supp}(\alpha) \neq \emptyset\}.$$

Definition 2.13 The dimension of a monoideal E is defined as

$$\dim(E) = \begin{cases} n, & \text{if } E = \emptyset, \\ 0, & \text{if } E = N^n, \\ n - \min\{\mathrm{card}(\sigma);\ \sigma \in V(E)\}, & \text{if } E \text{ is proper,} \end{cases}$$

where $\mathrm{card}(\sigma)$ is the number of elements of σ.

The key result for us in [1] is the following:

Theorem 2.14 Let $R = k\{x_1, \cdots x_n; Q, \preceq_\omega\}$ be a PBW k-algebra. Let $N \subseteq R^m$ be a left R-submodule of R^m and R^m/N. Then

$$\mathrm{GKdim}(M) = \dim(\mathrm{Exp}(N)).$$

3 Gelfand-Kirillov Dimension of Quantized Enveloping Algebra of Type D_4

In this section we compute the Gelfand-Kirillov dimension of quantized enveloping algebra $U_q(D_4)$. We choose following orientation for D_4:

3

2 1

4

Then the corresponding Cartan matrix A is

$$\mathbf{A} = \begin{bmatrix} 2 & -1 & -1 & -1 \\ -1 & 2 & 0 & 0 \\ -1 & 0 & 2 & 0 \\ -1 & 0 & 0 & 2 \end{bmatrix}.$$

Let q be a nonzero element of k so that is not a root of unity. The quantized enveloping algebra $U_q(D_4)$ is a free k-algebra with generators $\{E_i, K_i^{\pm 1}, F_i | 1 \le i, j \le 4\}$ subject to the relations

$$\begin{aligned} K = \quad & \{K_iK_j - K_jK_i,\ K_iK_i^{-1} - 1,\ K_i^{-1}K_i - 1,\ E_jK_i^{\pm 1} - q^{\mp d_i a_{ij}} K_i^{\pm 1} E_j, \\ & K_i^{\pm 1} F_j - q^{\mp d_i a_{ij}} F_j K_i^{\pm 1}\}; \\ T = \quad & \{E_iF_j - F_jE_i - \delta_{ij} \tfrac{K_i^2 - K_i^{-2}}{q^{2d_i} - q^{-2d_i}}\}; \\ S^+ = \quad & \{\sum_{v=0}^{1-a_{ij}} (-1)^v \begin{bmatrix} 1-a_{ij} \\ v \end{bmatrix}_t E_i^{1-a_{ij}-v} E_j E_i^v | i \ne j, t = q^{2d_i}\}; \\ S^- = \quad & \{\sum_{v=0}^{1-a_{ij}} (-1)^v \begin{bmatrix} 1-a_{ij} \\ v \end{bmatrix}_t F_i^{1-a_{ij}-v} F_j F_i^v | i \ne j, t = q^{2d_i}\} \end{aligned}$$

for all $1 \le i, j \le 4$ and

$$\begin{bmatrix} m \\ n \end{bmatrix}_\alpha = \begin{cases} \prod\limits_{i=1}^{n} \frac{t^{m-1+i} - t^{i-m-1}}{t^i - t^{-i}} & (\text{for } m > n > 0), \\ 1 & (\text{for } n = 0 \text{ or } n = m). \end{cases}$$

Let $U_q^0(D_4)$, $U_q^+(D_4)$ and $U_q^-(D_4)$ be the subalgebras of $U_q(D_4)$ generated by $\{K_i^{\pm 1} | 1 \le i \le 4\}$, $\{E_i | 1 \le i \le 4\}$ and $\{F_i | 1 \le i \le 4\}$, respectively. Then we have following triangular decomposition of $U_q(D_4)$:

$$U_q(D_4) \cong U_q^+(D_4) \otimes U_q^0(D_4) \otimes U_q^-(D_4).$$

Let

$$\begin{aligned} X = \quad & \{E_1, E_{12}, E_{13}, E_{14}, E_{21}, E_{22}, E_{23}, E_{24}, E_{31}, E_2, E_3, E_4, K_1, K_2, K_3, K_4, \\ & K_1^{-1}, K_2^{-1}, K_3^{-1}, K_4^{-1}, F_1, F_{12}, F_{13}, F_{14}, F_{21}, F_{22}, F_{23}, F_{24}, F_{31}, F_2, F_3, F_4\}, \end{aligned}$$

then the set X is also a generating set of $U_q(D_4)$, where

$$E_1, E_{12}, E_{13}, E_{14}, E_{21}, E_{22}, E_{23}, E_{24}, E_{31}, E_2, E_3, E_4$$

are the modified images of isomorphism classes of indecomposable representations of the type D_4 under canonical isomorphism of Ringel between the corresponding Ringel-Hall algebra $\mathcal{H}(D_4)$and the positive part of quantized enveloping algebra $U_q^+(D_4)$, and

$$F_1, F_{12}, F_{13}, F_{14}, F_{21}, F_{22}, F_{23}, F_{24}, F_{31}, F_2, F_3, F_4$$

are the images of the

$$E_1, E_{12}, E_{13}, E_{14}, E_{21}, E_{22}, E_{23}, E_{24}, E_{31}, E_2, E_3, E_4$$

under the convolution automorphism of quantized enveloping algebra $U_q(D_4)$ (for details see [2]).

