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LEAST-SQUARES SOLUTION OF INVERSE PROBLEM FOR HERMITIAN REFLEXIVE MATRICES

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Abstract: The least-square solution of inverse problem for Hermitian reflexive matrices and their optimal approximation are considered. By using the singular value decomposition method, the general expression of the least-square solution is provided. Also, the representation of its unique optimal approximation in the least-square solutions set is presented.

Keywords: Hermitian reflexive matrix; least-squares solution; optimal approximation

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1 Introduction

In electricity, control theory, and processing of digital signals, we often need to consider the following problem.

Problem I Given $X, B \in C^{n\times m}$, set $S \subseteq C^{n\times n}$, find $A \in S$ such that

$$\|AX - B\| = \min .$$

At the same time, in the process of testing or correcting given data, we also present an optimal approximation problem with some constraints as follows.

Problem II Given $\tilde{A} \in C^{n\times n}$, find $\hat{A} \in S_A$ such that $\left\|\tilde{A} - \hat{A}\right\| = \min\limits_{A\in S_A} \left\|\tilde{A} - A\right\|$, where S_A is the solution set of Problem I. $\|\cdot\|$ is the Frobenius norm.

For important results on Problems I and II for different sets of matrices S, we refer to [l–13].

In this paper, we discuss Problems I and II when S in Problem I consists of Hermitian reflexive matrices defined by the following definition.

Definition 1.1 A matrix $J \in C^{n\times n}$ is called a generalized reflection matrix if $J^H = J$ and $J^2 = I_n$.

Definition 1.2 Given a generalized reflection matrix J, a matrix $A \in C^{n\times n}$ is called a Hermitian reflexive matrix with respect to the generalized reflection matrix J if $A^H = A$ and $JAJ = A$. We denoted by $HC_r^{n\times n}(J)$ the set of all $n \times n$ Hermitian reflexive matrices with respect to the generalized reflection matrix J.

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Our first goal is to investigate the properties of the set of Hermitian reflexive matrices. We will deduce an expression for the general solutions of Problem I. For Problem II, we show the existence and uniqueness of the solution of the problem. In addition, we derive an expression of the solution of Problem II.

The paper is organized as follows. In Section 2, we first discuss the structure properties of the generalized reflection matrix J and the Hermitian reflexive matrices, and then using these properties, together with the singular value decomposition (SVD) and Moore-Penrose generalized inverse of matrix, we will derive the existence theorems of and the general expressions for the solution to Problem I. In Section 3, we prove the existence and uniqueness of the solution of Problem II and derive the expression of this unique solution.

Some notation is introduced as follows. Denote by $R^{n\times m}$ the set of all $n\times m$ real matrices, by $C^{n\times m}$ the set of all $n\times m$ complex matrices. Let $HC^{n\times n}$ stand for the set of all $n\times n$ Hermitian matrices and $UC^{n\times n}$ stand for the set of all $n\times n$ unitary matrices. Denote the conjugate transpose, Moore-Penrose generalized inverse and the Frobenius norm of a matrix A by A^H, A^+ and $\|A\|$, respectively. I_n stands for the identity matrix of size n. We define inner product in space $C^{n\times m}$, $(A,B)=\mathrm{trace}(B^TA)$ for all $A,B\in C^{n\times m}$. Then $C^{n\times m}$ is a complete inner product space and the norm of a matrix generated by this inner product is Frobenius norm. For $A=(a_{ij}),B=(b_{ij})\in C^{n\times m}$, the notation $A*B=(a_{ij}b_{ij})\in C^{n\times m}$ represents the Hadamard product of the matrices A and B.

2 The Expression of the Solution to Problem I

We first discuss the structure of the $n\times n$ generalized reflection matrix J and the set $HC_r^{n\times n}(J)$. Since $J^2=I_n$, the only possible eigenvalues of J are $+1$ and -1. Say, $+1$ is an eigenvalue of multiplicity r. Since $J^H=J$, the eigenspace associated with $+1$ has also size r and its orthogonal complement (obviously, of size $n-r$) is the eigenspace associated with -1. Thus we can easily show the following lemma.

