

STABILITY PROPERTIES FOR THE OPTIMAL CONTROL PROBLEM GOVERNED BY FRACTIONAL ORDER PARABOLIC EQUATIONS

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Abstract: In this paper, we study an optimal control problem governed by fractional order parabolic equations. We discuss the stability property for the optimal control and the optimal value. By the convex method, we obtain certain convergence properties for these controlled systems, and it is a general versions of the conclusion proved in reference [3].

Keywords: parabolic equations; fractional order; optimal control; stability property

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1 Introduction

Let us first state the optimal control problem studied in this paper. We begin with introducing the controlled equation. Let Ω be a bounded domain in \mathbb{R}^n , $n \geq 1$, with C^∞ smooth boundary. Let ω be an open and nonempty subset of Ω , and χ_ω denotes the characteristic function of the subset ω . We define an unbounded operator A in $L^2(\Omega)$,

$$\begin{cases} D(A) = H^2(\Omega) \cap H_0^1(\Omega), \\ Av = -\Delta v \text{ for any } v \in D(A). \end{cases}$$

Let $\{\lambda_i\}_{i=1}^\infty$, $0 < \lambda_1 < \lambda_2 \leq \dots$, be the eigenvalues of $A = -\Delta$, and $\{e_i\}_{i=1}^\infty$ be the corresponding eigenfunctions satisfied that $\|e_i\|_{L^2(\Omega)} = 1$, $i = 1, 2, 3, \dots$, which constitutes an orthonormal basis of $L^2(\Omega)$. It is well known that we can define a class of fractional order operator A^α ($\alpha > 0$) in $L^2(\Omega)$ as follows:

$$\begin{cases} D(A^\alpha) = \{v \in L^2(\Omega) \mid v = \sum_{i=1}^\infty v_i e_i \text{ and } \sum_{i=1}^\infty \lambda_i^{2\alpha} |v_i|^2 < \infty\}, \\ A^\alpha v = \sum_{i=1}^\infty \lambda_i^\alpha v_i e_i, \text{ where } v = \sum_{i=1}^\infty v_i e_i. \end{cases}$$

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Moreover, the operator A^α is a self-adjoint operator and $-A^\alpha$ is an infinitesimal generator of a strong continuous semigroup $\{S_\alpha(t)\}_{t \geq 0}$. In this paper, we consider the following linear controlled fractional orders parabolic equation:

$$\begin{cases} \partial_t y_\alpha(x, t) + A^\alpha y_\alpha(x, t) = Bu_\alpha(x, t), & (x, t) \in \Omega \times (0, T], \\ y_\alpha(x, 0) = y_0(x), & x \in \Omega, \end{cases} \quad (1.1)$$

where $\alpha > 0$, B is a linear bounded operator in $L^2(\Omega)$ defined by $Bu_\alpha = \chi_\omega u_\alpha$, $y_0 \in L^2(\Omega)$ and u_α is a control function taken from the space $\mathcal{U}_{ad} = L^2(0, T; L^2(\Omega))$. We denote $y_\alpha(\cdot; y_0, u_\alpha)$ to the unique solution of (1.1) corresponding to the control u_α and the initial value y_0 . We denote $\|\cdot\|$ and $\langle \cdot, \cdot \rangle$ to be the usual norm and the inner product in $L^2(\Omega)$, respectively. Besides, variables x and t for functions of (x, t) and variable x for functions of x will be omitted, provided that it is not going to cause any confusion.

In recent years, extensive research was devoted to the study of differential equations with fractional orders due to their importance for applications in various branches of applied sciences and engineering (see [5]). Many important phenomena in signal processing, electromagnetics, crowded systems, and fluid mechanics are well described by fractional orders differential equation. In this paper, we always discuss the fractional Laplacian.

Now, we discuss an optimal control problem of system (1.1). Let $f_d(\cdot) \in L^2(0, T; L^2(\Omega))$, which can be regarded as a target function. The optimal control problem reads as follows:

$$\begin{aligned} (\mathcal{P}_\alpha) : \quad & \inf \{ J_\alpha(u_\alpha, y_\alpha) : u_\alpha \in \mathcal{U}_{ad}, y_\alpha = y_\alpha(\cdot; y_0, u_\alpha) \text{ is the solution to (1.1)} \}, \\ & \text{where } J_\alpha(u_\alpha, y_\alpha) = \frac{1}{2} \int_0^T \int_\Omega (|y_\alpha - f_d|^2 + |u_\alpha|^2) dx dt. \end{aligned}$$

