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A NOTE ON FOUR-PARAMETRIZED QUARTIC THUE EQUATIONS

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Abstract: In this paper, we study four-parametric quartic Thue equations. An effective upper bound of the solutions (x, y) is obtained for the four-parametric quartic Thue equations by using simpler method to approximate certain algebraic numbers, which extends the number of parameters from 2 to 4.

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1 Introduction

Let $F(x, y) \in \mathbb{Z}[x, y]$ be an irreducible binary form of degree $n \geq 3$. Thue [1] proved that the number of solutions of Thue equations $F(x, y) = k$ is finite. Furthermore, an explicit Thue equations can be solved by the method of A. Baker's [2] linear form in logarithms of algebraic numbers. Therefore, researchers focused on the parametrized Thue equations and inequalities, and abundant results are obtained. For cubic Thue equations and inequalities, see Thomas [3], Xia-Chen-Zhang [4] and Hoshi [5]. For quartic case, see Chen-Voutier [6] and Xia-Chen-Zhang [7]. While for the sextic case, we refer to Wakabayashi [8].

However, all of the above Thue equations and inequalities have at most two parameters. An interesting question is that whether or not there exist a solvable strategy for Thue equations with more than two parameters. In this research, we use the method proposed in [7] to study the Thue equations which consists of four integral parameters. Our main result is the following:

Theorem Define

$$f(x, y) = sx^4 + 4tbdx^3y + 6b^2dsx^2y^2 + 4b^3d^2txy^3 + sb^4d^2y^4 = N, \tag{1.1}$$

and let θ be the real root of $f(x, 1) = 0$, then we have

$$|y|^{3-\lambda} \leq \frac{cN\varepsilon^2}{\rho^3 b^3 \sqrt{d}^3},$$

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where $\lambda = \frac{\log(8\sqrt{d}\varepsilon)}{\log(\varepsilon/(8\sqrt{d}s^2))}$, $c = 32.3144d\sqrt{d}\varepsilon(\frac{\sqrt{d}(b\sqrt{d}+1)\varepsilon}{4s^2})^\lambda$, $\varepsilon = t\sqrt{d} + \sqrt{t^2d - s^2}$, and $\rho = \sqrt{\varepsilon^2 - s^2}$. If $\varepsilon > 64ds^3$, this is an effective upper bound for $|y|$.

2 Some Lemmas

Let n be a positive integer, suppose $\alpha, w \in (0,1)$ are real numbers,

$$p_n(x) = \sum_{k=0}^{n} \binom{n-\alpha}{n-k}\binom{n+\alpha}{k} x^k, \; q_n(x) = \sum_{k=0}^{n} \binom{n-\alpha}{k}\binom{n+\alpha}{n-k} x^k,$$

and $R_n(x) = x^\alpha q_n(x) - p_n(x)$.

Lemma 1 Let $R_n(w)$ be defined above, we have $|R_n(w)| \le (1-w^\alpha)(1-\sqrt{w})^{2n}$.

proof Put $f(t) = \frac{(1-t)(w-t)}{t}$. By Lemma 1 in [7], we get

$$|R_n(w)| \le |(n+1)\binom{n+\alpha}{n}\binom{\alpha}{n+1}| \int_1^w f(t)^n t^{\alpha-1} dt.$$

Since $|f'(t)| = \frac{w-t^2}{t^2}$. It is easy to obtain $|f(t)| < |f(\sqrt{w})| = (1-\sqrt{w})^2$.

While

$$|(n+1)\binom{n+\alpha}{n}\binom{\alpha}{n+1}| = |\alpha \cdot \frac{(n^2-\alpha^2)((n-1)^2-\alpha^2)\cdots(1^2-\alpha^2)}{(n!)^2}| \le \alpha,$$

we obtain

$$|R_n(w)| \le \alpha \int_1^w |1-\sqrt{w}|^{2n} t^{\alpha-1} dt = (1-w^\alpha)(1-\sqrt{w})^{2n}.$$

Thus Lemma 1 is proved.

