

## A NEW METHOD FOR COMPACT EXTENSION OF INFORMATION ALGEBRAS

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**Abstract:** In this paper, we study a new method to obtain compact information algebras from general information algebras. By establishing an equivalence relation on the directed sets of an information algebra, we show that the set of all corresponding equivalence classes with two operations combination and focusing forms a compact extension of the original information algebra.

**Keywords:** information algebras; compact information algebras; way-below relation; compact extension

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The information algebra is a generic framework for approximating inference [1–6], which was first introduced by Shenoy. It was inspired by the formulation of simple axioms that enable local computation of information. In information algebras, the essential algebraic operations are combination, which corresponds to aggregation of information, and marginalization, which corresponds to focusing of information. Initially, in this study, information or knowledge is represented by assigning values to variables. Therefore, a class of information algebra is also known as a valuation-based system.

Compact information algebras, introduced in the study of representation of information, are a kind of information algebras with the background of theoretical computer science. In view of the particularity and importance of compact information algebras, reference [3] has shown a method to obtain compact information algebras by ideal completion of information algebras. In the present paper we offer a new way to construct compact information algebras. In fact, obtaining objects with good properties from general ones is a common phenomenon in many fields of mathematical research, such as [9–12].

### 1 Definitions and Notations

First we recall some definitions and notations discussed in the following content.

If  $(L, \leq)$  is a partially ordered set, for simplicity, it will always be denoted by  $L$  for short. We write  $\vee A$  and  $\wedge A$  for the least upper bound and the greatest lower bound of  $A$  in  $L$  respectively if they exist. If  $a \vee b$  and  $a \wedge b$  exist for all  $a, b \in L$ , then we say  $L$  is a lattice. If  $L$  is a partially ordered set, a set  $A \subseteq L$  is said to be directed, if for all

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$a, b \in A$ , there is a  $c \in A$  such that  $a, b \leq c$ . A subset  $A$  of  $L$  is called a lower set, if  $A = \{x \in L : x \leq a \text{ for some } a \in A\}$ . If  $A$  is a directed lower set, then it is called an ideal.

Let  $a, b$  be two elements of a partially ordered set  $L$ . We call  $a$  way-below  $b$  [13], in symbols  $a \ll b$ , if and only if for all directed subsets  $X \subseteq L$ , if  $\vee X$  exists and  $b \leq \vee X$ , then there exists an  $x \in X$  such that  $a \leq x$ . If  $a \ll a$ , we call  $a$  a compact or finite element.

Here we adopt the axiomatic definition of information algebras presented in [3], which was given by Kohlas. It should be noted that information algebras in this paper are often called domain-free information algebras.

**Definition 1.1** [3] Let  $(\Phi, D)$  be a tuple with two operations defined, where  $D$  is a lattice:

1. Combination:  $\Phi \times \Phi \rightarrow \Phi; (\phi, \psi) \mapsto \phi \otimes \psi$ ,
2. Focusing:  $\Phi \times D \rightarrow \Phi; (\phi, x) \mapsto \phi \Rightarrow^x$ .

The tuple  $(\Phi, D)$  is called an information algebra, if it satisfies the following axioms:

- (1) Semigroup:  $\Phi$  is associative and commutative under combination. There is an element  $e \in \Phi$  such that for all  $\phi \in \Phi$  with  $e \otimes \phi = \phi \otimes e = \phi$ . We call  $e$  a neutral element.
- (2) Transitivity: For  $\phi \in \Phi$  and  $x, y \in D$ ,  $(\phi \Rightarrow^y) \Rightarrow^x = \phi \Rightarrow^{x \wedge y}$ .
- (3) Combination: For  $\phi, \psi \in \Phi$  and  $x \in D$ ,  $(\phi \Rightarrow^x \otimes \psi) \Rightarrow^x = \phi \Rightarrow^x \otimes \psi \Rightarrow^x$ .
- (4) Neutrality: For  $x \in D$ ,  $e \Rightarrow^x = e$ .
- (5) Support: For  $\phi \in \Phi$ , there is an  $x$  such that  $\phi \Rightarrow^x = \phi$ .  $x$  is called a support set of  $\phi$ .
- (6) Idempotency: For  $\phi \in \Phi$  and  $x \in D$ ,  $\phi \otimes \phi \Rightarrow^x = \phi$ .

