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A SUFFICIENT AND NECESSARY CONDITION FOR THE STRONG CONVERGENCE OF ASYMPTOTICALLY HEMI-PSEUDONTRACTIVE MAPPING

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Abstract: In this paper, the asymptotically hemi-pseudocontractive mapping is studied in Banach space. By using the modified Ishikawa iterative scheme with errors, some sufficient and necessary conditions on the strong convergence for approximation of fixed points of the uniformly L-Lipschitzian asymptotically hemi-pseudocontractive mapping are obtained, which extend and improve some corresponding results.

Keywords: fixed point; Banach space; asymptotically pseudocontractive mapping; asymptotically hemi-pseudocontractive mapping; uniformly L-Lipshitzian mapping

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1 Introduction and Preliminaries

Throughout this work, we assume that E is a real Banach space, E^* is the dual space of E and $J : E \to 2^{E^*}$ is the normalized duality mapping defined by

$$J(x) = \{f \in E^* : \langle x, f\rangle = \|x\|\|f\|, \|f\| = \|x\|\}, \quad \forall x \in E,$$

where $\langle\cdot,\cdot\rangle$ denotes duality pairing between E and E^*. A single-valued normalized duality mapping is denoted by j.

Let C be a nonempty subset of E and $T : C \to C$ be a mapping. We denote the set of fixed points of T by $F(T)$, i.e., $F(T) = \{x \in C : Tx = x\}$.

Definition 1.1 (1) (see [1]) T is said to be pseudocontractive, if for all $x, y \in C$, there exists $j(x-y) \in J(x-y)$ such that $\langle Tx - Ty, j(x-y)\rangle \le \|x-y\|^2$.

(2) (see [2]) T is said to be uniformly L-Lipshitzian if there exists $L > 0$ such that

$$\|T^n x - T^n y\| \le L\|x-y\|$$

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for all $x, y \in C$ and $n \geq 1$.

(3) (see [3]) T is said to be asymptotically pseudocontractive if there exists a sequence $\{k_n\} \subset [1, \infty)$ with $\lim\limits_{n\to\infty} k_n = 1$, for any $x, y \in C$, there exists $j(x-y) \in J(x-y)$ such that

$$\langle T^n x - T^n y, j(x-y) \rangle \leq k_n \|x-y\|^2, \quad n \geq 1.$$

Recently, Guo Weiping and Guo Qi [4] introduced a new mapping as follows:

Definition 1.2 T is said to be asymptotically hemi-pseudocontractive if $F(T) \neq \emptyset$ and there exists a sequence $\{k_n\} \subset [1, \infty)$ with $\lim_{n\to\infty} k_n = 1$ such that, for any $x \in C$ and $p \in F(T)$, there exists $j(x-p) \in J(x-p)$ such that

$$\langle T^n x - p, j(x-p) \rangle \leq k_n \|x-p\|^2, \quad n \geq 1.$$

Remark 1.1 It is easy to see that if T is an asymptotically pseudocontractive mapping with $F(T) \neq \emptyset$, then T is an asymptotically hemi-pseudocontractive mapping. Conversely, Guo Weiping and Guo Qi [4] give an example to show that the converse is not true in general.

Let C be a nonempty convex subset of E with $C + C \subset C$ and $T : C \to C$ be a mapping. For any given $x_1 \in C$, the sequence $\{x_n\}$ defined by

$$\begin{cases} x_{n+1} = (1-\alpha_n)x_n + \alpha_n T^n y_n + u_n, \\ y_n = (1-\beta_n)x_n + \beta_n T^n x_n + v_n, \quad \forall n \geq 1, \end{cases} \tag{1.1}$$

where $\{\alpha_n\}$, $\{\beta_n\}$ are two real sequences in $[0,1]$ and $\{u_n\}$, $\{v_n\}$ are two bounded sequences in C.