We define an ordering

$$\begin{aligned} & F_1 < F_{12} < F_{13} < F_{14} < F_{21} < F_{22} < F_{23} < F_{24} < F_{31} < F_2 < F_3 < F_4 < K_1^{-1} \\ < \quad & K_2^{-1} < K_3^{-1} < K_4^{-1} < K_1 < K_2 < K_3 < K_4 < E_1 < E_{12} < E_{13} < E_{14} < E_{21} \\ < \quad & E_{22} < E_{23} < E_{24} < E_{31} < E_2 < E_3 < E_4 \end{aligned}$$

on the set X. The set S of following skew-commutator relations are compute in [2]:

$$\begin{aligned}
&E_{mn}E_{ij} = E_{ij}E_{mn}, && ((m,n)(i,j)) \in C_1 \\
&E_{mn}E_{ij} = \upsilon E_{ij}E_{mn}, && ((m,n)(i,j)) \in C_2 \cup C_3 \cup C_4, \\
&E_{mn}E_{ij} = \upsilon^{-1} E_{ij}E_{mn} + E_{1n}, && ((m,n)(i,j)) \in C_5, \\
&E_{mn}E_{ij} = \upsilon^{-1} E_{ij}E_{mn} + E_{2r}, && ((m,n)(i,j)) \in C_6, \\
&E_{mn}E_{ij} = \upsilon^{-1} E_{ij}E_{mn} + E_{m1}, && ((m,n)(i,j)) \in C_7, \\
&E_{mn}E_{ij} = E_{ij}E_{mn} + (\upsilon - \upsilon^{-1})E_{2r}E_{2s}, && ((m,n)(i,j)) \in C_8, \\
&E_{mn}E_{ij} = E_{ij}E_{mn} + (\upsilon - \upsilon^{-1})E_{ir}E_{is}, && ((m,n)(i,j)) \in C_9, \\
&E_{mn}E_{ij} = \upsilon E_{ij}E_{mn} + (\upsilon^2 - 2 + \upsilon^{-2})E_{i2}E_{i3}E_{i4}, && ((m,n)(i,j)) \in C_{10}, \\
&E_{mn}E_{ij} = \upsilon^{-1} E_{ij}E_{mn} + (\upsilon - 2\upsilon^{-1})E_{21} + (1 - \upsilon^{-2})E_{12}E_{22} \\
&\qquad\qquad + (1 - \upsilon^{-2})E_{13}E_{23} + (1 - \upsilon^{-2})E_{14}E_{24}, && ((m,n)(i,j)) \in C_{11}, \\
&F_{mn}F_{ij} = F_{ij}F_{mn}, && ((m,n)(i,j)) \in C_1 \\
&F_{mn}F_{ij} = \upsilon F_{ij}F_{mn}, && ((m,n)(i,j)) \in C_2 \cup C_3 \cup C_4, \\
&F_{mn}F_{ij} = \upsilon^{-1} F_{ij}F_{mn} + F_{1n}, && ((m,n)(i,j)) \in C_5, \\
&F_{mn}F_{ij} = \upsilon^{-1} F_{ij}F_{mn} + F_{2r}, && ((m,n)(i,j)) \in C_6, \\
&F_{mn}F_{ij} = \upsilon^{-1} F_{ij}F_{mn} + F_{m1}, && ((m,n)(i,j)) \in C_7, \\
&F_{mn}F_{ij} = F_{ij}F_{mn} + (\upsilon - \upsilon^{-1})F_{2r}F_{2s}, && ((m,n)(i,j)) \in C_8, \\
&F_{mn}F_{ij} = F_{ij}F_{mn} + (\upsilon - \upsilon^{-1})F_{ir}F_{is}, && ((m,n)(i,j)) \in C_9, \\
&F_{mn}F_{ij} = \upsilon F_{ij}F_{mn} + (\upsilon^2 - 2 + \upsilon^{-2})F_{i2}F_{i3}F_{i4}, && ((m,n)(i,j)) \in C_{10}, \\
&F_{mn}F_{ij} = \upsilon^{-1} F_{ij}F_{mn} + (\upsilon - 2\upsilon^{-1})F_{21} + (1 - \upsilon^{-2})F_{12}F_{22} \\
&\qquad\qquad + (1 - \upsilon^{-2})F_{13}F_{23} + (1 - \upsilon^{-2})F_{14}F_{24}, && ((m,n)(i,j)) \in C_{11}, \\
&K_iK_j = K_jK_i, K_iK_i^{-1} = 1, K_i^{-1}K_i = 1,
\end{aligned}$$