If  $(\Phi, D)$  is an information algebra and  $\phi, \psi \in \Phi$ , we write  $\psi \leq \phi$  if  $\psi \otimes \phi = \phi$ . The order relation means that the information  $\phi$  is more informative than another information  $\psi$ . This defines a partial order on  $\Phi$ , if  $(\Phi, D)$  is an information algebra [3]. The following lemma gives some basic properties of this ordering relation.

**Lemma 1.1** If  $(\Phi, D)$  is an information algebra, then for  $\phi, \psi \in \Phi$  and  $x, y \in D$ ,

- (1)  $\phi \Rightarrow^x \leq \phi$ ;
- (2)  $\phi \otimes \psi = \sup\{\phi, \psi\}$ ;
- (3)  $\phi \leq \psi$  implies  $\phi \Rightarrow^x \leq \psi \Rightarrow^x$ ;
- (4)  $x \leq y$  implies  $\phi \Rightarrow^x \leq \phi \Rightarrow^y$ ;
- (5)  $\phi_1 \leq \phi_2$  and  $\psi_1 \leq \psi_2$  imply  $\phi_1 \otimes \psi_1 \leq \phi_2 \otimes \psi_2$ .

Computers can treat only “finite” information. Therefore, a new kind of structures named compact information algebras was introduced by Kohlas. The main characteristic of a compact information algebra is that the focusing of each piece of information  $\phi$  on a set  $x$  can be approximated by a family of finite information with the support set  $x$ , which is coarser than  $\phi$ .

**Definition 1.2** [3, 7] A system  $(\Phi, \Phi_f, D)$ , where  $(\Phi, D)$  is an information algebra, the lattice  $D$  has top element,  $\Phi_f$  is closed under combination and contains the neutral element  $e$ , satisfying the three conditions of convergence, density and compactness formulated following, is called a compact information algebra:

1. Convergency: If  $X \subseteq \Phi_f$  is a directed set, then the supremum  $\vee X$  over  $X$  exists and belongs to  $\Phi$ .
2. Density: For  $\phi \in \Phi$  and  $x \in D$ ,  $\phi^{\Rightarrow x} = \vee\{\psi \in \Phi_f : \psi \leq \phi, \psi^{\Rightarrow x} = \psi\}$ .
3. Compactness: If  $X \subseteq \Phi_f$  is a directed set, and  $\phi \in \Phi_f$  such that  $\phi \leq \vee X$ , then there exists a  $\psi \in X$  such that  $\phi \leq \psi$ .

In [3], it was shown that, for a compact information algebra  $(\Phi, \Phi_f, D)$ , an information  $\phi \in \Phi$  belongs to  $\Phi_f$  if and only if  $\phi \ll \phi$ . More important properties on compact information algebras can be found in [3, 4, 7, 8]

In the following we provide two simple examples of compact information algebras.

**Example 1.1** Let  $L = [0, 1]$  and  $D = \{0, 1\}$ .  $\Phi = Id L$  is the set of all nonempty ideals. The operations on  $(\Phi, D)$  are defined as follows:  $\forall I_1, I_2, I \in \Phi$ ,

$$\begin{aligned} \text{combination: } I_1 \otimes I_2 &= \{x \in [0, 1] : x \leq a \text{ for some } a \in I_1 \cup I_2\}; \\ \text{focusing: } I^{\Rightarrow 1} &= I, I^{\Rightarrow 0} = \{0\}. \end{aligned}$$

We can progressively show that  $(\Phi, D)$  is an information algebra with these two operations. Let  $I_x = \{y \in [0, 1] : y \leq x\}$  and  $\Phi_f = \{I_x : x \in [0, 1]\}$ . Then the set  $\Phi_f$ , which is closed under combination, satisfies the axioms of compactness. We can obtain that  $(\Phi, \Phi_f, D)$  is a compact information algebra. Here we can refer to some conclusions shown in [13].