Letting $\beta_n = 0$, $v_n = 0$, $\forall n \geq 1$ in (1.1), then the sequence $\{x_n\}$ defined by (1.1) is reduced to the following iterative scheme:

$$x_{n+1} = (1-\alpha_n)x_n + \alpha_n T^n x_n + u_n, \quad \forall n \geq 1. \tag{1.2}$$

For any $z \in C$ and a set K in E, we denote the distance between z and K by $d(z, K) = \inf\limits_{y\in K} \|z - y\|$.

Recently, Tang et al. [5] proved some sufficient and necessary conditions for strong convergence of Lipschitzian pseudoncontractive mappings in Banach spaces.

In this work, we give a new method and prove some sufficient and necessary conditions for the strong convergence of the iterations sequences (1.1) and (1.2) to a fixed point of asymptotically hemi-pseudocontractive in Banach spaces.

Lemma 1.1 (see [6]) Let X be a Banach space and $x, y \in X$. Then $\|x\| \leq \|x + ry\|$ for all $r > 0$ if and only if there exists $j(x) \in J(x)$ such that $\langle y, j(x) \rangle \geq 0$.

Lemma 1.2 (see [7]) Let $\{a_n\}$, $\{\lambda_n\}$, $\{\sigma_n\}$ be three nonnegative sequences satisfying the following inequality:

$$a_{n+1} \leq (1+\lambda_n)a_n + \sigma_n, \forall n \geq n_0,$$

where n_0 is some nonnegative integer, $\sum\limits_{n=1}^{\infty} \lambda_n < \infty$ and $\sum\limits_{n=1}^{\infty} \sigma_n < \infty$. Then $\lim\limits_{n\to\infty} a_n$ exists. In particular, if $\liminf\limits_{n\to\infty} a_n = 0$, then $\lim\limits_{n\to\infty} a_n = 0$.

2 Main Results

First, we prove the following lemmas.

Lemma 2.1 Let C be a nonempty subset of a Banach space E and $T : C \to C$ be an asymptotically hemi-pseudocontractive mapping with the sequence $\{k_n\} \subset [1, \infty)$, $\lim\limits_{n\to\infty} k_n = 1$. Then

$$\|x-p\| \le \|x-p+r[(k_nI-T^n)x-(k_nI-T^n)p]\| \tag{2.1}$$

for all $x \in C$, $p \in F(T)$, $r > 0$ and $n \ge 1$, where I is identity mapping.

Proof Since T is an asymptotically hemi-pseudocontractive mapping with the sequence $\{k_n\}$, for all $x \in C$ and $p \in F(T)$, there exists $j(x-p) \in J(x-p)$, such that

$$\langle T^nx-p, j(x-p)\rangle \le k_n\|x-p\|^2 = k_n\langle x-p, j(x-p)\rangle, n \ge 1,$$

and so

$$\langle (k_nI-T^n)x-(k_nI-T^n)p, j(x-p)\rangle \ge 0.$$

Therefore, (2.1) holds by Lemma 1.1. This completes the proof.

Lemma 2.2 Let E be a real Banach space and C be a nonempty convex subset of E with $C+C \subset C$ and $T : C \to C$ be a uniformly L-Lipschitzian asymptotically hemi-pseudocontractive mapping with the sequence $\{k_n\} \subset [1, \infty)$, $\lim\limits_{n\to\infty} k_n = 1$. Let $\{\alpha_n\}$, $\{\beta_n\}$ be two real sequences in $[0, 1]$ and $\{u_n\}$, $\{v_n\}$ be two bounded sequences in C. Suppose that the sequence $\{x_n\}$ is defined by (1.1) satisfying the following conditions:

(i) $\sum\limits_{n=1}^{\infty} \alpha_n^2 < \infty$, $\sum\limits_{n=1}^{\infty} \alpha_n\beta_n < \infty$;

(ii) $\sum\limits_{n=1}^{\infty} \|u_n\| < \infty$, $\sum\limits_{n=1}^{\infty} \|v_n\| < \infty$;

(iii) $\sum\limits_{n=1}^{\infty} \alpha_n(k_n-1) < \infty$.