The problem of optimal control of parabolic equation was also the object of numerous studies, and it was widely studied in the past years. Extensive related reference can be found in [1, 3, 4, 11–13, 15] and the rich works cited therein. Especially, we refer to [11] for the property for the optimal control governed by parabolic equations which plays a key role to our study. As the development of the theory of optimal control for partial differential equations progression, the related theoretical results are expected to be used in the fields of applied science. However, the mathematical model which we set up is an approximation for a real system in general. Indeed, with any error on the model, or on the initial state, the optimal control or optimal cost will change dramatically in certain cases. Thus, it is important to study the stability or sensitivity of the optimal control and optimal cost.

The fractional Laplacian $-A^\alpha$, with $\alpha \in (0, 1]$, generates the rotationally invariant 2α stable Lévy process. For $\alpha = 1$, this process is the normal Brownian motion B_t on \mathbb{R}^n , see [10]. When $\alpha = 1$, we sometime rewrite the controlled systems the following linear controlled heat equation for convenient:

$$\begin{cases} \partial_t y(x, t) + Ay(x, t) = Bu(x, t), & (x, t) \in \Omega \times (0, T], \\ y(x, 0) = y_0(x), & x \in \Omega, \end{cases} \quad (1.2)$$

where u is a control function taken from the space $\mathcal{U}_{ad} = L^2(0, T; L^2(\Omega))$. We denote $y(\cdot; y_0, u)$ to the unique solution of (1.1) corresponding to the control u and the initial value y_0 . The associated optimal control problem is as follows:

$$(\mathcal{P}) : \quad \inf\{J(u, y) : u \in \mathcal{U}_{ad}, y = y(\cdot; y_0, u) \text{ is the solution to (1.2)}\}.$$

$$\text{Where } J(u, y) = \frac{1}{2} \int_0^T \int_{\Omega} (|y - f_d|^2 + |u|^2) dx dt.$$

Clearly, problem (\mathcal{P}_{α}) can be regarded as a perturbed problem of problem (\mathcal{P}) with the perturbed operator:

$$A - A^{\alpha}, \quad \alpha > 0,$$

which is not a bounded operator in $L^2(\Omega)$. This gives rise to the difficulty in application of the classical perturbation theory of C_0 -semigroups, see [8].

According to the strictly convexity of $J(\cdot, \cdot)$ and $J_{\alpha}(\cdot, \cdot)$, we can easily get the existence and uniqueness of the optimal controls for problem (\mathcal{P}) and (\mathcal{P}_{α}) , see [4]. Assume that (u^*, y^*) and $(u_{\alpha}^*, y_{\alpha}^*)$ are the optimal pairs for problem (\mathcal{P}) and (\mathcal{P}_{α}) , respectively. The main result of the paper is presented as follows:

Theorem 1.1 Let $y_0 \in L^2(\Omega)$. Suppose that (u^*, y^*) is the optimal pair for problem (\mathcal{P}) and $(u_{\alpha}^*, y_{\alpha}^*)$ is the optimal pair for problem (\mathcal{P}_{α}) . Then

$$u_{\alpha}^* \rightarrow u^* \text{ strongly in } L^2(0, T; L^2(\Omega)) \text{ as } \alpha \rightarrow 1 \quad (1.3)$$

and

$$J_{\alpha}(u_{\alpha}^*, y_{\alpha}^*) \rightarrow J(u^*, y^*) \text{ as } \alpha \rightarrow 1. \quad (1.4)$$

In this paper, we study a kind of optimal control problem governed by fractional orders parabolic equations. How do these optimal solutions differ from each other. In this lecture, we study the relationship for these optimal solutions in certain cases. Indeed, Theorem 1.1 can be regarded as a stability of such a solution in optimal control. To the best of my knowledge, this problem has not been studied in the past publications. Some interesting articles on control problem governed by fractional orders parabolic equations can be viewed in [6, 7, 9].

The rest of the paper is structured as follows: In Section 2, we give some lemmas which will be used in the proof of the main result. With the aid of the properties presented in Section 2, we will provide the proof of Theorem 1.1 in Section 3.