Lemma 2.1 Let J be the $n\times n$ generalized reflection matrix. Then there exists an $n\times n$ unitary matrix U such that

$$J=U\begin{bmatrix} I_r & 0\\ 0 & -I_{n-r}\end{bmatrix}U^H. \tag{2.1}$$

By Definition 1.2 in the previous section and the above lemma, we have the following result for the structure of the set $HC_r^{n\times n}(J)$.

Lemma 2.2 Let $A\in C^{n\times n}$ and the spectral decomposition of the $n\times n$ generalized reflection matrix J be given as (2.1). Then $A\in HC_r^{n\times n}(J)$ if and only if

$$A=U\begin{bmatrix} A_{11} & 0\\ 0 & A_{22}\end{bmatrix}U^H, \tag{2.2}$$

where $A_{11}\in HC^{r\times r}$, $A_{22}\in HC^{(n-r)\times(n-r)}$.

Proof If $A \in HC_r^{n\times n}(J)$, then by Definition 1.2 and (2.1), we obtain

$$A = U\begin{bmatrix} I_r & 0 \\ 0 & -I_{n-r} \end{bmatrix} U^H AU \begin{bmatrix} I_r & 0 \\ 0 & -I_{n-r} \end{bmatrix} U^H. \tag{2.3}$$

Since $A^H = A$, then $U^H AU \in HC^{n\times n}$. Let $A = U\begin{bmatrix} A_{11} & A_{12} \\ A_{12}^H & A_{22} \end{bmatrix} U^H$, where $A_{11} \in HC^{r\times r}$, $A_{22} \in HC^{(n-r)\times(n-r)}$. Substituting it in (2.3) yields (2.2).

On the other hand, if A can be expressed as (2.2), then, obviously, $A^H = A$ and, by a direct computation, $A = J^H AJ$. By Definition 1.2, $A \in HC_r^{n\times n}(J)$.

Lemma 2.3 (see [14]) Given $C, D \in C^{n\times l}$, if the singular value decomposition of matrix C is given by

$$C = U\begin{bmatrix} \Sigma & 0 \\ 0 & 0 \end{bmatrix} V^H = U_1\Sigma V_1^H, \tag{2.4}$$

where $U = (U_1, U_2) \in UC^{n\times n}$, $U_1 \in C^{n\times r}$, $V = (V_1, V_2) \in UC^{l\times l}$, $V_1 \in C^{l\times r}$, $r = \mathrm{rank}(C)$, $\Sigma = \mathrm{diag}(\sigma_1, \sigma_2, \ldots \sigma_r)$, $\sigma_i > 0$, $i = 1, 2, \cdots, r$.

Denote $S_1 \equiv \{X \in HC^{n\times n} | f_1(X) = \|XC - D\| = \min\}$, then the elements of S_1 can be expressed as

$$X = U\begin{bmatrix} \Phi * (U_1^H DV_1\Sigma + \Sigma V_1^H D^H U_1) & \Sigma^{-1}V_1^H D^H U_2 \\ U_2^H DV_1\Sigma^{-1} & X_{22} \end{bmatrix} U^H, \tag{2.5}$$

where $\forall X_{22} \in HC^{(n-r)\times(n-r)}$, $\Phi = (\phi_{ij}) \in C^{r\times r}$, $\phi_{ij} = \frac{1}{\sigma_i^2+\sigma_j^2}, i, j = 1, 2, \ldots, r$.

Assume the given generalized reflection matrix J in Problem I has the form of (2.1). Let

$$U^H X = \begin{bmatrix} X_1 \\ X_2 \end{bmatrix}, U^H B = \begin{bmatrix} B_1 \\ B_2 \end{bmatrix}, \tag{2.6}$$

where $X_1, B_1 \in C^{r\times m}$, $X_2, B_2 \in C^{(n-r)\times m}$, and the SVDs of X_1, X_2 are, respectively,

$$X_1 = W\begin{bmatrix} \Sigma_1 & 0 \\ 0 & 0 \end{bmatrix} V^H = W_1\Sigma_1 V_1^H, \tag{2.7}$$

where $W = (W_1, W_2) \in UC^{r\times r}$, $W_1 \in C^{r\times r_1}$, $V = (V_1, V_2) \in UC^{m\times m}$, $V_1 \in C^{m\times r_1}$, $r_1 = \mathrm{rank}(X_1)$, $\Sigma_1 = \mathrm{diag}(\alpha_1, \alpha_2, \ldots \alpha_{r_1})$, $\sigma_i > 0$, $i = 1, 2, \ldots, r_1$.