Then (1) there exist two sequences $\{r_n\}, \{s_n\} \subset [0, \infty)$, such that $\sum\limits_{n=1}^{\infty} r_n < \infty$, $\sum\limits_{n=1}^{\infty} s_n < \infty$ and

$$\|x_{n+1}-p\| \le (1+r_n)\|x_n-p\| + s_n \tag{2.2}$$

for all $p \in F(T)$ and $n \ge 1$.

(2) The limit $\lim\limits_{n\to\infty} d(x_n, F(T))$ exists.

Proof (1) Let $p \in F(T)$, by (1.1), we have

$$\begin{aligned} x_n &= x_{n+1} + \alpha_nx_n - \alpha_nT^ny_n - u_n \\ &= (1+\alpha_n)x_{n+1} + \alpha_n(k_nI-T^n)x_{n+1} - (1+k_n)\alpha_nx_{n+1} \\ &\quad + \alpha_nx_n + \alpha_n(T^nx_{n+1}-T^ny_n) - u_n \\ &= (1+\alpha_n)x_{n+1} + \alpha_n(k_nI-T^n)x_{n+1} - (1+k_n)\alpha_n[x_n+\alpha_n(T^ny_n-x_n)+u_n] \\ &\quad + \alpha_nx_n + \alpha_n(T^nx_{n+1}-T^ny_n) - u_n \end{aligned}$$

$$\begin{aligned}
&= (1+\alpha_n)x_{n+1} + \alpha_n(k_nI - T^n)x_{n+1} - (1+k_n)\alpha_n x_n + (1+k_n)\alpha_n^2(x_n - T^ny_n)\\
&\quad + \alpha_n x_n + \alpha_n(T^nx_{n+1} - T^ny_n) - [(1+k_n)\alpha_n + 1]u_n\\
&= (1+\alpha_n)x_{n+1} + \alpha_n(k_nI - T^n)x_{n+1} - k_n\alpha_n x_n + (1+k_n)\alpha_n^2(x_n - T^ny_n)\\
&\quad + \alpha_n(T^nx_{n+1} - T^ny_n) - [(1+k_n)\alpha_n + 1]u_n \qquad (2.3)
\end{aligned}$$

and

$$p = (1+\alpha_n)p + \alpha_n(k_nI - T^n)p - k_n\alpha_n p. \qquad (2.4)$$

Together with (2.3) and (2.4), we can obtain

$$\begin{aligned}
x_n - p =& (1+\alpha_n)(x_{n+1} - p) + \alpha_n[(k_nI - T^n)x_{n+1} - (k_nI - T^n)p] - k_n\alpha_n(x_n - p)\\
&+ (1+k_n)\alpha_n^2(x_n - T^ny_n) + \alpha_n(T^nx_{n+1} - T^ny_n) - [(1+k_n)\alpha_n + 1]u_n. \qquad (2.5)
\end{aligned}$$

Notice that

$$\begin{aligned}
& (1+\alpha_n)(x_{n+1} - p) + \alpha_n[(k_nI - T^n)x_{n+1} - (k_nI - T^n)p]\\
= \;& (1+\alpha_n)[(x_{n+1} - p) + \frac{\alpha_n}{1+\alpha_n}((k_nI - T^n)x_{n+1} - (k_nI - T^n)p)].
\end{aligned}$$

Using Lemma 2.1, we obtain that

$$\|(1+\alpha_n)(x_{n+1} - p) + \alpha_n(k_nI - T^n)(x_{n+1} - p)\| \geq (1+\alpha_n)\|x_{n+1} - p\|. \qquad (2.6)$$

It follows from (2.5) and (2.6) that

$$\begin{aligned}
\|x_n - p\| \geq& (1+\alpha_n)\|x_{n+1} - p\| - k_n\alpha_n\|x_n - p\| - (1+k_n)\alpha_n^2\|x_n - T^ny_n\|\\
&- \alpha_n\|T^nx_{n+1} - T^ny_n\| - [(1+k_n)\alpha_n + 1]\|u_n\|.
\end{aligned}$$