2 Preliminary Results

Based on classical semigroup theory, we see that the operator $-A^{\alpha}$ is the generator of a semigroup of contraction in $L^2(\Omega)$, which we denote by $S_{\alpha}(t)$, $\alpha > 0$. Indeed, the semigroup can be written as follows:

$$S_{\alpha}(t)\phi = \sum_{i=1}^{\infty} e^{-\lambda_i^{\alpha} t} \langle \phi, e_i \rangle e_i.$$

We also have

$$\|S_\alpha(t)\phi\|_{L^2(\Omega)} \leq e^{-\lambda_1^\alpha t} \|\phi\|_{L^2(\Omega)} \leq C\|\phi\|_{L^2(\Omega)} \tag{2.1}$$

for any $\alpha \in (\frac{1}{2}, 2)$. Here and in what follows, C is a positive number, which is independent on α . This constant varies in different contexts.

In the evolution problem, standard semigroup theory implies that there exists a unique solution to both of the systems (1.1) and (1.2) in the above functional framework, for $\alpha \in (\frac{1}{2}, 2)$. More precisely, we can show the following conclusions:

Lemma 2.1 The solution of (1.1) and (1.2) satisfies the following estimates:

$$\|y(t; y_0, u)\|_{C([0,T];L^2(\Omega))} \leq C(\|y_0\|_{L^2(\Omega)} + \|u(t)\|_{L^2(0,T;L^2(\Omega))})$$

and

$$\|y_\alpha(t; y_0, u_\alpha)\|_{C([0,T];L^2(\Omega))} \leq C(\|y_0\|_{L^2(\Omega)} + \|u_\alpha(t)\|_{L^2(0,T;L^2(\Omega))})$$

for $\alpha \in (\frac{1}{2}, 2)$. Here the constant C only depends on Ω and T .

We can deduce this lemma by the classical semigroup theory, see [2, 8, 14].

Lemma 2.2 For any $\phi \in L^2(\Omega)$ and any $t \in [0, T]$,

$$S_\alpha(t)\phi \rightarrow S(t)\phi \text{ strongly in } L^2(\Omega) \text{ as } \alpha \rightarrow 1. \tag{2.2}$$

Proof For any $\phi \in L^2(\Omega)$,

$$S_\alpha(t)\phi = \sum_{i=1}^{\infty} e^{-\lambda_i^\alpha t} \langle \phi, e_i \rangle e_i$$

and

$$S(t)\phi = \sum_{i=1}^{\infty} e^{-\lambda_i t} \langle \phi, e_i \rangle e_i.$$

On one hand, for any $\epsilon > 0$, there exists a natural number N , which only depend on ϵ and $\varphi \in L^2(\Omega)$, such that

$$\left\| \sum_{i=N+1}^{\infty} e^{-\lambda_i^\alpha t} \langle \phi, e_i \rangle e_i \right\|_{L^2(\Omega)} \leq \sum_{i=N+1}^{\infty} |\langle \phi, e_i \rangle|^2 < \epsilon$$

and

$$\left\| \sum_{i=N+1}^{\infty} e^{-\lambda_i t} \langle \phi, e_i \rangle e_i \right\|_{L^2(\Omega)} \leq \sum_{i=N+1}^{\infty} |\langle \phi, e_i \rangle|^2 < \epsilon$$

for $\alpha \in (\frac{1}{2}, 2)$.

On the other hand,

$$\sum_{i=1}^N e^{-\lambda_i^\alpha t} \langle \phi, e_i \rangle e_i \rightarrow \sum_{i=1}^N e^{-\lambda_i t} \langle \phi, e_i \rangle e_i \text{ in } L^2(\Omega) \text{ as } \alpha \rightarrow 1.$$

Namely, there exist a positive number $\delta > 0$, which only depend on N , such that

$$\left\| \sum_{i=1}^N e^{-\lambda^\alpha t} \langle \phi, e_i \rangle e_i - \sum_{i=1}^N e^{-\lambda t} \langle \phi, e_i \rangle e_i \right\|_{L^2(\Omega)} < \epsilon,$$

as $\alpha \in U(1, \delta) \equiv (1 - \delta, 1 + \delta) \setminus \{1\}$.

In summary, we conclude that (2.2) stands. This completes the proof of the lemma.