where

$$
\begin{aligned}
C_1 &= \{((m,n)(i,j)) \mid m = i \in \{1,2,3\},\ n \in \{3,4\},\ j \in \{2,3\} \text{ and } n > j\},\\
C_2 &= \{((m,n)(i,j)) \mid m = i \in \{1,2,3\},\ n \in \{2,3,4\},\ j = 1\ \},\\
C_3 &= \{((m,n)(i,j)) \mid m = 3,\ i = 1,\ n = j \in \{2,3,4\}\},\\
C_4 &= \{((m,n)(i,j)) \mid m \in \{2,3\},\ i = m-1,\ n \in \{1,2,3,4\},\ j \in \{2,3,4\} \text{ and } n \neq j\},\\
C_5 &= \{((m,n)(i,j)) \mid m = 3,\ i = 1,\ n =\in \{2,3,4\},\ j = 1\},\\
C_6 &= \{((m,n)(i,j)) \mid m = 3,\ i = 1,\ n,j \in \{2,3,4\} \text{ and } n \neq j\},\\
C_7 &= \{((m,n)(i,j)) \mid m \in \{2,3\},\ i = m-1,\ n = j \in \{2,3,4\}\},\\
C_8 &= \{((m,n)(i,j)) \mid m = 3,\ i = 1,\ n = 1,\ j \in \{2,3,4\}\},\\
C_9 &= \{((m,n)(i,j)) \mid m \in \{2,3\},\ i = m-1,\ n \in \{2,3,4\},\ j = 1\},\\
C_{10} &= \{((m,n)(i,j)) \mid m \in \{2,3\},\ i = m-1,\ n = j = 1\},\\
C_{11} &= \{((m,n)(i,j)) \mid m = 3,\ i = n = j = 1\},
\end{aligned}
$$

where $i = 1,2,3,4; j = 1,2,3,4$. We set $E_1 = E_{11}, E_2 = E_{32}, E_3 = E_{33}, E_4 = E_{34}$, and $v^2 = q$. The main result in [2] says that the set S is minimal Gröbner-Shirshov basis (see [4] for the definition) of quantized enveloping algebra $U_q(D_4)$ with respect to the above ordering.

In order to prove that $U_q(D_4)$ is a quotient of a PBW algebra and hence we are able to compute its Gelfand-Kirillov dimension, we need following additional relations

$$
\begin{aligned}
&E_m K_n^{\pm1} = q^{\pm1} K_n^{\pm1} E_m && (m = 1, n \in \{2,3,4\} \text{ or } m \in \{2,3,4\}, n = 1);\\
&E_m K_n^{\pm1} = K_n^{\pm1} E_m && (m,n \in \{2,3,4\}, m \neq n);\\
&E_m K_n^{\pm1} = q^{\mp2} K_n^{\pm1} E_m && (m = n \in \{1,2,3,4\});\\
&E_{1m} K_1^{\pm1} = q^{\mp1} K_1^{\pm1} E_{1m} && (m \in \{2,3,4\});\\
&E_{1m} K_m^{\pm1} = q^{\mp1} K_m^{\pm1} E_{1m} && (m \in \{2,3,4\});\\
&E_{1m} K_n^{\pm1} = q^{\pm1} K_n^{\pm1} E_{1m} && (m,n \in \{2,3,4\}, m \neq n);\\
&E_{21} K_1^{\pm1} = q^{\mp1} K_1^{\pm1} E_{21}; &&\\
&E_{21} K_n^{\pm1} = K_n^{\pm1} E_{21} && (n \in \{2,3,4\});\\
&E_{2m} K_1^{\pm1} = K_1^{\pm1} E_{2m} && (m \in \{2,3,4\});\\
&E_{2m} K_m^{\pm1} = q^{\pm1} K_m^{\pm1} E_{2m} && (m \in \{2,3,4\});\\
&E_{2m} K_n^{\pm1} = q^{\mp1} K_n^{\pm1} E_{2m} && (m,n \in \{2,3,4\}, m \neq n);\\
&E_{31} K_1^{\pm1} = q^{\pm1} K_1^{\pm1} E_{31}; &&\\
&E_{31} K_n^{\pm1} = q^{\mp1} K_n^{\pm1} E_{31} && (n \in \{2,3,4\});\\
&K_m^{\pm1} F_n = q^{\pm1} F_n K_m^{\pm1} && (m = 1, n \in \{2,3,4\} \text{ or } m \in \{2,3,4\}, n = 1);\\
&K_m^{\pm1} F_n = F_n K_m^{\pm1} && (m,n \in \{2,3,4\}, m \neq n);\\
&K_m^{\pm1} F_n = q^{\mp2} F_n K_m^{\pm1} && (m = n \in \{1,2,3,4\});\\
&K_1^{\pm1} F_{1m} = q^{\mp1} F_{1m} K_1^{\pm1} && (m \in \{2,3,4\});\\
&K_m^{\pm1} F_{1m} = q^{\mp1} F_{1m} K_m^{\pm1} && (m \in \{2,3,4\});\\
&K_m^{\pm1} F_{1n} = q^{\pm1} F_{1n} K_m^{\pm1} && (m,n \in \{2,3,4\}, m \neq n);\\
&K_1^{\pm1} F_{21} = q^{\mp1} F_{21} K_1^{\pm1}; &&\\
&K_n^{\pm1} F_{21} = F_{21} K_n^{\pm1} && (n \in \{2,3,4\});
\end{aligned}
$$