This implies that

$$\begin{aligned}
(1+\alpha_n)\|x_{n+1} - p\| \leq& (1+k_n\alpha_n)\|x_n - p\| + (1+k_n)\alpha_n^2\|x_n - T^ny_n\|\\
&+ \alpha_n\|T^nx_{n+1} - T^ny_n\| + [(1+k_n)\alpha_n + 1]\|u_n\|. \qquad (2.7)
\end{aligned}$$

Next, we make the following estimations:

$$\begin{aligned}
\|y_n - p\| &= \|(1-\beta_n)(x_n - p) + \beta_n(T^nx_n - p) + v_n\|\\
&\leq (1-\beta_n)\|x_n - p\| + \beta_n\|T^nx_n - p\| + \|v_n\|\\
&\leq (1-\beta_n)\|x_n - p\| + L\beta_n\|x_n - p\| + \|v_n\|\\
&\leq (1-\beta_n + L\beta_n)\|x_n - p\| + \|v_n\|,\\
\|x_n - T^ny_n\| &\leq \|x_n - p\| + \|p - T^ny_n\|\\
&\leq \|x_n - p\| + L\|y_n - p\|\\
&\leq [1 + L(1-\beta_n + L\beta_n)]\|x_n - p\| + L\|v_n\|, \qquad (2.8)
\end{aligned}$$

$$\begin{aligned}\|x_n - y_n\| &\le \beta_n\|x_n - T^n x_n\| + \|v_n\| \\ &\le \beta_n\|x_n - p\| + \beta_n\|T^n x_n - p\| + \|v_n\| \\ &\le \beta_n(1+L)\|x_n - p\| + \|v_n\|, \qquad (2.9)\\ \|T^n x_{n+1} - T^n y_n\| &\le L\|x_{n+1} - y_n\| \\ &= L\|x_n - y_n + \alpha_n(T^n y_n - x_n) + u_n\| \\ &\le L\|x_n - y_n\| + \alpha_n L\|T^n y_n - x_n\| + L\|u_n\|. \qquad (2.10)\end{aligned}$$

Substituting (2.8) and (2.9) into (2.10), we have

$$\begin{aligned}\|T^n x_{n+1} - T^n y_n\| \le{}& [\alpha_n L + \alpha_n L^2(1-\beta_n + L\beta_n) + \beta_n L(1+L)]\|x_n - p\| \\ &+ L(1+\alpha_n L)\|v_n\| + L\|u_n\|. \qquad (2.11)\end{aligned}$$

Substituting (2.8) and (2.11) into (2.7), we have

$$\begin{aligned}(1+\alpha_n)\|x_{n+1} - p\| \le{}& (1+k_n\alpha_n)\|x_n - p\| + (1+k_n)\alpha_n^2[(1+L(1-\beta_n+L\beta_n))\|x_n - p\| + L\|v_n\|] \\ &+ \alpha_n[\alpha_n L + \alpha_n L^2(1-\beta_n + L\beta_n) + \beta_n L(1+L)]\|x_n - p\| \\ &+ \alpha_n L(1+\alpha_n L)\|v_n\| + \alpha_n L\|u_n\| + [(1+k_n)\alpha_n + 1]\|u_n\|.\end{aligned}$$