**Example 1.2** For any set  $X$ , let the operations combination and focusing be defined as follows:

$$\begin{aligned} \text{combination: } \forall A, B \subseteq X, A \otimes B &= A \cup B; \\ \text{focusing: } \forall A, B \subseteq X, A^{\Rightarrow B} &= A \cap B. \end{aligned}$$

Then  $(\mathcal{P}(X), \mathcal{P}_f(X), \mathcal{P}(X))$  is a compact information algebra according to Definition 1.1, where  $\mathcal{P}_f(X)$  is the set of all finite subsets of  $X$ .

## 2 Compact Extension of Information Algebras

In this section we give the method named compact extension for obtaining compact information algebras from general information algebras. Here we say a compact information algebra  $(\Psi, D)$  is a compact extension of an information algebra  $(\Phi, D)$ , if there exists an injective  $f : (\Phi, D) \rightarrow (\Psi, D)$  such that  $f(\phi \otimes \psi) = f(\phi) \otimes f(\psi)$ ,  $f(\phi^{\Rightarrow x}) = [f(\phi)]^{\Rightarrow x}$ , where  $\phi, \psi \in \Phi$  and  $x \in D$ . We often call  $f$  a embedding map.

Let  $\mathcal{D}_\Phi$  denote the set of all the directed subsets of an information algebra  $(\Phi, D)$ . We define the operations combination and focusing on  $\mathcal{D}_\Phi$  as follows: For all  $A, B \in \mathcal{D}_\Phi$ ,

$$\begin{aligned} \text{combination: } A \otimes B &= \{\phi \otimes \psi : \phi \in A, \psi \in B\}; \\ \text{focusing: } A^{\Rightarrow x} &= \{\phi^{\Rightarrow x} : \phi \in A\}. \end{aligned}$$

**Lemma 2.1** If  $A, B \in \mathcal{D}_\Phi$ , then  $A \otimes B, A^{\Rightarrow x} \in \mathcal{D}_\Phi$ .

**Proof** Assume that  $\phi_1 \otimes \psi_1, \phi_2 \otimes \psi_2 \in A \otimes B$ , where  $\phi_i \in A, \psi_i \in B (i = 1, 2)$ . There exist  $\phi \in A, \psi \in B$  such that  $\phi_i \leq \phi, \psi_i \leq \psi$ , since  $A, B \in \mathcal{D}_\Phi$ . Then  $\phi_i \otimes \psi_i \leq \phi \otimes \psi \in A \otimes B$  by Lemma 1.1(5). Hence  $A \otimes B \in \mathcal{D}_\Phi$ .

By the directness of  $A$  and Lemma 1.1(3), the directness of  $A^{\Rightarrow x}$  also can be obtained easily.

An equivalence relation  $\theta$  on  $\mathcal{D}_\Phi$  is defined as follows: For  $A, B \in \mathcal{D}_\Phi$ ,

$$\begin{aligned} A \equiv B \pmod{\theta} \text{ iff for any } \phi \in A, \text{ there exists a } \psi \in B \text{ such that } \phi \leq \psi, \\ \text{and meanwhile, for any } \beta \in B, \text{ there exists a } \alpha \in A \text{ such that } \beta \leq \alpha. \end{aligned}$$

The characteristic of  $\theta$  demonstrates that, in a sense, information content of  $A$  and  $B$  are equivalent. Next we define the operations combination and focusing on the quotient object  $\mathcal{D}_\Phi/\theta$ : For  $A, B \in \mathcal{D}_\Phi$ ,

$$\begin{aligned} \text{combination: } [A]_\theta \otimes [B]_\theta &= [A \otimes B]_\theta; \\ \text{focusing: } [A]_\theta^{\Rightarrow x} &= [A^{\Rightarrow x}]_\theta. \end{aligned}$$

The two operations defined as above are well-defined on  $\mathcal{D}_\Phi/\theta$ .