Lemma 2.3 For any $t \in [0, T]$ and $u \in \mathcal{U}_{ad}$,

$$y_\alpha(t; y_0, u) \rightarrow y(t; y_0, u) \text{ strongly in } L^2(\Omega) \text{ as } \alpha \rightarrow 1.$$

Proof Indeed,

$$y(t; y_0, u) = S(t)y_0 + \int_0^t S(t-s)\chi_\omega u(s)ds$$

and

$$y_\alpha(t; y_0, u) = S_\alpha(t)y_0 + \int_0^t S_\alpha(t-s)\chi_\omega u(s)ds.$$

By (2.2), we can derive that $S_\alpha(t)y_0 \rightarrow S(t)y_0$, strongly in $L^2(\Omega)$, as $\alpha \rightarrow 1$. It follows from Lemma 2.2 that

$$S_\alpha(t-s)\chi_\omega u(s) \rightarrow S(t-s)\chi_\omega u(s) \text{ strongly in } L^2(\Omega) \text{ as } \alpha \rightarrow 1 \text{ for any } s \in [0, t].$$

By (2.1), we can get

$$\|S_\alpha(t-s)\chi_\omega u(s)\| \leq \|u(s)\| \text{ for any } s \in [0, t].$$

Combining $u \in \mathcal{U}_{ad}$ with the Dominated Convergence Theorem, it shows that

$$\int_0^t S_\alpha(t-s)\chi_\omega u(s)ds \rightarrow \int_0^t S(t-s)\chi_\omega u(s)ds \text{ strongly in } L^2(\Omega) \text{ as } \alpha \rightarrow 1.$$

This completes the proof of the lemma.

For any $\alpha \in (\frac{1}{2}, 2)$, we define two linear bounded operators as follows:

$$\Lambda_1^\alpha : L^2(0, T; L^2(\Omega)) \rightarrow L^2(0, T; L^2(\Omega)),$$

by setting

$$u(\cdot) \rightarrow \int_0^t S_\alpha(t-s)\chi_\omega u(s)ds$$

and

$$\Lambda_2^\alpha : L^2(0, T; L^2(\Omega)) \rightarrow L^2(0, T; L^2(\Omega)),$$

by setting

$$u(\cdot) \rightarrow \int_s^T \chi_\omega S_\alpha(T-t)u(t)dt.$$

When $\alpha = 1$, we sometimes rewrite these two operators as Λ_1 and Λ_2 , for convenient. Now, we will present the following result for these operators.

Lemma 2.4 The adjoint operator of Λ_1^α is Λ_2^α . For any $f(\cdot) \in L^2(0, T; L^2(\Omega))$, we have

$$(\Lambda_2^\alpha f)(s) \rightarrow (\Lambda_2 f)(s) \text{ strongly in } L^2(0, T; L^2(\Omega)) \text{ as } \alpha \rightarrow 1. \tag{2.3}$$

Proof For any $f(\cdot), g(\cdot) \in L^2(0, T; L^2(\Omega))$,

$$\begin{aligned} & \langle \Lambda_1^\alpha f(t), g(t) \rangle_{L^2(0, T; L^2(\Omega))} \\ &= \int_0^T \langle g(t), \int_0^t S_\alpha(t-s)\chi_\omega f(s) ds \rangle dt \\ &= \int_0^T \int_0^t \langle g(t), S_\alpha(t-s)\chi_\omega f(s) \rangle ds dt \\ &= \int_0^T \int_0^T \chi_{[0, t]}(s) \langle g(t), S_\alpha(t-s)\chi_\omega f(s) \rangle ds dt \\ &= \int_0^T \int_0^T \chi_{[0, t]}(s) \langle \chi_\omega S_\alpha(t-s)g(t), f(s) \rangle ds dt. \end{aligned}$$

The last step is based on the fact that $S_\alpha(t)$ is a self-adjoint operator. According to Fubini Theorem, we can derive that

$$\begin{aligned} & \langle \Lambda_1^\alpha f(t), g(t) \rangle_{L^2(0, T; L^2(\Omega))} \\ &= \int_0^T \int_0^T \chi_{[s, T]}(t) \langle \chi_\omega S_\alpha(t-s)g(t), f(s) \rangle dt ds \\ &= \int_0^T \int_s^T \langle \chi_\omega S_\alpha(t-s)g(t), f(s) \rangle dt ds \\ &= \int_0^T \langle \int_s^T \chi_\omega S_\alpha(t-s)g(t) dt, f(s) \rangle ds \\ &= \langle f(t), \Lambda_2^\alpha g(t) \rangle_{L^2(0, T; L^2(\Omega))}. \end{aligned}$$

By this, it follows that $(\Lambda_1^\alpha)^* = \Lambda_2^\alpha$. By Lemma 2.2 and Dominated Convergence Theorem, it shows that

$$\begin{aligned} (\Lambda_2^\alpha f)(s) &= \int_s^T \chi_\omega S_\alpha(T-t)f(t) dt \\ &\rightarrow \int_s^T \chi_\omega S(T-t)f(t) dt \\ &= (\Lambda_2 f)(s) \text{ strongly in } L^2(\Omega) \text{ as } \alpha \rightarrow 1. \end{aligned}$$

From this and Dominated Convergence Theorem, we can obtain (2.3). This completes the proof of this lemma.

