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A NOTE ON QUASI-d-KOSZUL MODULES

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Abstract: In this paper, we give a sufficient condition for the minimal horseshoe lemma to be true in [4]. By using quasi-d-Koszul modules, we obtain a necessary and sufficient condition and provide some applications of minimal horseshoe lemma.

Keywords: quasi-d-Koszul modules; minimal horseshoe lemma

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1 Introduction and Main Results

In [4], Lü and Zhao proved the following theorem, which is one of the main results of that paper.

Theorem 1.1 Let $0 \longrightarrow K \longrightarrow M \longrightarrow N \longrightarrow 0$ be an exact sequence in the category of quasi-d-Koszul modules such that $JK = K \cap JM$. Then the minimal Horseshoe lemma holds.

One of the aims of this note is to prove that the above result is a necessary and sufficient condition. More precisely, we obtain

Theorem 1.2 Let $\xi : 0 \longrightarrow K \longrightarrow M \longrightarrow N \longrightarrow 0$ be a short exact sequence in the category of quasi-d-Koszul modules. Then $JK = K \cap JM$ if and only if the "minimal Horseshoe lemma" holds with respect to ξ.

Moreover, we provide some applications of minimal Horseshoe lemma。

Theorem 1.3 Let $\xi : 0 \longrightarrow K \longrightarrow M \longrightarrow N \longrightarrow 0$ be a short exact sequence of finitely generated R-modules, where R denotes a Noetherian semiperfect augmented algebra and J denotes the Jacobson radical of R. Denote $\mathcal{Q}^d(R)$ the category of quasi-d-Koszul modules. Then we have the following statements:

(1) if minimal Horseshoe lemma is true with respect to ξ, then $K \in \mathcal{Q}^d(R)$ provided that $M,\ N \in \mathcal{Q}^d(R)$;

(2) if minimal Horseshoe lemma is true with respect to ξ and we have $J^{d-1}\Omega^i(K) = \Omega^i(K) \cap J^{d-1}\Omega^i(M)$ and $J^d\Omega^i(K) = \Omega^i(K) \cap J^d\Omega^i(M)$ for all positive odd integers i, and $J^2\Omega^j(K) = \Omega^j(K) \cap J^2\Omega^j(M)$ for all nonnegative even integers j, then $N \in \mathcal{Q}^d(R)$ provided that $K,\ M \in \mathcal{Q}^d(R)$.

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We end this section with the following definition:

Definition 1.4 Let R be a Noetherian semiperfect augmented algebra and J be the Jacobson radical of R. A finitely generated R-module M is called a quasi-d-Koszul module provided that M has a minimal graded projective resolution

$$\cdots \longrightarrow P_n \xrightarrow{d_n} P_{n-1} \xrightarrow{d_{n-1}} \cdots \longrightarrow P_1 \xrightarrow{d_1} P_0 \xrightarrow{d_0} M \longrightarrow 0$$

such that $J\ker d_i = \ker d_i \cap J^2P_i$ for i being even, and $J\ker d_i = \ker d_i \cap J^dP_i$ for i being odd, where $d \geq 2$ is a fixed integer.

2 Proof of Theorem 1.2

Lemma 2.1 Let $\xi : 0 \longrightarrow K \longrightarrow M \longrightarrow N \longrightarrow 0$ be a short exact sequence of finitely generated R-modules, where R denotes a Noetherian semiperfect augmented algebra and J denotes its Jacobson radical. Then $J\Omega^i(K) = \Omega^i(K) \cap J\Omega^i(M)$ for all $i \geq 0$ if and only if for any given commutative diagram

$$\begin{array}{ccccccccc}
 & & \mathcal{P}_* & & & & \mathcal{Q}_* & & \\
 & & \downarrow & & & & \downarrow & & \\
0 & \longrightarrow & K & \longrightarrow & M & \longrightarrow & N & \longrightarrow & 0 \\
 & & \downarrow & & & & \downarrow & & \\
 & & 0 & & & & 0 & &
\end{array}$$