By $1 + \alpha_n \ge 1$, this implies that

$$\begin{aligned}\|x_{n+1} - p\| \le{}& (1 + (k_n - 1)\alpha_n)\|x_n - p\| + (1+k_n)\alpha_n^2[(1+L(1-\beta_n+L\beta_n))\|x_n - p\| + L\|v_n\|] \\ &+ \alpha_n[\alpha_n L + \alpha_n L^2(1-\beta_n + L\beta_n) + \beta_n L(1+L)]\|x_n - p\| \\ &+ \alpha_n L(1+\alpha_n L)\|v_n\| + \alpha_n L\|u_n\| + [(1+k_n)\alpha_n + 1]\|u_n\| \\ ={}& [1 + (k_n-1)\alpha_n + (1+k_n)(1+L(1-\beta_n+L\beta_n))\alpha_n^2 + \alpha_n^2 L(1+L(1-\beta_n+L\beta_n)) \\ &+ \alpha_n\beta_n L(1+L)]\|x_n - p\| + L[(1+k_n)\alpha_n^2 + \alpha_n(1+\alpha_n L)]\|v_n\| \\ &+ [1 + (1+k_n)\alpha_n + \alpha_n L]\|u_n\| \\ ={}& (1+r_n)\|x_n - p\| + s_n,\end{aligned}$$

where

$$\begin{aligned}r_n &= (k_n-1)\alpha_n + (1+k_n)(1+L(1-\beta_n+L\beta_n))\alpha_n^2 \\ &\quad + \alpha_n^2 L(1+L(1-\beta_n+L\beta_n)) + \alpha_n\beta_n L(1+L), \\ s_n &= L[(1+k_n)\alpha_n^2 + \alpha_n(1+\alpha_n L)]\|v_n\| + [1 + (1+k_n)\alpha_n + \alpha_n L]\|u_n\|,\end{aligned}$$

and $\sum_{n=1}^{\infty} r_n < \infty$, $\sum_{n=1}^{\infty} s_n < \infty$ by conditions (i)–(iii). This completes the proof of (1).

(2) Taking the infimum over all $p \in F(T)$ on both sides in (2.2), we get

$$d(x_{n+1}, F(T)) \le (1+r_n)d(x_n, F(T)) + s_n.$$

It follows from Lemma 1.2 that the limit $\lim_{n\to\infty} d(x_n, F(T))$ exists.

Theorem 2.1 Let E be a real Banach space and C be a nonempty closed convex subset of E with $C+C\subset C$ and $T: C\rightarrow C$ be a uniformly L-Lipschitzian asymptotically hemi-pseudocontractive mapping with the sequence $k_n\subset[1,\infty)$, $\lim\limits_{n\rightarrow\infty}k_n=1$. Let $\{\alpha_n\}$, $\{\beta_n\}$ be two real sequences in $[0,1]$ and $\{u_n\}$, $\{v_n\}$ be two bounded sequences in C. Suppose that the sequence $\{x_n\}$ is defined by (1.1) satisfying the following conditions:

(i) $\sum\limits_{n=1}^{\infty}\alpha_n^2<\infty$, $\sum\limits_{n=1}^{\infty}\alpha_n\beta_n<\infty$;

(ii) $\sum\limits_{n=1}^{\infty}\|u_n\|<\infty$, $\sum\limits_{n=1}^{\infty}\|v_n\|<\infty$;

(iii) $\sum\limits_{n=1}^{\infty}\alpha_n(k_n-1)<\infty$.

Then $\{x_n\}$ converges strongly to a fixed point of T if and only if $\liminf\limits_{n\rightarrow\infty}d(x_n,F(T))=0$.

Proof The necessary of Theorem 2.1 is obvious. We just need to prove the sufficiency. Assume that $\liminf\limits_{n\rightarrow\infty}d(x_n,F(T))=0$, by Lemma 1.2, then $\lim\limits_{n\rightarrow\infty}d(x_n,F(T))=0$.

Next, we show that $\{x_n\}$ is a Cauchy sequence. In fact, for any $p\in F(T)$ and any positive integers m,n, $m>n$, from inequality $1+x\leq e^x$, $x\geq 0$ and Lemma 2.2, we have

$$\begin{aligned}\|x_m-p\|&\leq\prod_{j=n}^{m-1}(1+r_j)\|x_n-p\|+\sum_{j=n}^{m-1}s_j\prod_{j=n}^{m-1}(1+r_j)\\&\leq e^{\sum\limits_{j=n}^{m-1}r_j}\|x_n-p\|+e^{\sum\limits_{j=n}^{m-1}r_j}\sum_{j=n}^{m-1}s_j\leq M\|x_n-p\|+M\sum_{j=n}^{m-1}s_j,\end{aligned}$$

where $M=e^{\sum\limits_{j=1}^{\infty}r_j}$. Thus, we have

$$\|x_n-x_m\|\leq\|x_n-p\|+\|x_m-p\|\leq(1+M)\|x_n-p\|+M\sum_{j=n}^{\infty}s_j.$$