$$X_2 = P\begin{bmatrix} \Sigma_2 & 0 \\ 0 & 0 \end{bmatrix} Q^H = P_1\Sigma_2 Q_1^H, \tag{2.8}$$

where $P = (P_1, P_2) \in UC^{(n-r)\times(n-r)}$, $P_1 \in C^{(n-r)\times r_2}$, $Q = (Q_1, Q_2) \in UC^{m\times m}$, $Q_1 \in C^{m\times r_2}$, $r_2 = \mathrm{rank}(X_2)$, $\Sigma_2 = \mathrm{diag}(\beta_1, \beta_2, \ldots \beta_{r_2})$, $\sigma_i > 0$, $i = 1, 2, \ldots, r_2$.

Let

$$\Phi_1 = (\phi_{ij}^{(1)}) \in C^{r_1 \times r_1}, \phi_{ij}^{(1)} = \frac{1}{\alpha_i^2 + \alpha_j^2}, i, j = 1, 2, \ldots, r_1,$$

$$\Phi_2 = (\phi_{ij}^{(2)}) \in C^{r_2 \times r_2}, \phi_{ij}^{(2)} = \frac{1}{\beta_i^2 + \beta_j^2}, i, j = 1, 2, \ldots, r_2.$$

Then we can establish the existence theorems of Problem I as follows.

Theorem 2.1 Given $X, B \in C^{n \times m}$, and a generalized reflection matrix J of size n, suppose the spectral decomposition of J is given by (2.1), $U^H X$ and $U^H B$ have the partition forms of (2.6), and the SVDs of X_1 and X_2 are (2.7), (2.8). Then Problem I has a solution $A \in HC_r^{n \times n}(J)$, and its general solution can be expressed as

$$A = A_0 + U \begin{bmatrix} W_2 G_1 W_2^H & 0 \\ 0 & P_2 G_2 P_2^H \end{bmatrix} U^H, \tag{2.9}$$

where $\forall G_1 \in HC^{(r-r_1)\times(r-r_1)}$, $\forall G_2 \in HC^{(n-r-r_2)\times(n-r-r_2)}$,

$$A_0 = U \begin{bmatrix} A_{11}^{(0)} & 0 \\ 0 & A_{22}^{(0)} \end{bmatrix} U^H, \tag{2.10}$$

$$A_{11}^{(0)} = W \begin{bmatrix} \Phi_1 * (W_1^H B_1 V_1 \Sigma_1 + \Sigma_1 V_1^H B_1^H W_1) & \Sigma_1^{-1} V_1^H B_1^H W_2 \\ W_2^H B_1 V_1 \Sigma_1^{-1} & 0 \end{bmatrix} W^H,$$

$$A_{22}^{(0)} = P \begin{bmatrix} \Phi_2 * (P_1^H B_2 Q_1 \Sigma_2 + \Sigma_2 Q_1^H B_2^H P_1) & \Sigma_2^{-1} Q_1^H B_2^H P_2 \\ P_2^H B_2 Q_1 \Sigma_2^{-1} & 0 \end{bmatrix} P^H.$$

Proof Using the invariance of the Frobenius norm under unitary transformations, we have from Lemma 2.2 that

$$A = U \begin{bmatrix} A_{11} & 0 \\ 0 & A_{22} \end{bmatrix} U^H, \tag{2.11}$$

where $A_{11} \in HC^{r \times r}$, $A_{22} \in HC^{(n-r)\times(n-r)}$, then

$$\|AX - B\|^2 = \left\| \begin{bmatrix} A_{11} & 0 \\ 0 & A_{22} \end{bmatrix} U^H X - U^H B \right\|^2 = \|A_{11}X_1 - B_1\|^2 + \|A_{22}X_2 - B_2\|^2.$$