Lemma 2 Let $q_n(w)$ be defined above, then we have

$$|q_n(w)| < \frac{(1+\sqrt{w})^{2n}}{\sqrt{w}^\alpha}(1 + \frac{1-\sqrt{w}}{\pi}).$$

Proof By Lemma 2 in [7], we get

$$q_n(w) = \frac{(-1)^n}{2\pi i}\oint_c (1-wt)^{n-\alpha}(1-t)^{n+\alpha} t^{-n-1} dt,$$

where c denotes the integration path encircles the origin once in the positive sense.

In order to get the estimation of $q_n(w)$, we cut the complex plain from 1 to $1/\sqrt{w}$, and consider the integration path $c = c_1 + c_2 + c_3$, where c_1 denotes the path that starts from 1 and proceeds along the positive real axis to $1/\sqrt{w}$, c_2 denotes the path that circles the origin in positive sense by a circle of radius $1/\sqrt{w}$, and c_3 denotes the path that starts from $1/\sqrt{w}$ back to 1 along the lower part of real axis.

While putting $f_1(t) = \frac{(1-wt)(1-t)}{t}$ and $g_1(t) = \frac{(1-wt)^{1-\alpha}(1-t)^{1+\alpha}}{t^2}$, we have

$$\begin{aligned} |q_n(w)| &= \frac{1}{2\pi}|\oint_c f_1(t)^{n-1} g_1(t) dt| \\ &\le \frac{1}{2\pi}(|\int_{c_1} f_1(t)^{n-1} g_1(t) dt| + |\int_{c_2} f_1(t)^{n-1} g_1(t) dt| + |\int_{c_3} f_1(t)^{n-1} g_1(t) dt|). \end{aligned} \quad (2.1)$$

On the upper part of real axis, we have argt=0, so it leads to

$$|\int_{c_1} f_1(t)^{n-1}g_1(t)dt| = \int_1^{1/\sqrt{w}} |f_1(t)|^{n-1}|g_1(t)|dt.$$

It's easy to get

$$|f_1(t)| < |f_1(\frac{1}{\sqrt{w}})| = (1-\sqrt{w})^2,$$

and

$$|g_1(t)| = (\frac{1}{t} - w)^{1-\alpha}(1-\frac{1}{t})^{1+\alpha} < g(\frac{1}{\sqrt{w}}) = \sqrt{w}^{1-\alpha}(1-\sqrt{w})^2.$$

Straight forward computation shows that

$$|\int_{c_1} f_1(t)^{n-1}g_1(t)dt| \leq \sqrt{w}^{-\alpha}(1-\sqrt{w})^{2n+1}.$$

While on the lower part of real axis, we have argt=2π. Similarly, it leads to

$$|\int_{c_3} f_1(t)^{n-1}g_1(t)dt| = |\int_{1/\sqrt{w}}^{1} |f_1(te^{2\pi i})|^{n-1}|g_1(te^{2\pi i})|e^{2\pi i}dt|,$$

and

$$|\int_{c_3} f_1(t)^{n-1}g_1(t)dt| \leq \sqrt{w}^{-\alpha}(1-\sqrt{w})^{2n+1}.$$

On the circle, after putting $t = \frac{e^{i\theta}}{\sqrt{w}}$, we have

$$\begin{aligned} |\int_{c_2} f_1(t)^{n-1}g_1(t)dt| &= |i\int_0^{2\pi} f_1(\frac{e^{i\theta}}{\sqrt{w}})^{n-1}g_1(\frac{e^{i\theta}}{\sqrt{w}})\frac{e^{i\theta}}{\sqrt{w}}d\theta| \\ &= |\int_0^{2\pi} f_1(\frac{e^{i\theta}}{\sqrt{w}})^{n-1}g_1(\frac{e^{i\theta}}{\sqrt{w}})\frac{1}{\sqrt{w}}d\theta|. \end{aligned}$$