3 Proof of Main Results

Proof We shall prove Theorem 1.1 in a series of steps as follows.

Step 1: There exists a positive number δ such that $J_\alpha(u_\alpha^*, y_\alpha^*) \leq \frac{3}{2}J(u^*, y^*)$ for $\alpha \in \hat{U}(1, \delta) \equiv (1 - \delta, 1 + \delta) \setminus \{1\}$.

According to Lemma 2.1 it follows that

$$\begin{aligned} & |J_\alpha(u^*, y_\alpha(\cdot; y_0, u^*)) - J(u^*, y^*)| \\ = & \frac{1}{2} \left| \int_0^T \int_\Omega (|y(t; y_0, u^*) - f_d|^2 + |u^*|^2) dxdt - \int_0^T \int_\Omega (|y_\alpha(t; y_0, u^*) - f_d|^2 + |u^*|^2) dxdt \right| \\ = & \frac{1}{2} \left| \int_0^T \int_\Omega (y(t; y_0, u^*) - y_\alpha(t; y_0, u^*))(y(t; y_0, u^*) + y_\alpha(t; y_0, u^*) - 2f_d) dxdt \right| \\ \leq & C \left(\int_0^T \int_\Omega |y(t; y_0, u^*) - y_\alpha(t; y_0, u^*)|^2 dxdt \right)^{\frac{1}{2}} \\ & (\|y_0\|_{L^2(\Omega)} + \|u^*\|_{L^2(0, T; L^2(\Omega))} + \|f_d\|_{L^2(0, T; L^2(\Omega))}) \end{aligned}$$

for $\alpha \in (\frac{1}{2}, 2)$. Along with Lemma 2.3 and Dominated Convergence Theorem, it indicates that

$$J_\alpha(u^*, y_\alpha(\cdot; y_0, u^*)) \rightarrow J(u^*, y^*) \quad \text{as } \alpha \rightarrow 1. \quad (3.1)$$

Thus, there exists a positive number $\delta > 0$, such that

$$J_\alpha(u_\alpha^*, y_\alpha^*) \leq J_\alpha(u^*, y_\alpha(\cdot; y_0, u^*)) \leq \frac{3}{2}J(u^*, y^*)$$

for $\alpha \in U(1, \delta) \equiv (1 - \delta, 1 + \delta) \setminus \{1\} \subset (\frac{1}{2}, 2)$.

Step 2: From Step 1, we can get that the families $\{u_\alpha^*\}_{\alpha \in \hat{U}(1, \delta)}$ and $\{y_\alpha^*\}_{\alpha \in \hat{U}(1, \delta)}$ are bounded in $L^2(0, T; L^2(\Omega))$. Thus, we can take a sequence of positive numbers $\alpha_j \in \hat{U}(1, \delta)$ with $\lim_{j \rightarrow \infty} \alpha_j = 1$, such that when $j \rightarrow \infty$,

$$\begin{aligned} u_{\alpha_j}^* & \rightharpoonup \bar{u}, \text{ weakly in } L^2(0, T; L^2(\Omega)), \\ y_{\alpha_j}^* & \rightharpoonup \bar{y}, \text{ weakly in } L^2(0, T; L^2(\Omega)). \end{aligned} \quad (3.2)$$

Now, we claim that $\bar{y} = y(\cdot; y_0, \bar{u})$. In fact,

$$y_{\alpha_j}^*(t; y_0, u_{\alpha_j}^*) = S_{\alpha_j}(t)y_0 + \int_0^t S_{\alpha_j}(t-s)\chi_\omega u_{\alpha_j}^*(s)ds$$

and

$$y(t; y_0, \bar{u}) = S(t)y_0 + \int_0^t S(t-s)\chi_\omega \bar{u}(s)ds.$$

On one hand, by Lemma 2.2, it shows that

$$S_{\alpha_j}(t)y_0 \rightarrow S(t)y_0 \text{ strongly in } L^2(\Omega) \text{ as } j \rightarrow \infty.$$