with $\mathcal{P}_*$ and $\mathcal{Q}_*$ being minimal projective resolutions of K and N, respectively. Then we can complement the above diagram into the following commutative diagram with exact rows and columns

$$\begin{array}{ccccccccc}
0 & \longrightarrow & \mathcal{P}_* & \longrightarrow & \mathcal{L}_* & \longrightarrow & \mathcal{Q}_* & \longrightarrow & 0 \\
 & & \downarrow & & \downarrow & & \downarrow & & \\
0 & \longrightarrow & K & \longrightarrow & M & \longrightarrow & N & \longrightarrow & 0 \\
 & & \downarrow & & \downarrow & & \downarrow & & \\
 & & 0 & & 0 & & 0 & &
\end{array}$$

such that $\mathcal{L}_* \longrightarrow M \longrightarrow 0$ is also a minimal projective resolution and for all $n \geq 0$, $L_n \cong P_n \oplus Q_n$. That is, the minimal Horseshoe lemma holds.

Proof First we claim that, for ξ, $JK = K \cap JM$ if and only if we have the following commutative diagram with exact rows and columns

$$\begin{array}{ccccccccc}
 & & 0 & & 0 & & 0 & & \\
 & & \downarrow & & \downarrow & & \downarrow & & \\
0 & \longrightarrow & \Omega^1(K) & \longrightarrow & \Omega^1(M) & \longrightarrow & \Omega^1(N) & \longrightarrow & 0 \\
 & & \downarrow & & \downarrow & & \downarrow & & \\
0 & \longrightarrow & P_0 & \longrightarrow & L_0 & \longrightarrow & Q_0 & \longrightarrow & 0 \\
 & & \downarrow & & \downarrow & & \downarrow & & \\
0 & \longrightarrow & K & \longrightarrow & M & \longrightarrow & N & \longrightarrow & 0 \\
 & & \downarrow & & \downarrow & & \downarrow & & \\
 & & 0 & & 0 & & 0 & &
\end{array}$$

such that $P_0 \to K \to 0$, $L_0 \to M \to 0$ and $Q_0 \to N \to 0$ are projective covers. In fact, we obtain the exact sequence

$$0 \longrightarrow K/JK \longrightarrow M/JM \longrightarrow N/JN \longrightarrow 0$$

since $JK = K \cap JM$. Note that for any finitely generated R-module M, $R \otimes_{R/J} M/JM \longrightarrow M \longrightarrow 0$ is a projective cover. Now put $P_0 := R \otimes_{R/J} K/JK$, $L_0 := R \otimes_{R/J} M/JM$ and $Q_0 := R \otimes_{R/J} N/JN$. We have the following exact sequence

$$0 \longrightarrow P_0 \longrightarrow L_0 \longrightarrow Q_0 \longrightarrow 0$$

since R/J is semisimple. Now by Snake lemma, we get the exact sequence

$$0 \longrightarrow \Omega^1(K) \longrightarrow \Omega^1(M) \longrightarrow \Omega^1(N) \longrightarrow 0,$$

which implies the desired diagram. Conversely, suppose that we have the above diagram. Note that the projective cover of a module is unique up to isomorphisms. We may assume that $P_0 := R \otimes_{R/J} K/JK$, $L_0 := R \otimes_{R/J} M/JM$ and $Q_0 := R \otimes_{R/J} N/JN$. From the middle row of the diagram, we have the following exact sequence

$$0 \longrightarrow R \otimes_{R/J} K/JK \longrightarrow R \otimes_{R/J} M/JM \longrightarrow R \otimes_{R/J} N/JN \longrightarrow 0.$$

Note that R/J is semisimple, we have $JK = K \cap JM$ since we have the short exact sequence as R/J-modules

$$0 \longrightarrow K/JK \longrightarrow M/JM \longrightarrow N/JN \longrightarrow 0.$$

Now we prove the claim.