**Lemma 2.2** The system  $(\mathcal{D}_\Phi/\theta, D)$  is an information algebra, where  $(\Phi, D)$  is an information algebra and  $D$  has a top element  $\top$ .

**Proof** (1) Semigroup: It is easy to see that  $\mathcal{D}_\Phi/\theta$  is associative and commutative under combination, since  $(\Phi, D)$  is an information algebra.  $[\{e\}]_\theta$  is the neutral element such that  $[\{e\}]_\theta \otimes [A]_\theta = [A]_\theta$ , where  $e \in \Phi$  is the neutral element of  $(\Phi, D)$ . Meanwhile, the axiom of neutrality also can be obtained easily.

(2) Combination: For  $A, B \in \mathcal{D}_\Phi$  and  $x \in D$ , we have

$$\begin{aligned} (A^{\Rightarrow x} \otimes B)^{\Rightarrow x} &= \{(\phi^{\Rightarrow x} \otimes \psi)^{\Rightarrow x} : \phi \in A, \psi \in B\} \\ &= \{\phi^{\Rightarrow x} \otimes \psi^{\Rightarrow x} : \phi \in A, \psi \in B\} \\ &= \{\phi^{\Rightarrow x} : \phi \in A\} \otimes \{\psi^{\Rightarrow x} : \psi \in B\} \\ &= A^{\Rightarrow x} \otimes B^{\Rightarrow x}. \end{aligned}$$

Then  $([A]_\theta^{\Rightarrow x} \otimes [B]_\theta)^{\Rightarrow x} = [(A^{\Rightarrow x} \otimes B)^{\Rightarrow x}]_\theta = [A^{\Rightarrow x} \otimes B^{\Rightarrow x}]_\theta = [A]_\theta^{\Rightarrow x} \otimes [B]_\theta^{\Rightarrow x}$ .

(3) Transitivity: For  $A \in \mathcal{D}_\Phi$  and  $x, y \in D$ ,

$$\begin{aligned} (A^{\Rightarrow x})^{\Rightarrow y} &= \{\phi^{\Rightarrow x} : \phi \in A\}^{\Rightarrow y} \\ &= \{(\phi^{\Rightarrow x})^{\Rightarrow y} : \phi \in A\} \\ &= \{\phi^{\Rightarrow x \wedge y} : \phi \in A\} \\ &= A^{\Rightarrow x \wedge y}. \end{aligned}$$

Then  $([A]_\theta^{\Rightarrow x})^{\Rightarrow y} = [(A^{\Rightarrow x})^{\Rightarrow y}]_\theta = [A^{\Rightarrow x \wedge y}]_\theta = [A]_\theta^{\Rightarrow x \wedge y}$ .

(4) Support: For  $A \in \mathcal{D}_\Phi$ ,  $A^{\Rightarrow \top} = A$ . Then  $[A]_\theta^{\Rightarrow \top} = [A]_\theta$ .

(5) Idempotency: For  $A \in \mathcal{D}_\Phi$  and  $x \in D$ , we need to verify that  $A \otimes A^{\Rightarrow x} \equiv A \pmod{\theta}$ .

In fact, for  $\phi, \psi \in A$ , since  $A$  is directed, then there exists a  $\varphi \in A$  such that  $\phi, \psi \leq \varphi$ . So  $\phi \otimes \psi^{\Rightarrow x} \leq \phi \otimes \psi \leq \varphi$ . Meanwhile, for  $\phi \in A$ , we have  $\phi = \phi \otimes \phi^{\Rightarrow x} \in A \otimes A^{\Rightarrow x}$ . Then  $A \otimes A^{\Rightarrow x} \equiv A \pmod{\theta}$  by the definition of  $\theta$ . Thus

$$[A]_\theta \otimes [A]_\theta^{\Rightarrow x} = [A \otimes A^{\Rightarrow x}]_\theta = [A]_\theta.$$

**Lemma 2.3** Let  $A, B \in \mathcal{D}_\Phi$ . Then,  $[A]_\theta \leq [B]_\theta$  if and only if for any  $\phi \in A$ , there exists a  $\psi \in B$  such that  $\phi \leq \psi$ .