It is easy to get

$$|f_1(\frac{e^{i\theta}}{\sqrt{w}})| \leq |f_1(\frac{-1}{\sqrt{w}})| = (1+\sqrt{w})^2$$

and

$$\begin{aligned} |g_1(\frac{e^{i\theta}}{\sqrt{w}})| &= |(\sqrt{w}e^{-i\theta} - w)^{1-\alpha}(\sqrt{w}e^{-i\theta} - 1)^{1+\alpha}| \\ &= \sqrt{(w - 2w\sqrt{w}\cos\theta + w^2)^{1-\alpha}(w - 2\sqrt{w}\cos\theta + 1)^{1+\alpha}} \\ &= \sqrt{w}^{1-\alpha}(w + 1 - 2\sqrt{w}\cos\theta) \\ &\leq \sqrt{w}^{1-\alpha}(1+\sqrt{w})^2. \end{aligned}$$

So we have

$$|\int_{c_2} f_1(t)^{n-1}g_1(t)dt| \leq \frac{2\pi}{\sqrt{w}^{\alpha}}(1+\sqrt{w})^{2n}.$$

By (2.1), we complete the proof of Lemma 2.

Lemma 3 Let $p_n(w)$ be denoted above, then we have

$$|p_n(w)| < |(1+\sqrt{w})^{2n}\sqrt{w}^{\alpha}(1+\frac{1-\sqrt{w}}{\pi})|.$$

Proof Note that

$$p_n(w) = w^n q_n(\frac{1}{w}),$$

and we know that the estimation of $|p_n(w)|$ is decided by that of $|q_n(\frac{1}{w})|$.

By Lemma 2 in [7], we have

$$|p_n(w)| = w^n|\frac{(-1)^n}{2\pi i}\oint_c (1-\frac{t}{w})^{n-\alpha}(1-t)^{n+\alpha}t^{-n-1}dt|,$$

where $c = c_1 + c_2 + c_3$. In detail, c_1 denotes the path that starts from w and proceeds along the positive real axis to $\sqrt{w}$, c_2 denotes the path that circles the origin in positive sense by a circle of radius $\sqrt{w}$, and c_3 denotes the path that starts from $\sqrt{w}$ and return to w along the lower part of real axis.

Now we put

$$f_2(t) = \frac{(1-\frac{t}{w})(1-t)}{t} \text{ and } g_2(t) = \frac{(1-\frac{t}{w})^{1-\alpha}(1-t)^{1+\alpha}}{t^2},$$

then we have

$$\begin{aligned} |p_n(w)| &= w^n\frac{1}{2\pi}|\oint_c f_2(t)^{n-1}g_2(t)dt| \qquad (2.2)\\ &\le w^n\frac{1}{2\pi}(|\int_{c_1} f_2(t)^{n-1}g_2(t)dt| + |\int_{c_2} f_2(t)^{n-1}g_2(t)dt| + |\int_{c_3} f_2(t)^{n-1}g_2(t)dt|). \end{aligned}$$

In a similar way as the proof in Lemma 2, we have

$$\begin{aligned} |\int_{c_2} f_2(t)^{n-1}g_2(t)dt| &= |\int_w^{\sqrt{w}} f_2(t)^{n-1}g_2(t)dt| \\ &\le |\int_w^{\sqrt{w}} (\frac{(1-\sqrt{w})^2}{w})^{n-1}(\frac{1}{\sqrt{w}}-1)^2(\frac{1}{\sqrt{w}})^{1-\alpha}dt| \\ &\le \frac{(1-\sqrt{w})^{2n}}{w^n}(1-\sqrt{w})\sqrt{w}^{\alpha}, \end{aligned}$$

and also

$$|\int_{c_3} f_2(t)^{n-1}g_2(t)dt| \le \frac{(1-\sqrt{w})^{2n}}{w^n}(1-\sqrt{w})\sqrt{w}^{\alpha}.$$

On the circle, putting $t = \sqrt{w}e^{i\theta}$, we have

$$\begin{aligned} |\int_{c_2} f_2(t)^{n-1}g_2(t)dt| &= |i\int_0^{2\pi} f_2(\sqrt{w}e^{i\theta})^{n-1}g_2(\sqrt{w}e^{i\theta})\sqrt{w}e^{i\theta}d\theta| \\ &= |\int_0^{2\pi} f_2(\sqrt{w}e^{i\theta})^{n-1}g_2(\sqrt{w}e^{i\theta})\sqrt{w}d\theta| \\ &\le |\int_0^{2\pi} f_2(-\sqrt{w})^{n-1}g_2(-\sqrt{w})\sqrt{w}d\theta| \\ &= 2\pi\frac{(1+\sqrt{w})^{2n}}{w^n}\sqrt{w}^{\alpha}. \end{aligned}$$

From (2.2), it is direct to prove Lemma 3.