$\Rightarrow$ By the claim, $J\Omega^i(K) = \Omega^i(K) \cap J\Omega^i(M)$ for all $i \geq 0$ if and only if, for all $i \geq 0$, we have the following commutative diagram with exact rows and columns

$$\begin{array}{ccccccccc}
 & & 0 & & 0 & & 0 & & \\
 & & \downarrow & & \downarrow & & \downarrow & & \\
0 & \longrightarrow & \Omega^{i+1}(K) & \longrightarrow & \Omega^{i+1}(M) & \longrightarrow & \Omega^{i+1}(N) & \longrightarrow & 0 \\
 & & \downarrow & & \downarrow & & \downarrow & & \\
0 & \longrightarrow & P_i & \longrightarrow & L_i & \longrightarrow & Q_i & \longrightarrow & 0 \\
 & & \downarrow & & \downarrow & & \downarrow & & \\
0 & \longrightarrow & \Omega^{i}(K) & \longrightarrow & \Omega^{i}(M) & \longrightarrow & \Omega^{i}(N) & \longrightarrow & 0 \\
 & & \downarrow & & \downarrow & & \downarrow & & \\
 & & 0 & & 0 & & 0 & &
\end{array}$$

such that P_i, L_i and Q_i are projective covers of $\Omega^i(K)$, $\Omega^i(M)$ and $\Omega^i(N)$, respectively. Now putting these commutative diagrams together, we finish the proof of necessity.

$\Leftarrow$ By the claim, it is easy to see that minimal Horseshoe lemma is true if and only if we have $J\Omega^i(K) = \Omega^i(K) \cap J\Omega^i(M)$ for all $i \geq 0$.

Proof of Theorem 1.2 By Theorem 3.1 of [4], it is enough to prove the sufficiency. In fact, by Lemma 2.1, the minimal Horseshoe lemma is true if and only if $J\Omega^i(K) = \Omega^i(K) \cap J\Omega^i(M)$ for all $i \geq 0$. In particular, we have $JK = K \cap JM$ for the case of $i = 0$.

In fact, Theorem 2.8 (see [6]) is also a necessary and sufficient condition, which is immediate from Theorem 1.2. That is, we have

Corollary 2.2 Let $0 \longrightarrow K \longrightarrow M \longrightarrow N \longrightarrow 0$ be an exact sequence of nice modules. Then $JK = K \cap JM$ if and only if the minimal Horseshoe lemma holds with respect to such an exact sequence.

3 Applications

As an application of minimal Horseshoe lemma, we give some sufficient conditions such that the category of quasi-d-Koszul modules $\mathcal{Q}^d(R)$ preserves kernels of epimorphisms and cokernels of monomorphisms.

Lemma 3.1 Let $\xi : 0 \longrightarrow K \longrightarrow M \longrightarrow N \longrightarrow 0$ be a short exact sequence of finitely generated R-modules, where R denotes a Noetherian semiperfect augmented algebra and J denotes the Jacobson radical of R. If minimal Horseshoe lemma is true with respect to ξ, then $K \in \mathcal{Q}^d(R)$ provided that $M,\ N \in \mathcal{Q}^d(R)$.

Proof By Lemma 2.1, we have the commutative diagram

$$\begin{array}{ccccccccc}
 & & 0 & & 0 & & 0 & & \\
 & & \downarrow & & \downarrow & & \downarrow & & \\
0 & \longrightarrow & \Omega^{i+1}(K) & \longrightarrow & \Omega^{i+1}(M) & \longrightarrow & \Omega^{i+1}(N) & \longrightarrow & 0 \\
 & & \downarrow & & \downarrow & & \downarrow & & \\
0 & \longrightarrow & P_i & \longrightarrow & L_i & \longrightarrow & Q_i & \longrightarrow & 0 \\
 & & \downarrow & & \downarrow & & \downarrow & & \\
0 & \longrightarrow & \Omega^{i}(K) & \longrightarrow & \Omega^{i}(M) & \longrightarrow & \Omega^{i}(N) & \longrightarrow & 0 \\
 & & \downarrow & & \downarrow & & \downarrow & & \\
 & & 0 & & 0 & & 0 & &
\end{array}$$