$$
\begin{aligned}
&K_1^{\pm1}F_{2m}=F_{2m}K_1^{\pm1} && (m\in\{2,3,4\});\\
&K_m^{\pm1}F_{2m}=q^{\pm1}F_{2m}K_m^{\pm1} && (m\in\{2,3,4\});\\
&K_m^{\pm1}F_{2n}=q^{\mp1}F_{2n}K_m^{\pm1} && (m,n\in\{2,3,4\},m\neq n);\\
&K_1^{\pm1}F_{31}=q^{\pm1}F_{31}K_1^{\pm1}; && \\
&K_n^{\pm1}F_{31}=q^{\mp1}F_{31}K_n^{\pm1} && (n\in\{2,3,4\});\\
&E_mF_n=F_nE_m && (m,n\in\{1,2,3,4\},m\neq n);\\
&E_mF_m=F_mE_m+\tfrac{K_m-K_m^{-1}}{q-q^{-1}} && (m\in\{1,2,3,4\});\\
&E_1F_{1m}=F_{1m}E_1+\tfrac{1-q^{\frac12}}{q-q^{-1}}F_mK_1+\tfrac{q^{-\frac32}-1}{q-q^{-1}}F_mK_1^{-1} && (m\in\{2,3,4\});\\
&E_mF_{1n}=F_{1n}E_m && (m,n\in\{2,3,4\},m\neq n);\\
&E_mF_{1m}=F_{1m}E_m+\tfrac{q-q^{-\frac12}}{q-q^{-1}}F_1K_m\tfrac{q^{-\frac12}-q^{-1}}{q-q^{-1}}F_1K_m^{-1} && (m\in\{2,3,4\});\\
&E_{1m}F_1=F_1E_{1m}+\tfrac{q-q^{-\frac12}}{q-q^{-1}}K_1E_m+\tfrac{q^{-\frac12}-q^{-1}}{q-q^{-1}}K_1^{-1}E_m && (m\in\{2,3,4\});\\
&E_{1m}F_n=F_nE_{1m} && (m,n\in\{2,3,4\},m\neq n);\\
&E_{1m}F_m=F_mE_{1m}+\tfrac{1-q^{\frac12}}{q-q^{-1}}K_mE_1\tfrac{q^{-\frac32}-1}{q-q^{-1}}K_m^{-1}E_1 && (m\in\{2,3,4\});\\
&E_1F_{2m}=F_{2m}E_1+\tfrac{1-2q^{\frac12}+q}{q-q^{-1}}F_sF_tK_1 && \\
&\qquad+\tfrac{2q^{-\frac32}-1-q^{-3}}{q-q^{-1}}F_sF_tK_1^{-1} && (m,s,t\in\{2,3,4\},t>s,m\neq t,m\neq s);\\
&E_mF_{2n}=F_{2n}E_m+\tfrac{q-q^{-\frac12}}{q-q^{-1}}F_{1t}K_m+\tfrac{q^{-\frac12}-q^{-1}}{q-q^{-1}}F_{1t}K_m^{-1} && (m,n,t\in\{2,3,4\},m\neq t,n\neq t);\\
&E_mF_{2m}=F_{2m}E_m && (m\in\{2,3,4\});\\
&E_{2m}F_1=F_1E_{2m}+\tfrac{q^2-q-q^{\frac12}+q^{-\frac12}}{q-q^{-1}}K_1E_sE_t && \\
&\qquad+\tfrac{q^{-\frac32}-q^{-\frac12}-q^{-2}+q^{-1}}{q-q^{-1}}K_1^{-1}E_sE_t && (m,s,t\in\{2,3,4\},t>s,m\neq t,m\neq s);\\
&E_{2m}F_n=F_nE_{2m}+\tfrac{1-q^{\frac12}}{q-q^{-1}}K_mE_{1t}+\tfrac{q^{-\frac32}-1}{q-q^{-1}}K_m^{-1}E_{1t} && (m,n,t\in\{2,3,4\},m\neq t,n\neq t);\\
&E_{2m}F_m=F_mE_{2m} && (m\in\{2,3,4\});\\
&E_1F_{31}=F_{31}E_1+\tfrac{1+2q^{\frac32}-2q^{\frac12}-q^2}{q-q^{-1}}F_2F_3F_4K_1 && \\
&\qquad+\tfrac{2q^{-\frac32}-1-2q^{-\frac52}-q^{-3}+q^{-1}+q^{-4}}{q-q^{-1}}F_2F_3F_4K_1^{-1}; && \\
&E_mF_{31}=F_{31}E_m+\tfrac{q-q^{\frac12}}{q-q^{-1}}F_{2m}K_m+\tfrac{q^{-\frac12}-q^{-1}}{q-q^{-1}}F_{2m}K_m^{-1} && (m\in\{2,3,4\});\\
&E_{31}F_1=F_1E_{31}+\tfrac{q^3-q^2-2q^{\frac32}+2q^{\frac12}+1-q^{-1}}{q-q^{-1}}K_1E_2E_3E_4 && \\
&\qquad+\tfrac{2q^{-\frac52}-2q^{-\frac32}-q^{-3}+q^{-1}}{q-q^{-1}}K_1^{-1}E_2E_3E_4; && \\
&E_{31}F_m=F_mE_{31}+\tfrac{1-q^{\frac12}}{q-q^{-1}}K_mE_{2m}+\tfrac{q^{-\frac32}-1}{q-q^{-1}}K_m^{-1}E_{2m} && (m\in\{2,3,4\}).
\end{aligned}
$$