Combining (2.1) with Dominated Convergence Theorem, it follows that

$$S_{\alpha_j}(t)y_0 \rightarrow S(t)y_0 \text{ strongly in } L^2(0, T; L^2(\Omega)) \text{ as } j \rightarrow \infty. \tag{3.3}$$

On the other hand, we also get that for any $f(\cdot) \in L^2(0, T; L^2(\Omega))$,

$$\begin{aligned} &< \int_0^t S_{\alpha_j}(t-s)\chi_\omega u_{\alpha_j}^*(s)ds, f(t) >_{L^2(0,T;L^2(\Omega))} \\ &= < \Lambda_1^{\alpha_j} u_{\alpha_j}^*(s), f(t) >_{L^2(0,T;L^2(\Omega))} \\ &= < u_{\alpha_j}^*(s), \Lambda_2^{\alpha_j} f(t) >_{L^2(0,T;L^2(\Omega))}. \end{aligned}$$

This, together with Lemma 2.4 and (3.2), shows that

$$< u_{\alpha_j}^*(s), \Lambda_2^{\alpha_j} f(t) >_{L^2(0,T;L^2(\Omega))} \rightarrow < \bar{u}(s), \Lambda_2 f(t) >_{L^2(0,T;L^2(\Omega))} \text{ as } j \rightarrow \infty.$$

Namely,

$$\begin{aligned} &< \int_0^t S_{\alpha_j}(t-s)\chi_\omega u_{\alpha_j}^*(s)ds, f(t) >_{L^2(0,T;L^2(\Omega))} \\ &\rightarrow < \int_0^t S(t-s)\chi_\omega \bar{u}(s)ds, f(t) >_{L^2(0,T;L^2(\Omega))} \text{ as } j \rightarrow \infty. \end{aligned}$$

In summary, we can get that

$$y_{\alpha_j}^* = y_{\alpha_j}^*(t; y_0, u_{\alpha_j}^*) \rightharpoonup y(t; y_0, \bar{u}) \text{ weakly in } L^2(0, T; L^2(\Omega)) \text{ as } j \rightarrow \infty.$$

This, together with (3.2), shows that $\bar{y} = y(\cdot; y_0, \bar{u})$.

Step 3: Based on Step 2, the strickly convexity of $J(u, y)$ and (3.1), it follows that

$$J(\bar{u}, \bar{y}) \leq \liminf_{j \rightarrow \infty} J_{\alpha_j}(u_{\alpha_j}^*, y_{\alpha_j}^*) \leq \lim_{j \rightarrow \infty} J_{\alpha_j}(u^*, y_{\alpha_j}(\cdot; y_0, u^*)) = J(u^*, y^*).$$

According to the optimality of u^* and the uniqueness of optimal control, we derive that

$$u^* = \bar{u}, J(\bar{u}, \bar{y}) = J(u^*, y^*)$$

and

$$\lim_{j \rightarrow \infty} J_{\alpha_j}(u_{\alpha_j}^*, y_{\alpha_j}^*) = J(u^*, y^*). \tag{3.4}$$

From these, we can proof (1.4) because for any sequence $\{\alpha_k\} \subset \hat{U}(1, \delta)$ there is a subsequence $\{\alpha_{k_j}\}$ such that (3.4) holds. Then, we derive that

$$\|u_{\alpha_j}^*\|_{L^2(0,T;L^2(\Omega))} \rightarrow \|u^*\|_{L^2(0,T;L^2(\Omega))}$$

and

$$\|y_{\alpha_j}^* - f_d\|_{L^2(0,T;L^2(\Omega))} \rightarrow \|y^* - f_d\|_{L^2(0,T;L^2(\Omega))} \text{ as } j \rightarrow \infty.$$

This, together with (3.2), shows that

$$u_{\alpha_j}^* \rightarrow u^* \text{ strongly in } L^2(0, T; L^2(\Omega)) \text{ as } j \rightarrow \infty. \tag{3.5}$$

This leads to (1.3), because for any sequence $\{\alpha_k\} \subset \hat{U}(1, \delta)$ there is a subsequence $\{\alpha_{k_j}\}$ such that (3.5) holds. Now, we can complete the proof of this theorem.

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分数阶抛物方程控制系统最优控制的稳定性

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摘要: 本文研究了一类分数阶抛物方程的最优控制问题, 主要讨论了其最优控制与最优值的稳定性. 利用了凸方法获得了获得了这类问题最优控制的稳定性结果, 并且推广了在参考文献[3]中的最优控制稳定性结论.

关键词: 抛物方程; 分数阶; 最优控制; 稳定性

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