**Proof** If  $[A]_\theta \leq [B]_\theta$ , then  $A \otimes B \equiv B \pmod{\theta}$ . For any  $\phi \in A$  and  $\eta \in B$ , by the definition of  $\theta$ , there exists a  $\psi \in B$  such that  $\phi \otimes \eta \leq \psi$ . Then  $\phi \leq \psi$ .

Conversely, we need to show  $A \otimes B \equiv B \pmod{\theta}$ . On one hand, let  $\phi \in A$  and  $\psi \in B$ . By the assumption here, there exists a  $\eta \in B$  such that  $\phi \leq \eta$ . Then  $\phi \otimes \psi \leq \eta \otimes \psi$ . Since  $B$  is directed, there exists a  $\gamma \in B$  such that  $\eta, \psi \leq \gamma$ . Thus  $\phi \otimes \psi \leq \gamma$ . On the other hand, for any  $\psi \in B$  and  $\phi \in A$ , we have  $\psi \leq \phi \otimes \psi \in A \otimes B$ . Thus  $A \otimes B \equiv B \pmod{\theta}$ . It implies that  $[A]_\theta \leq [B]_\theta$ .

Let  $\Gamma = \{[\{\phi\}]_\theta : \phi \in \Phi\}$ . To facilitate the writing,  $[\{\phi\}]_\theta$  is simply written as  $[\phi]_\theta$ .

**Theorem 2.1** The system  $(\mathcal{D}_\Phi/\theta, \Gamma, D)$  is a compact information algebra, where  $(\Phi, D)$  is an information algebra and  $D$  has a top element  $\top$ .

**Proof** Clearly  $[e]_\theta \in \Gamma$  is the empty information and  $\Gamma$  is closed under combination. Following we show the axioms of convergence, density and compactness of  $(\mathcal{D}_\Phi/\theta, \Gamma, D)$ . Then the conclusion is proved.

(1) Convergence: Assume that  $\{[\phi_i]_\theta : i \in I\} \subseteq \Gamma$  is a directed subset, we claim that  $\bigvee_{i \in I} [\phi_i]_\theta = [E]_\theta$ , where  $E = \{\phi_{i_1} \otimes \phi_{i_2} \otimes \cdots \otimes \phi_{i_n} : i_1, i_2, \dots, i_n \in I, n \in \mathbf{N}\}$ . Obviously,  $E$  is a directed subset of  $\Phi$  and  $[\phi_i]_\theta \leq [E]_\theta$  for all  $i \in I$ . Next suppose that  $[A]_\theta$  is another upper bound of  $\{[\phi_i]_\theta : i \in I\}$ . For each  $i \in I$ , there exists a  $\psi_i \in A$  such that  $\phi_i \leq \psi_i$ . Then

$$\phi_{i_1} \otimes \phi_{i_2} \otimes \cdots \otimes \phi_{i_n} \leq \psi_{i_1} \otimes \psi_{i_2} \otimes \cdots \otimes \psi_{i_n}.$$

Since  $A$  is directed, there is a  $\psi \in A$  such that  $\psi_{i_j} \leq \psi (j = 1, 2, \dots, n)$ . Thus  $\phi_{i_1} \otimes \phi_{i_2} \otimes \cdots \otimes \phi_{i_n} \leq \psi$ . So  $[E]_\theta \leq [A]_\theta$ . We obtain that  $\bigvee_{i \in I} [\phi_i]_\theta = [E]_\theta$ .