Taking the infimum over all $p\in F(T)$, we have

$$\|x_n-x_m\|\leq(1+M)d(x_n,F(T))+M\sum_{j=n}^{\infty}s_j.$$

It follows from $\sum\limits_{j=1}^{\infty}s_j<\infty$ and $\lim\limits_{n\rightarrow\infty}d(x_n,F)=0$ that $\{x_n\}$ is a Cauchy sequence. Since C is a nonempty closed convex subset of E, so there exists a $p_0\in C$ such that $x_n\rightarrow p_0$ as $n\rightarrow\infty$. Further, since T is uniformly L-Lipschitzian, it is easy to prove that $F(T)$ is closed. Again since $\lim\limits_{n\rightarrow\infty}d(x_n,F(T))=0$ and so $p_0\in F(T)$. This shows that $\{x_n\}$ converges strongly to a fixed point of T. This completes the proof.

Corollary 2.1 Under the assumptions of Theorem 2.1. Then $\{x_n\}$ converges strongly to a fixed point p of T if and only if there exists a subsequence $\{x_{n_k}\}$ of $\{x_n\}$ which converges strongly to p.

Proof It follows from $\liminf\limits_{n\rightarrow\infty}d(x_n,F)\leq\liminf\limits_{k\rightarrow\infty}d(x_{n_k},F)\leq\lim\limits_{k\rightarrow\infty}\|x_{n_k}-p\|=0$ and Theorem 2.1 that Corollary 2.1 holds. This completes the proof.

Letting $\beta_n = 0$ and $v_n = 0$ for all $n \geq 1$ in Theorem 2.1, we obtain the following results:

Theorem 2.2 Let E be a real Banach space and C be a nonempty closed convex subset of E with $C + C \subset C$ and $T : C \to C$ be a uniformly L-Lipschitzian asymptotically hemi-pseudocontractive mapping with the sequence $\{k_n\} \subset [1, \infty)$, $\lim\limits_{n\to\infty} k_n = 1$. Let $\{\alpha_n\}$ be a real sequence in $[0, 1]$ and $\{u_n\}$ be a bounded sequence in C. Suppose that the sequence $\{x_n\}$ is defined by (1.2) satisfying the following conditions:

(i) $\sum\limits_{n=1}^{\infty} \alpha_n^2 < \infty$; (ii) $\sum\limits_{n=1}^{\infty} \|u_n\| < \infty$; (iii) $\sum\limits_{n=1}^{\infty} \alpha_n(k_n - 1) < \infty$.

Then $\{x_n\}$ converges strongly to a fixed point of T if and only if $\liminf\limits_{n\to\infty} d(x_n, F(T)) = 0$.

Corollary 2.2 Under the assumptions of Theorem 2.2. Then $\{x_n\}$ converges strongly to a fixed point p of T if and only if there exists a subsequence $\{x_{n_k}\}$ of $\{x_n\}$ which converges strongly to p.

Remark 2.1 By Remark 1.1, clearly Theorems 2.1, 2.2 and Corollaries 2.1, 2.2 hold for uniformly L-Lipschitzian and asymptotically pseudoncontractive mappings with $F(T) \neq \emptyset$.

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渐近半伪压缩映射的强收敛的充分必要条件

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摘要: 本文研究了渐近半伪压缩映射. 应用带误差项的修改的Ishikawa迭代序列, 得到了一致L-Lipschitzian渐近半伪压缩映射逼近其不动点的强收敛的充分必要条件.

关键词: 不动点; Banach 空间; 渐近伪压缩映射; 渐近半伪压缩映射; 一致L-Lipschitzian映射

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