such that $P_i \to \Omega^i(K) \to 0$, $Q_i \to \Omega^i(M) \to 0$ and $L_i \to \Omega^i(N) \to 0$ are projective covers, respectively. Note that M and N are in $\mathcal{Q}^d(R)$, we have the following commutative diagram with exact rows and columns

$$\begin{array}{ccccccccc}
 & & 0 & & 0 & & 0 & & \\
 & & \downarrow & & \downarrow & & \downarrow & & \\
0 & \longrightarrow & \Omega^{i+1}(K) & \longrightarrow & \Omega^{i+1}(M) & \longrightarrow & \Omega^{i+1}(N) & \longrightarrow & 0 \\
 & & \downarrow & & \downarrow & & \downarrow & & \\
0 & \longrightarrow & J^{d-1}P_i & \longrightarrow & J^{d-1}Q_i & \longrightarrow & J^{d-1}L_i & \longrightarrow & 0
\end{array}$$

for all positive odd integers i and the following commutative diagram with exact rows and columns

$$\begin{array}{ccccccccc}
 & & 0 & & 0 & & 0 & & \\
 & & \downarrow & & \downarrow & & \downarrow & & \\
0 & \longrightarrow & \Omega^{i+1}(K) & \longrightarrow & \Omega^{i+1}(M) & \longrightarrow & \Omega^{i+1}(N) & \longrightarrow & 0 \\
 & & \downarrow & & \downarrow & & \downarrow & & \\
0 & \longrightarrow & JP_i & \longrightarrow & JQ_i & \longrightarrow & JL_i & \longrightarrow & 0
\end{array}$$

for all nonnegative even integers i.

Apply the functor $R/J \otimes_R -$ to the above two diagrams, we have the following commutative diagram with exact rows and columns

$$\begin{array}{ccccccccc} & & & & 0 & & & & \\ & & & & \downarrow & & & & \\ 0 & \longrightarrow & R/J\otimes_R \Omega^{i+1}(K) & \longrightarrow & R/J\otimes_R \Omega^{i+1}(M) & \longrightarrow & R/J\otimes_R \Omega^{i+1}(N) & \longrightarrow & 0 \\ & & \downarrow{\scriptstyle\theta} & & \downarrow & & \downarrow & & \\ 0 & \longrightarrow & R/J\otimes_R J^{d-1}P_i & \longrightarrow & R/J\otimes_R J^{d-1}Q_i & \longrightarrow & R/J\otimes_R J^{d-1}L_i & \longrightarrow & 0 \end{array}$$

for all positive odd integers i and the following commutative diagram with exact rows and columns

$$\begin{array}{ccccccccc} & & & & 0 & & & & \\ & & & & \downarrow & & & & \\ 0 & \longrightarrow & R/J\otimes_R \Omega^{i+1}(K) & \longrightarrow & R/J\otimes_R \Omega^{i+1}(M) & \longrightarrow & R/J\otimes_R \Omega^{i+1}(N) & \longrightarrow & 0 \\ & & \downarrow{\scriptstyle\theta} & & \downarrow & & \downarrow & & \\ 0 & \longrightarrow & R/J\otimes_R JP_i & \longrightarrow & R/J\otimes_R JQ_i & \longrightarrow & R/J\otimes_R JL_i & \longrightarrow & 0 \end{array}$$

for all nonnegative even integers i. Therefore, θ is a monomorphism. Thus we have $J\Omega^{i+1}(K) = \Omega^{i+1}(K)\cap J^dP_i$ for all positive odd integers i and $J\Omega^{i+1}(K) = \Omega^{i+1}(K)\cap J^2P_i$ for all nonnegative even integers i, which implies that $K \in \mathcal{Q}^d(R)$, as desired.