(2) Density: For  $A \in \mathcal{D}_\Phi$  and  $x \in D$ , we show that

$$[A]_\theta^{\Rightarrow x} = \bigvee \{[\phi]_\theta : [\phi]_\theta \leq [A]_\theta, [\phi]_\theta = [\phi]_\theta^{\Rightarrow x}\}.$$

Let  $\Theta = \{[\phi]_\theta : [\phi]_\theta \leq [A]_\theta, [\phi]_\theta = [\phi]_\theta^{\Rightarrow x}\}$ . For any  $[\phi]_\theta \in \Theta$ , there exists a  $\psi \in A$  such that  $\phi \leq \psi$ . Meanwhile, by Lemma 2.3 we have  $\phi = \phi^{\Rightarrow x}$  for  $[\phi]_\theta \in \Theta$ . Thus  $\phi = \phi^{\Rightarrow x} \leq \psi^{\Rightarrow x}$ . By Lemma 2.3 again,  $[\phi]_\theta \leq [A]_\theta^{\Rightarrow x}$ . This implies that  $[A]_\theta^{\Rightarrow x}$  is an upper bound of  $\Theta$ .

Assume that  $[B]_\theta$  is another upper bound of  $\Theta$ . For  $\psi \in A$ , we have  $[\psi^{\Rightarrow x}]_\theta \in \Theta$ , because  $\psi^{\Rightarrow x} = (\psi^{\Rightarrow x})^{\Rightarrow x}$  and  $\psi^{\Rightarrow x} \leq \psi$ . Then  $[\psi^{\Rightarrow x}]_\theta \leq [B]_\theta$ . By Lemma 2.3 there is a  $\eta \in B$  such that  $\psi^{\Rightarrow x} \leq \eta$ . Thus we obtain that  $[A]_\theta^{\Rightarrow x} \leq [B]_\theta$ . So  $[A]_\theta^{\Rightarrow x} = \bigvee \Theta$ .

(3) Compactness: Suppose that  $\{[\phi_i]_\theta : i \in I\} \subseteq \Gamma$  is a directed subset. Let  $[\phi]_\theta \in \Gamma$  and  $[\phi]_\theta \leq \bigvee \{[\phi_i]_\theta : i \in I\}$ . By what we have proven in the convergency, there exists a set  $\{i_1, i_2, \dots, i_n\} \subseteq I$  such that  $\phi \leq \phi_{i_1} \otimes \phi_{i_2} \otimes \cdots \otimes \phi_{i_n}$ . Since  $\{\phi_i : i \in I\}$  is directed, there exists a  $j \in I$  such that  $\phi \leq \phi_j$ . So  $[\phi]_\theta \leq [\phi_j]_\theta$ . The compactness is true.

**Theorem 2.2** The system  $(\mathcal{D}_\Phi/\theta, D)$  is a compact extension of  $(\Phi, D)$ , where the embedding map  $f : (\Phi, D) \rightarrow (\mathcal{D}_\Phi/\theta, D)$  is defined as  $f(\phi) = [\phi]_\theta$ .

**Proof** Let  $\phi, \psi \in \Phi$ . If  $f(\phi) = f(\psi)$ , then

$$[\phi]_{\theta} = [\psi]_{\theta}.$$

By Lemma 2.3, we have  $\phi \leq \psi$  and  $\psi \leq \phi$ . Thus  $\phi = \psi$ , that is,  $f$  is injective. Also, we can show that  $f$  preserves these two operations combination and focusing, and preserves the neutral element  $e \in \Phi$  directly.

At last, let us see an example of compact extension of an information algebra.

**Example 2.1** Let  $\Phi = [0, 1]$ ,  $D = \{0, 1\}$ . The operations on  $(\Phi, D)$  are defined as follows:

$$\text{Combination: } \forall \phi, \psi \in \Phi, \phi \otimes \psi = \max\{\phi, \psi\};$$

$$\text{Focusing: } \forall \phi \in \Phi, \phi^{\Rightarrow 1} = \phi;$$

$$\phi^{\Rightarrow 0} = \begin{cases} \phi, & \text{if } \phi \in [0, \frac{1}{2}]; \\ \frac{1}{2}, & \text{if } \phi \in [\frac{1}{2}, 1]. \end{cases}$$

We have  $\phi \ll \psi$  if and only if  $\phi < \psi$  or  $\phi = \psi = 0$ . Then the tuple  $(\Phi, D)$  is an information algebra but not compact [7].