The following relations has too many terms and we only need the leading term, so for convenience, we only write the leading term with their coefficients a_i $(1\leq i\leq 24)$:

$$
\begin{aligned}
&E_1F_{21}=F_{21}E_1+a_1F_{12}F_3F_4K_1+\text{other terms};\\
&E_mF_{21}=F_{21}E_m+a_2F_{1s}F_{1t}K_m+\text{other terms}\\
&\qquad(m,s,t\in\{2,3,4\},t>s,m\neq t,m\neq s);\\
&E_{21}F_1=F_1E_{21}+a_3K_1E_{12}E_3E_4+\text{other terms};\\
&E_{21}F_m=F_mE_{21}+a_4K_mE_{1s}E_{1t}+\text{other terms}\\
&\qquad(m,s,t\in\{2,3,4\},t>s,m\neq t,m\neq s);\\
&E_{1m}F_{1m}=F_{1m}E_{1m}+a_5F_mK_mE_m+\text{other terms}\ \ (m\in\{2,3,4\});\\
&E_{1m}F_{1n}=F_{1n}E_{1m}+a_6F_nK_1E_m+\text{other terms}\ \ (m,n\in\{2,3,4\},m\neq n);
\end{aligned}
$$

$$\begin{aligned}
E_{1m}F_{2m} &= F_{2m}E_{1m} + a_7F_sF_tK_1E_m + \text{other terms} \\
&\qquad (m, s, t \in \{2,3,4\}, t > s, m \neq t, m \neq s); \\
E_{1m}F_{2n} &= F_{2n}E_{1m} + a_8F_sF_tK_1E_m + \text{other terms} \\
&\qquad (m, n, s, t \in \{2,3,4\}, t > s, n \neq t, n \neq s, m \neq n); \\
E_{2m}F_{1m} &= F_{1m}E_{2m} + a_9F_mK_1E_sE_t + \text{other terms} \\
&\qquad (m, s, t \in \{2,3,4\}, t > s, m \neq t, m \neq s); \\
E_{2m}F_{1n} &= F_{1n}E_{2m} + a_{10}F_nK_1E_sE_t + \text{other terms} \\
&\qquad (m, n, s, t \in \{2,3,4\}, t > s, m \neq t, m \neq s, m \neq n); \\
E_{1m}F_{21} &= F_{21}E_{1m} + a_{11}F_{12}F_3F_4K_1E_m + \text{other terms} \ \ (m \in \{2,3,4\}); \\
E_{21}F_{1m} &= F_{1m}E_{21} + a_{12}F_mK_1E_{12}E_3E_4 + \text{other terms} \ \ (m \in \{2,3,4\}); \\
E_{1m}F_{31} &= F_{31}E_{1m} + a_{13}F_2F_3F_4K_1E_m + \text{other terms} \ \ (m \in \{2,3,4\}); \\
E_{31}F_{1m} &= F_{1m}E_{31} + a_{14}F_mK_1E_2E_3E_4 + \text{other terms} \ \ (m \in \{2,3,4\}); \\
E_{2m}F_{2m} &= F_{2m}E_{2m} + a_{15}F_sF_tK_1E_sE_t + \text{other terms} \\
&\qquad (m, s, t \in \{2,3,4\}, s < t, m \neq s, m \neq t); \\
E_{2m}F_{2n} &= F_{2n}E_{2m} + a_{16}F_nF_tK_1E_sE_{t'} + \text{other terms} \\
&\qquad (m, n, s, t, t' \in \{2,3,4\}, m \neq n, m \neq t, n \neq t, n \neq s, n \neq t'); \\
E_{2m}F_{21} &= F_{21}E_{2m} + a_{17}F_{12}F_3F_4K_1E_sE_t + \text{other terms} \\
&\qquad (m, s, t \in \{2,3,4\}, s < t, m \neq t, m \neq s); \\
E_{21}F_{2m} &= F_{2m}E_{21} + a_{18}F_sF_tK_1E_{12}E_3E_4 + \text{other terms} \\
&\qquad (m, s, t \in \{2,3,4\}, s < t, m \neq t, m \neq s); \\
E_{2m}F_{31} &= F_{31}E_{2m} + a_{19}F_2F_3F_4K_1E_sE_t + \text{other terms} \\
&\qquad (m, s, t \in \{2,3,4\}, s < t, m \neq t, m \neq s); \\
E_{31}F_{2m} &= F_{2m}E_{31} + a_{20}F_sF_tK_1E_2E_3E_4 + \text{other terms} \\
&\qquad (m, s, t \in \{2,3,4\}, s < t, m \neq t, m \neq s); \\
E_{21}F_{21} &= F_{21}E_{21} + a_{21}F_2F_3F_4K_1E_2E_3E_4 + \text{other terms}; \\
E_{21}F_{31} &= F_{31}E_{21} + a_{22}F_2F_3F_4K_1E_{12}E_3E_4 + \text{other terms}; \\
E_{31}F_{21} &= F_{21}E_{31} + a_{23}F_{12}F_3F_4K_1E_2E_3E_4 + \text{other terms}; \\
E_{31}F_{31} &= F_{31}E_{31} + a_{24}F_2F_3F_4K_1E_2E_3E_4 + \text{other terms}.
\end{aligned}$$