Lemma 4 (see [6]) Let θ be an algebraic number. Suppose that there exists $k_0 > 0$, l_0, $Q > 1$, $E > 1$ such that for all n there are rational integers P_n and y_n with $|Q_n| < k_0 Q^n$ and $|Q_n\theta - P_n| \le l_0 E^{-n}$ and suppose further that $P_n Q_{n+1} \ne Q_n P_{n+1}$. Then, for any rational integers x and y, $y \ge e/(2l_0)$, we have

$$|x - y\theta| > \frac{1}{cy^{\lambda}},\ \text{where } c = 2k_0 Q(2l_0 E)^{\lambda},\ \lambda = \frac{\log Q}{\log E}.$$

3 Proof of Theorem

Let $f(x, y)$, ε be as in Theorem and assume $\varepsilon > 64dt^3, d > 1$, where $d, s, t, b, d \in Z$ and $t > 0$ without loss of generality. Then we have

$$f(x, 1) = sx^4 + 4tbdx^3 + 6b^2dsx^2 + 4b^3d^2tx + sb^4d^2$$

denoted by $f(x)$. If denoting the root of $f(x) = 0$ as θ, it is easy to show that θ satisfies

$$(\frac{\theta + b\sqrt{d}}{\theta - b\sqrt{d}})^4 = \frac{t\sqrt{d} - s}{t\sqrt{d} + s}.$$

For simplicity, we denote $z = t\sqrt{d} - s, u = t\sqrt{d} + s$, and $w = \frac{z}{u}$.

It is straightforward to get

$$\sqrt{w} = \frac{t\sqrt{d} + \sqrt{t^2d - s^2} - s}{t\sqrt{d} + \sqrt{t^2d - s^2} + s} = \frac{\varepsilon - s}{\varepsilon + s}.$$

Putting $\rho = \sqrt{\varepsilon^2 - s^2}$, we have $\sqrt[4]{w} = \pm\frac{\varepsilon - s}{\rho}, \pm\frac{\varepsilon - s}{\rho}i$. Hence, the roots of $f(x) = 0$ are

$$\begin{aligned}
\theta_0 &= b\sqrt{d}\frac{\frac{\varepsilon-s}{\rho} + 1}{\frac{\varepsilon-s}{\rho} - 1} = b\sqrt{d}\,\frac{\varepsilon - s + \rho}{\varepsilon - s - \rho},\\
\theta_1 &= b\sqrt{d}\frac{-\frac{\varepsilon-s}{\rho} + 1}{-\frac{\varepsilon-s}{\rho} - 1} = b\sqrt{d}\,\frac{\varepsilon - s - \rho}{\varepsilon - s + \rho},\\
\theta_2 &= b\sqrt{d}\frac{\frac{\varepsilon-s}{\rho}i + 1}{\frac{\varepsilon-s}{\rho}i - 1} = b\sqrt{d}\,\frac{s - \rho i}{\varepsilon},\\
\theta_3 &= b\sqrt{d}\frac{-\frac{\varepsilon-s}{\rho}i + 1}{-\frac{\varepsilon-s}{\rho}i - 1} = b\sqrt{d}\,\frac{s + \rho i}{\varepsilon}.
\end{aligned}$$

Let $\delta_i = |x - y\theta_i|\ \ (i = 0, 1, 2, 3)$, then we have

$$\delta_2 = |x - \theta_2 y| = |x - b\sqrt{d}\frac{s - \rho i}{\varepsilon}y| > \frac{\rho}{\varepsilon}b\sqrt{d}|y|.$$