Let  $\theta$  be the equivalence relation defined as above. For any directed subset  $\Upsilon$  of  $[0, 1]$  with a maximum element  $\phi \in \Upsilon$ , we have  $\Upsilon \equiv \{\phi\}(\text{mod } \theta)$ . While, for any two directed subsets  $\Upsilon_1$  and  $\Upsilon_2$  with a same supremum  $d$  and  $d \notin \Upsilon_1, \Upsilon_2$ , we have  $\Upsilon_1 \equiv \Upsilon_2(\text{mod } \theta)$ . Then the quotient object

$$\mathcal{D}_{\Phi}/\theta = \{[\phi]_{\theta} : \phi \in [0, 1]\} \cup \{[\{\phi - \frac{1}{n} : n > [\frac{1}{\phi}], n \in \mathbf{Z}\}]_{\theta} : \phi \in [0, 1]\},$$

where  $[x]$  denotes the largest integer not greater than  $x$ . The set  $\mathcal{D}_{\Phi}/\theta$  has more finite elements than  $\Phi$ . In fact, according to the proof in Theorem 2.1, we have  $[\phi]_{\theta} \ll [\phi]_{\theta}$  for all  $\phi \in [0, 1]$ . Hence  $(\mathcal{D}_{\Phi}/\theta, \Gamma, \{0, 1\})$  is compact clearly, where  $\Gamma = \{[\phi]_{\theta} : \phi \in [0, 1]\}$ .

## References

- [1] Shenoy P P. A valuation-based language for expert systems[J]. International Journal of Approximate Reasoning, 1989, 3(2): 383–411.
- [2] Kohlas J, Shenoy P P. Computation in valuation algebras[A]. Kohlas J, Moral S. Handbook of Defeasible Reasoning and Uncertainty Management Systems, Vol. 5, Algorithms for Uncertainty and Defeasible Reasoning[C]. Dordrecht: Kluwer Academic Publishers, 2000.
- [3] Kohlas J. Information algebras: generic structures for inference[M]. London: Springer-Verlag, 2003.
- [4] Kohlas J. Lecture notes on the algebraic theory of information[EB/OL]. <http://diuf.unifr.ch/drupal/tns/sites/diuf.unifr.ch.drupal.tns/files/file/kohlas/main.pdf>, 2012-03-25.
- [5] Haenni R. Ordered valuation algebras: a generic framework for approximating inference[J]. International Journal of Approximate Reasoning, 2004, 37(1): 1–41.
- [6] Pouly M. A generic framework for local computation[D]. Fribourg: University of Fribourg, 2008.
- [7] Guan Xuechong, Li Yongming. On two types of continuous information algebras[J]. Int. J. Uncertainty, Fuzziness and Knowledge-Based Systems, 2012, 20(5): 655–671.

- [8] Guan Xuechong. A study on some problems of valuation algebras[D]. Xi'an: Shaanxi Normal University, 2011.
- [9] Johnstone P T. Wallman compactification of locales[J]. Houston Journal of Mathematics, 1984, 10(2): 201–206.
- [10] Li Yongming, Li Zhihui. Free semilattices and strongly free semilattices generated by partially ordered sets[J]. Northeastern Mathematics Journal, 1993, 9(3): 359–366.
- [11] Guo Zhilian, Zhao Bin. Structures and properties of free Dcpo and free join-complete lattices[J]. Journal of Shaanxi Normal University(Natural Science Edition), 2005, 33(4): 11–14.
- [12] Meng Peiyuan, Zhou Wuneng. A  $T_1$ -Compactification of  $L$ -fuzzy topological spaces[J]. J. of Math. (PRC), 1993, 13(4): 66–71.
- [13] Gierz G, et al.. Continuous lattices and domains: encyclopedia of mathematics and its applications[M]. Cambridge: Cambridge University Press, 2003.

## 信息代数紧扩张的一种新方法

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**摘要:** 本文研究了从一般信息代数获取紧信息代数的一种新方法. 通过在一个信息代数的定向集族上建立一个等价关系, 证得相应的等价类全体之集关于联合和聚焦两种运算构成了原信息代数的一个紧扩张.

**关键词:** 信息代数; 紧信息代数; 双小于关系; 紧扩张.

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