Lemma 3.2 Let $\xi : 0 \longrightarrow K \longrightarrow M \longrightarrow N \longrightarrow 0$ be a short exact sequence of finitely generated R-modules, where R denotes a Noetherian semiperfect augmented algebra and J denotes the Jacobson radical of R. If minimal Horseshoe Lemma is true with respect to ξ and we have $J^{d-1}\Omega^i(K) = \Omega^i(K)\cap J^{d-1}\Omega^i(M)$ and $J^d\Omega^i(K) = \Omega^i(K)\cap J^d\Omega^i(M)$ for all positive odd integers i, and $J^2\Omega^j(K) = \Omega^j(K)\cap J^2\Omega^j(M)$ for all nonnegative even integers j, then $N \in \mathcal{Q}^d(R)$ provided that $K,\ M \in \mathcal{Q}^d(R)$.

Proof Note that minimal Horseshoe lemma is true for ξ, which is equivalent to $J\Omega^i(K) = \Omega^i(K)\cap J\Omega^i(M)$ for all $i \geq 0$ by the claim in the proof of Lemma 2.1. By assumption, $J\Omega^i(K) = \Omega^i(K)\cap J\Omega^i(M)$ for all positive odd integers i, thus we have the exact sequences

$$0 \longrightarrow J^{d-1}\Omega^i(K) \longrightarrow J^{d-1}\Omega^i(M) \longrightarrow J^{d-1}\Omega^i(N) \longrightarrow 0$$

for all positive odd integers i and

$$0 \longrightarrow J\Omega^i(K) \longrightarrow J\Omega^i(M) \longrightarrow J\Omega^i(N) \longrightarrow 0$$

for all $i \geq 0$.

Similar to the proof of (1), we have the following commutative diagrams with exact rows and columns

$$\begin{array}{ccccccccc}
 & & 0 & & 0 & & 0 & & \\
 & & \downarrow & & \downarrow & & \downarrow & & \\
0 & \longrightarrow & \Omega^{i+1}(K) & \longrightarrow & \Omega^{i+1}(M) & \longrightarrow & \Omega^{i+1}(N) & \longrightarrow & 0 \\
 & & \downarrow & & \downarrow & & \downarrow & & \\
0 & \longrightarrow & J^{d-1}P_i & \longrightarrow & J^{d-1}Q_i & \longrightarrow & J^{d-1}L_i & \longrightarrow & 0 \\
 & & \downarrow & & \downarrow & & \downarrow & & \\
0 & \longrightarrow & J^{d-1}\Omega^i(K) & \longrightarrow & J^{d-1}\Omega^i(M) & \longrightarrow & J^{d-1}\Omega^i(N) & \longrightarrow & 0 \\
 & & \downarrow & & \downarrow & & \downarrow & & \\
 & & 0 & & 0 & & 0 & &
\end{array}$$

for all positive odd integers i and

$$\begin{array}{ccccccccc}
 & & 0 & & 0 & & 0 & & \\
 & & \downarrow & & \downarrow & & \downarrow & & \\
0 & \longrightarrow & \Omega^{i+1}(K) & \longrightarrow & \Omega^{i+1}(M) & \longrightarrow & \Omega^{i+1}(N) & \longrightarrow & 0 \\
 & & \downarrow & & \downarrow & & \downarrow & & \\
0 & \longrightarrow & JP_i & \longrightarrow & JQ_i & \longrightarrow & JL_i & \longrightarrow & 0 \\
 & & \downarrow & & \downarrow & & \downarrow & & \\
0 & \longrightarrow & J\Omega^i(K) & \longrightarrow & J\Omega^i(M) & \longrightarrow & J\Omega^i(N) & \longrightarrow & 0 \\
 & & \downarrow & & \downarrow & & \downarrow & & \\
 & & 0 & & 0 & & 0 & &
\end{array}$$

for all nonnegative even integers i.