Now, we prove the following one case, and the proofs of other cases are similar. If $m \in \{2,3,4\}$, then

$$\begin{aligned}
E_1F_{1m} &= E_1(F_mF_1 - v^{-1}F_1F_m) = E_1F_mF_1 - v^{-1}E_1F_1F_m \\
&= F_mE_1F_1 - v^{-1}(F_1E_1 + \tfrac{K_1-K_1^{-1}}{q-q^{-1}})F_m \\
&= F_m(F_1E_1 + \tfrac{K_1-K_1^{-1}}{q-q^{-1}}) - v^{-1}F_1E_1F_m - v^{-1}(\tfrac{K_1-K_1^{-1}}{q-q^{-1}})F_m \\
&= F_mF_1E_1 - v^{-1}F_1F_mE_1 + \tfrac{F_mK_1-F_mK_1^{-1}}{q-q^{-1}} - v^{-1}(\tfrac{K_1F_m-K_1^{-1}F_m}{q-q^{-1}}) \\
&= (F_mF_1 - v^{-1}F_1F_m)E_1 + \tfrac{F_mK_1-F_mK_1^{-1}}{q-q^{-1}} - v^{-1}(\tfrac{qF_mK_1-q^{-1}F_mK_1^{-1}}{q-q^{-1}}) \\
&= F_{1m}E_1 + \tfrac{1-q^{\frac{1}{2}}}{q-q^{-1}}F_mK_1 + \tfrac{q^{-\frac{3}{2}}-1}{q-q^{-1}}F_mK_1^{-1} \ \ (v^{-1} = q^{-\frac{1}{2}}).
\end{aligned}$$

From the equivalent conditions of Gröbner-Shirshov basis, we know that the following monomials forms a k-basis of $U_q(D_4)$:

$$F_1^{\,n_1}F_{12}^{\,n_2}\cdots F_4^{\,n_{12}}K_1^{\,a_1}\cdots K_4^{\,a_4}E_1^{\,m_1}E_{12}^{\,m_2}\cdots E_4^{\,m_{12}},$$

where n_i, $m_i \in N$, and $a_i \in Z$.

In order to compute the Gelfand-Kirillov dimension of an algebra, first we had to prove this algebra is a PBW-algebra. For this, we need find to a vector with strictly positive components which is an exponent vector of some standard monomial (or equivalently, some basis element). This fact does not allow us to use the negative exponents (for details see [1]). So we need to introduce a new algebra generated by

$$F_1, F_{12}, F_{13}, F_{14}, F_{21}, F_{22}, F_{23}, F_{24}, F_{31}, F_2, F_3, F_4, L_1, L_2, L_3, L_4, K_1, K_2, K_3, K_4, E_1, E_{12}, E_{13}, E_{14}, E_{21}, E_{22}, E_{23}, E_{24}, E_{31}, E_2, E_3, E_4$$

and subject to the relations obtained from the relations of $U_q(D_4)$ by just replacing the K_i^{-1} in $U_q(D_4)$ with L_i for $i \in \{1,2,3,4\}$ and exclude the relations $K_iL_i - 1$, $L_iK_i - 1$ for $i \in \{1,2,3,4\}$. We denote this algebra by $V_q(D_4)$. By direct computation using the skew-commutator relations between all generators above, we know that the monomials

$$F_1{}^{n_1} F_{12}{}^{n_2} \cdots F_4{}^{n_{12}} L_1{}^{b_1} \cdots L_4{}^{b_4} K_1{}^{a_1} \cdots K_4{}^{a_4} E_1{}^{m_1} E_{12}{}^{m_2} \cdots E_4{}^{m_{12}}$$

form a k-basis for $V_q(D_4)$ with $n_i, a_i, b_i, m_i \in N$.

Now, we prove that the algebra $V_q(D_4)$ is a PBW algebra. From the definition of the PBW algebra, we know that we only need to find a weight vector ω with strictly positive components such that satisfies conditions (1) and (2) in Definition 2.11. Condition (2) is obvious. By [5] we know that we can take the vector ω as follows:

$$\omega = (\omega_1, \omega_2, \omega_3, \cdots \omega_{11}, \omega_{12}, 1,1,1,1,1,1,1,1, \omega_1, \omega_2, \omega_3, \cdots, \omega_{11}, \omega_{12})$$

and by simple calculation we know that condition (1) is equivalent to satisfies the following inequalities:

$$\begin{aligned}
&1+2w_{10} < 2w_2, \quad 1+2w_{11} < 2w_3, \quad 1+2w_{12} < 2w_4,\\
&w_3+w_4+1 < w_5+w_{10}, \quad w_2+w_4+1 < w_5+w_{11},\\
&w_{10}+w_{11}+1 < w_2+w_3, \quad w_{10}+w_{12}+1 < w_2+w_4\\
&w_{11}+w_{11}+1 < w_1+w_8, \quad w_{11}+w_{12}+1 < w_3+w_4,\\
&w_{11}+w_{12}+1 < w_1+w_6, \quad w_{10}+w_{12}+1 < w_1+w_7,\\
&1+2w_{10}+w_{12} < w_2+w_7, \quad 1+2w_{10}+w_{11} < w_2+w_8,\\
&1+2w_{11}+w_{12} < w_3+w_6, \quad 1+2w_{11}+w_{10} < w_3+w_8,\\
&1+2w_{12}+w_{11} < w_4+w_6, \quad 1+2w_{12}+w_{10} < w_4+w_7,\\
&w_2+w_3+1 < w_5+w_{12}, \quad w_{10}+w_{11}+w_{12}+1 < w_2+w_6,\\
&1 < w_1+w_1, \quad 1 < w_{10}+w_{10}, \quad 1 < w_{11}+w_{11}, \quad 1 < w_{12}+w_{12},\\
&w_{10}+1 < w_1+w_2, \quad w_{11}+1 < w_1+w_3, \quad w_{12}+1 < w_1+w_4,\\
&w_1+1 < w_2+w_{10}, \quad w_1+1 < w_3+w_{11}, \quad w_1+1 < w_4+w_{12},\\
&w_4+1 < w_7+w_{10}, \quad w_3+1 < w_8+w_{10}, \quad w_4+1 < w_6+w_{11},\\
&w_2+1 < w_8+w_{11}, \quad w_3+1 < w_6+w_{12}, \quad w_2+1 < w_7+w_{12},\\
&w_{10}+w_{11}+w_{12}+1 < w_1+w_9, \quad w_2+w_{11}+w_{12}+1 < w_1+w_5,
\end{aligned}$$

$$w_6+1<w_9+w_{10},\quad w_7+1<w_9+w_{11},\quad w_8+1<w_9+w_{12},$$
$$w_{10}+w_{11}+w_{12}+1<w_3+w_7,\quad w_{10}+w_{11}+w_{12}+1<w_4+w_8,$$
$$2w_{10}+w_{11}+w_{12}+1<w_2+w_9,\quad w_{10}+2w_{11}+w_{12}+1<w_3+w_9,$$
$$2w_{11}+w_{12}+w_2+1<w_3+w_5,\quad w_{11}+2w_{12}+w_2+1<w_4+w_5,$$
$$2w_{10}+w_{11}+w_{12}+1<w_7+w_8,\quad w_{10}+2w_{11}+w_{12}+1<w_6+w_8,$$
$$w_{10}+w_{11}+2w_{12}+1<w_6+w_7,\quad 2w_{11}+2w_{12}+w_2+1<w_5+w_6,$$
$$w_{10}+2w_{11}+2w_{12}+1<w_6+w_9,\quad 2w_{10}+w_{11}+2w_{12}+1<w_7+w_9,$$
$$2w_{10}+2w_{11}+w_{12}+1<w_8+w_9,\quad 2w_{10}+2w_{11}+2w_{12}+1<2w_5,$$
$$2w_{10}+2w_{11}+2w_{12}+1<2w_9,\quad w_{10}+2w_{11}+2w_{12}+w_2+1<w_5+w_9,$$
$$w_{10}+w_{11}+2w_{12}+1<w_4+w_9,\quad w_{10}+w_{11}+w_{12}+w_2+1<w_2+w_5,$$
$$1+2w_{11}+2w_{12}<2w_6,\quad 1+2w_{10}+2w_{12}<2w_7,\quad 1+2w_{10}+2w_{11}<2w_8,$$
$$2w_{10}+w_{11}+2w_{12}+w_2+1<w_5+w_7,\quad w_{10}+2w_{11}+2w_{12}+w_2+1<w_5+w_8.$$

As an example we prove the first inequality. Since

$$E_1F_{12}=F_{12}E_1+\tfrac{1-q^{\frac{1}{2}}}{q-q^{-1}}F_2K_1+\tfrac{q^{-\frac{3}{2}}-1}{q-q^{-1}}F_2K_1^{-1},$$

we have

$$\begin{aligned} &\exp(E_1F_{12}-F_{12}E_1)=\exp(\tfrac{1-q^{\frac{1}{2}}}{q-q^{-1}}F_2K_1)\\ =\ &(0,0,0,0,0,0,0,0,0,1,0,0,0,0,0,0,1,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,),\\ &\varepsilon_{10}+\varepsilon_{17}=(0,0,0,0,0,0,0,0,0,1,0,0,0,0,0,0,1,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,).\end{aligned}$$

Since the weight vector ω satisfies $\exp(E_1F_{12}-F_{12}E_1)_\omega\prec\varepsilon_{10}+\varepsilon_{17}$, we get $1+w_{10}<w_1+w_2$.

By solving these inequalities, we get

$$\omega=(1,2,2,2,5,3,3,3,7,1,1,1,1,1,1,1,1,1,1,1,1,2,2,2,5,3,3,3,7,1,1,1).$$

And for this ω the algebra $V_q(D_4)$ is a PBW algebra with respect to the ordering $\preceq_\omega$.