Similarly, we have $\delta_3 > \frac{\rho}{\varepsilon}b\sqrt{d}|y|$. If $\delta_0 < \delta_1$, then we know that

$$\delta_1 > \frac{\delta_0 + \delta_1}{2} > \frac{|x - y\theta_0 - (x - y\theta_1)|}{2} = |y|\frac{|\theta_0 - \delta_1|}{2} = \frac{\rho b\sqrt{d}}{s}|y|.$$

If $\delta_1 < \delta_0$, we similarly have $\delta_0 > \frac{\rho b\sqrt{d}}{s}|y|$. Since $|f(x,y)| = s\delta_0\delta_1\delta_2\delta_3$, we obtain that

$$\min\{\delta_0,\delta_1\} < \frac{N}{s(\frac{\rho}{\varepsilon}b\sqrt{d}|y|)^2\ \frac{\rho b\sqrt{d}}{s}|y|} = \frac{N\varepsilon^2}{\rho^3 b^3\sqrt{d}^3|y^3|}. \tag{3.1}$$

Thus we obtain an upper bound for δ_0 or δ_1. Thereafter, we will get a lower bound for them by proving Theorem.

Proof of Theorem Since

$$(\frac{\theta+b\sqrt{d}}{\theta-b\sqrt{d}})^4 = \frac{t\sqrt{d}-s}{t\sqrt{d}+s} = w,$$

it is easy to write θ_0,θ_1 as $\theta_i = b\sqrt{d}\frac{aw^{\frac{1}{4}}+a'}{aw^{\frac{1}{4}}-a'}$, where $a=1$, if $i=0$; $a=\sqrt{d}$, if $i=1$, and $'$ denote conjugate in $\mathbb{Q}[\sqrt{d}]$ that maps $Z+Z\sqrt{d}$ into $Z-Z\sqrt{d}$.

Since $0<w<1$, the lemmas can be applied into Theorem 1 after putting $\alpha = 1/4 \in (0,1)$. Now we use the Padè approximation method to formulate rational integers approximation so as to give an effective measure of the irrationality of θ.

Let $p_n(w), q_n(w), R_n(w)$ be defined above and θ is one of θ_0,θ_1. We know that

$$\begin{aligned}\theta &= b\sqrt{d}\frac{aw^{\frac{1}{4}}+a'}{aw^{\frac{1}{4}}-a'} \\ &= b\sqrt{d}\frac{aw^{\frac{1}{4}}q_n(w)+a'q_n(w)}{aw^{\frac{1}{4}}q_n(w)-a'q_n(w)} \\ &= \frac{b\sqrt{d}aR_n(w)+b\sqrt{d}ap_n(w)+b\sqrt{d}a'q_n(w)}{aR_n(w)+ap_n(w)-a'q_n(w)} \\ &= \frac{4^n(\sqrt{d})^{n+1}u^n b\sqrt{d}aR_n(w)+4^n(\sqrt{d})^{n+1}u^n(b\sqrt{d}ap_n(w)+b\sqrt{d}a'q_n(w))}{4^n(\sqrt{d})^{n+1}u^n aR_n(w)+4^n(\sqrt{d})^{n+1}u^n(ap_n(w)-a'q_n(w))}.\end{aligned}$$

After putting

$$P_n = 4^n(\sqrt{d})^{n+1}u^n(b\sqrt{d}ap_n(w)+b\sqrt{d}a'q_n(w))$$

and

$$Q_n = 4^n(\sqrt{d})^{n+1}u^n(ap_n(w)-a'q_n(w)),$$

we have

$$|P_n - Q_n\theta| = |4^n(\sqrt{d})^{n+1}u^n aR_n(w)(b\sqrt{d}-\theta)|,$$

and denoted it as R_n.