Note that we have

$$\begin{aligned}
J^{d-1}\Omega^i(K) \cap J(J^{d-1}\Omega^i(M)) &= J^{d-1}\Omega^i(K) \cap J^d\Omega^i(M) \\
&= J^{d-1}\Omega^i(K) \cap J^d\Omega^i(M) \cap \Omega^i(K) \\
&= J^{d-1}\Omega^i(K) \cap J^d\Omega^i(K) \\
&= J^{d-1}\Omega^i(K)
\end{aligned}$$

for all positive odd integers i and

$$\begin{aligned}
J\Omega^i(K) \cap J(J\Omega^i(M)) &= J\Omega^i(K) \cap J^2\Omega^i(M) \\
&= J\Omega^i(K) \cap J^2\Omega^i(M) \cap \Omega^i(K) \\
&= J\Omega^i(K) \cap J^2\Omega^i(K) \\
&= J\Omega^i(K)
\end{aligned}$$

for all nonnegative even integers i.

Now apply the functor $R/J \otimes_R -$ to the above two diagrams, we have the following commutative diagrams with exact rows and columns

$$\begin{array}{ccccccccc}
 & & 0 & & 0 & & & & \\
 & & \downarrow & & \downarrow & & & & \\
0 & \longrightarrow & R/J \otimes_R \Omega^{i+1}(K) & \longrightarrow & R/J \otimes_R \Omega^{i+1}(M) & \longrightarrow & R/J \otimes_R \Omega^{i+1}(N) & \longrightarrow & 0 \\
 & & \downarrow & & \downarrow & & \downarrow{\scriptstyle \zeta} & & \\
0 & \longrightarrow & R/J \otimes_R J^{d-1}P_i & \longrightarrow & R/J \otimes_R J^{d-1}Q_i & \longrightarrow & R/J \otimes_R J^{d-1}L_i & \longrightarrow & 0 \\
 & & \downarrow & & \downarrow & & \downarrow & & \\
0 & \to & R/J \otimes_R J^{d-1}\Omega^i(K) & \to & R/J \otimes_R J^{d-1}\Omega^i(M) & \to & R/J \otimes_R J^{d-1}\Omega^i(N) & \to & 0 \\
 & & \downarrow & & \downarrow & & \downarrow & & \\
 & & 0 & & 0 & & 0 & &
\end{array}$$

for all positive odd integers i and

$$\begin{array}{ccccccccc}
 & & 0 & & 0 & & & & \\
 & & \downarrow & & \downarrow & & & & \\
0 & \to & R/J \otimes_R \Omega^{i+1}(K) & \to & R/J \otimes_R \Omega^{i+1}(M) & \to & R/J \otimes_R \Omega^{i+1}(N) & \to & 0 \\
 & & \downarrow & & \downarrow & & \downarrow{\scriptstyle \zeta} & & \\
0 & \longrightarrow & R/J \otimes_R JP_i & \longrightarrow & R/J \otimes_R JQ_i & \longrightarrow & R/J \otimes_R JL_i & \longrightarrow & 0 \\
 & & \downarrow & & \downarrow & & \downarrow & & \\
0 & \to & R/J \otimes_R J\Omega^i(K) & \to & R/J \otimes_R J\Omega^i(M) & \to & R/J \otimes_R J\Omega^i(N) & \to & 0 \\
 & & \downarrow & & \downarrow & & \downarrow & & \\
 & & 0 & & 0 & & 0 & &
\end{array}$$

for all nonnegative even integers i.

Now by "3×3" lemma, we have ζ is a monomorphism, which implies that

$$J\Omega^{i+1}(N) = \Omega^{i+1}(N) \cap J^d L_i$$

for all positive odd integers i and $J\Omega^{i+1}(N) = \Omega^{i+1}(N) \cap J^2 L_i$ for all nonnegative even integers i, which implies that $N \in \mathcal{Q}^d(R)$, as desired.

Now Theorem 1.3 is immediate from Lemmas 3.1 and 3.2.

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关于拟d-Koszul模的一个注记

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摘 要: 本文研究了文献[4]中给出的极小马蹄型引理成立的充分条件. 借助拟d-Koszul模给出了一个充要条件并给出了一个极小马蹄型引理的应用.

关 键 词: 拟d-Koszul模; 极小马蹄型引理

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