Now, we define a map $\varphi:\ V_q(D_4)\to U_q(D_4)$

$$F_i\to F_i,\ \ E_i\to E_i,\ \ K_i\to K_i,\ \ L_i\to K_i^{-1},\ \ F_{mn}\to F_{mn},\ \ E_{mn}\to E_{mn},$$

where $i=1,2,3,4$. $mn=12,13,14,21,22,23,24,31$. Obviously, φ is an epimorphism, and $ker(\varphi)=\{K_iL_i-1, i=1,2,3,4\}$. Since $K_1L_1,\ K_2L_2,\ K_3L_3,\ K_4L_4$ are central elements, that is, $\forall\ r\in\ V_q(D_4)$, we have $rK_iL_i=K_iL_ir$, so $I=\langle K_iL_i-1\rangle$ is a two-sided ideal of $V_q(D_4)$. It follows that $U_q(D_4)$ is homomorphic image of the algebra $V_q(D_4)$. We have

$$U_q(D_4)\cong\frac{V_q(D_4)}{I}.$$

This isomorphism allows us to compute the Gelfand-Kirillov dimension of finitely generated $U_q(D_4)$-module. Since I is two-sided ideal, so $G=\{K_1L_1-1,K_2L_2-1,K_3L_3-1,K_4L_4-1\}$ is reduced Gröbner-Shirshov basis of I. Thus

$$\begin{aligned}\mathrm{Exp}(I)\ &=((0,0,0,0,0,0,0,0,0,0,0,0,1,0,0,0,1,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,)+N^{32})\\ &\cup((0,0,0,0,0,0,0,0,0,0,0,0,0,1,0,0,0,1,0,0,0,0,0,0,0,0,0,0,0,0,0,0,)+N^{32})\\ &\cup((0,0,0,0,0,0,0,0,0,0,0,0,0,0,1,0,0,0,1,0,0,0,0,0,0,0,0,0,0,0,0,0,)+N^{32})\\ &\cup((0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,1,0,0,0,1,0,0,0,0,0,0,0,0,0,0,0,0,)+N^{32}).\end{aligned}$$

Set

$$\alpha_1 = (0,0,0,0,0,0,0,0,0,0,0,0,1,0,0,0,1,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,) + N^{32}$$
$$\alpha_2 = (0,0,0,0,0,0,0,0,0,0,0,0,0,1,0,0,0,1,0,0,0,0,0,0,0,0,0,0,0,0,0,0,) + N^{32}$$
$$\alpha_3 = (0,0,0,0,0,0,0,0,0,0,0,0,0,0,1,0,0,0,1,0,0,0,0,0,0,0,0,0,0,0,0,0,) + N^{32}$$
$$\alpha_4 = (0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,1,0,0,0,1,0,0,0,0,0,0,0,0,0,0,0,0,) + N^{32},$$

then

$$\mathrm{supp}(\alpha_1) = \{13, 17\},\ \mathrm{supp}(\alpha_2) = \{14, 18\},\ \mathrm{supp}(\alpha_3) = \{15, 19\},\ \mathrm{supp}(\alpha_4) = \{16, 20\}.$$

Thus we get

$$\min\{\mathrm{card}(\sigma), \sigma \ \in \ V(\mathrm{Exp}(I))\} = 4.$$

By Definition 2.13, we know

$$\dim(\mathrm{Exp}(I)) = 32 - 4 = 28.$$

By Theorem 2.14, we have the main result of this paper.

Thoerem 3.1 $\mathrm{GKdim}(U_q(D_4)) = \mathrm{GKdim}(\frac{V_q(D_4)}{I}) = \dim(\mathrm{Exp}(I)) = 28.$

References

[1] Buese J, Gómez-Torrecillas J, Verschoren A. Algorithmic methods in non-commutative algebra: applications to the quantum groups (Mathematical modelling: Theory and applications, Vol.17) (1st ed.)[M]. Dordrecht: Kluwer Academic Publishers, 2003.

[2] Yunus G, Obul A. Gröbner-Shirshov basis of quantum group of type D_4[J]. Chin. Ann. Math., 2011, 32B(5): 581–592.

[3] Krause G R, Lenagan T H. Growth of algebras and Gelfand-Kirillov dimension[M]. Providence, Rhode Island: Graduate Studies in Math., Volume 22, American Mathematical Society, 1991.

[4] Bokut L A, Malcolmson K P. Gröbner basis for quantum enveloping algebras[J]. Israel J. Math., 1996, 96: 97–113.

[5] Torrecillals J G. Gelfand-Kirillov dimension of Multi-Filtered Algebras[J]. Proc. Edinburgh Math., 1999, 52: 155–158.

D_4 型量子包络代数的Gelfand-Kirillov 维数

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摘要: 本文研究了 D_4 型量子包络代数的 Gelfand-Kirillov 维数的计算问题. 利用文献 [1] 中给出的Gelfand-Kirillov 维数的计算方法和文献 [2] 中给出的 D_4 型量子包络代数的 Groebner-Shirshov 基计算了 D_4型量子包络代数的 Gelfand-Kirillov 维数, 得到的主要结果是 D_4 型量子包络代数的Gelfand-Kirillov 维数为28. 希望此结果为计算 D_n 型量子包络代数的Gelfand-Kirillov 维数提供一些思路.

关键词: Groebner-Shirshov 基; Poincare-Birkhoff-Witt 代数; 权向量; Gelfand-Kirillov 维数

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