Since $P_n, Q_n \in Z$, from the estimation of $p_n(w)$ and $q_n(w)$ in Lemma 2 and Lemma 3, we obtain

$$\begin{aligned}
|Q_n| &= |4^n(\sqrt{d})^{n+1}u^n(ap_n(w)-a'q_n(w))| \\
&\leq 4^n(\sqrt{d})^{n+1}u^n|a|(|p_n(w)|+|q_n(w)|) \\
&\leq 4^n\sqrt{d}^{n+1}u^n(1+\sqrt{w})^{2n}\,|a||(\sqrt{w}^{\alpha}(1+\frac{1-\sqrt{w}}{\pi})+\sqrt{w}^{-\alpha}(1+\frac{1-\sqrt{w}}{\pi}))| \\
&\leq 4^n\sqrt{d}^{n+1}u^n|a||(1+\sqrt{w})^{2n}(\sqrt{w}^{\alpha}+\sqrt{w}^{-\alpha})(1+\frac{1-\sqrt{w}}{\pi})| \\
&= C_Q(4\sqrt{d}u(1+\sqrt{w})^2)^n \\
&= C_Q(8\sqrt{d}\varepsilon)^n,
\end{aligned}$$

where

$$C_Q=\sqrt{d}|a|(\sqrt{w}^{\alpha}+\sqrt{w}^{-\alpha})(1+\frac{1-\sqrt{w}}{\pi}).$$

Since $|a|\leq\sqrt{d}$ and $\varepsilon>64ds^3$, we have $\sqrt{w}=\frac{\varepsilon-s}{\varepsilon+s}>63/65$, so we can estimate that $\sqrt{w}^{\alpha}+\sqrt{w}^{-\alpha}<2.00006$, and $1+\frac{1-\sqrt{w}}{\pi}<1.00979$. Hence $C_Q<2.01965d$.

On the other hand, from Lemma 1, we have

$$\begin{aligned}
|R_n| &= |4^n(\sqrt{d})^{n+1}u^n aR_n(w)(b\sqrt{d}-\theta)| \\
&< 4^n(\sqrt{d})^{n+1}u^n a|(1-w^{\alpha})(1-\sqrt{w})^{2n}(b\sqrt{d}-\theta)| \\
&= C_R(4\sqrt{d}u(1-\sqrt{w})^2)^n=C_R(8\sqrt{d}s^2/\varepsilon)^n,
\end{aligned}$$

where

$$C_R=\sqrt{d}|a(1-w^{\alpha})(b\sqrt{d}-\theta)|\leq\sqrt{d}|a|(b\sqrt{d}+1)\leq d(b\sqrt{d}+1).$$

Let $Q=8\sqrt{d}\varepsilon, k_0=2.01965b, E=\frac{\varepsilon}{8\sqrt{d}s^2}, l_0=d(b\sqrt{d}+1)$, result in Lemma 4 yields to

$$\delta_0,\delta_1>\frac{1}{c|y|^{\lambda}}, \tag{3.2}$$

where $\lambda=\frac{\log(8\sqrt{d}\varepsilon)}{\log(\varepsilon/(8\sqrt{d}s^2))}$ and $c=32.3144d\sqrt{d}(\frac{\sqrt{d}(b\sqrt{d}+1)\varepsilon}{4s^2})^{\lambda}$.

By (3.1) and (3.2), we obtain $|y|^{3-\lambda}\leq\frac{cN\varepsilon^2}{\rho^3b^3\sqrt{d}^3}$.

Thus Theorem follows.

Actually, when $\varepsilon>64ds^3$, it directly leads to $\lambda<3$, so we are able to derived an effective upper bound for y.

In this research, a four-parametrized quartic Thue equations is solved by approximation certain crucial algebraic numbers in an elementary way. A computable upper bound for solutions is obtained as well, which is quite effective when ε is much greater than $64ds^3$. In the mean time, the value of $\frac{1}{3-\lambda}$ decreases dramatically when w approximate to 1.

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含四个参数的四次Thue方程

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摘要: 本文研究了含四个参数的四次Thue方程. 利用简单的代数数有理逼近方法给出了该方程解的有效上界, 从而将参数个数由两个推广到四个.

关键词: Thue方程; 含参丢番图方程; 有理逼近

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