Thus, there exists $A \in HC_r^{n \times n}(J)$ such that $\|AX - B\| = \min$ is equivalent to the existence of X_{11}, X_{22} such that $\|A_{11}X_1 - B_1\| = \min$ and $\|A_{22}X_2 - B_2\| = \min$. It follows Lemma 2.3 that

$$A_{11} = W \begin{bmatrix} \Phi_1 * (W_1^H B_1 V_1 \Sigma_1 + \Sigma_1 V_1^H B_1^H W_1) & \Sigma_1^{-1} V_1^H B_1^H W_2 \\ W_2^H B_1 V_1 \Sigma_1^{-1} & G_1 \end{bmatrix} W^H, \tag{2.12}$$

where $\forall G_1 \in HC^{(r-r_1)\times(r-r_1)}$,

$$A_{22} = P \begin{bmatrix} \Phi_2 * (P_1^H B_2 Q_1 \Sigma_2 + \Sigma_2 Q_1^H B_2^H P_1) & \Sigma_2^{-1} Q_1^H B_2^H P_2 \\ P_2^H B_2 Q_1 \Sigma_2^{-1} & G_2 \end{bmatrix} P^H, \tag{2.13}$$

where $\forall G_2 \in HC^{(n-r-r_2)\times(n-r-r_2)}$.

Substituting (2.12) and (2.13) in (2.11), we know that the general solution in $HC_r^{n\times n}(J)$ of Problem I can be expressed as (2.9).

3 The Expression of the Solution to Problem II

Lemma 3.1 (see [15]) Let $E \in C^{n\times n}$, then $\forall G \in HC^{n\times n}$ we have

$$\left\| E - \frac{E+E^H}{2} \right\| \le \|E-G\|.$$

It is easy to show that the solution set S_A of Problem I is a closed convex set. Since $C^{n\times n}$ is a Hilbert space, then Problem II has a unique solution.

Theorem 3.1 Let $\tilde{A} \in C^{n\times n}$, and the conditions and symbols be the same as that in Theorem 2.1. Let

$$U^H\tilde{A}U = \begin{bmatrix} \tilde{A}_{11} & \tilde{A}_{12} \\ \tilde{A}_{21} & \tilde{A}_{22} \end{bmatrix}, \tilde{A}_{11} \in C^{r\times r}. \tag{3.1}$$

Then there is a unique solution $\hat{A}$ for Problem II and $\hat{A}$ can be represented as

$$\hat{A} = A_0 + U \begin{bmatrix} \hat{A}_{11} & 0 \\ 0 & \hat{A}_{22} \end{bmatrix} U^H, \tag{3.2}$$

where $\hat{A}_{11} = \frac{1}{2}(I-X_1X_1^+)(\tilde{A}_{11}+\tilde{A}_{11}^H)(I-X_1X_1^+)$, $\hat{A}_{22} = \frac{1}{2}(I-X_2X_2^+)(\tilde{A}_{22}+\tilde{A}_{22}^H)(I-X_2X_2^+)$.

Proof When S_A is nonempty, it is easy to verify from (2.9) that S_A is a closed convex set. Since $C^{n\times n}$ is a uniformly convex banach space under Frobenius norm, there exists a unique solution for Problem II. Let

$$W^H(\tilde{A}_{11} - A_{11}^{(0)})W = \begin{bmatrix} \tilde{A}_{11}^{(1)} & \tilde{A}_{12}^{(1)} \\ \tilde{A}_{21}^{(1)} & \tilde{A}_{22}^{(1)} \end{bmatrix}, \tag{3.3}$$

$$P^H(\tilde{A}_{22} - A_{22}^{(0)})P = \begin{bmatrix} \tilde{A}_{11}^{(2)} & \tilde{A}_{12}^{(2)} \\ \tilde{A}_{21}^{(2)} & \tilde{A}_{22}^{(2)} \end{bmatrix}, \tag{3.4}$$

where $\tilde{A}_{11}^{(1)} \in C^{r_1\times r_1}$, $\tilde{A}_{11}^{(2)} \in C^{r_2\times r_2}$. For $A \in S_A$, we have

$$\begin{aligned} A &= A_0 + U \begin{bmatrix} W_2G_1W_2^H & 0 \\ 0 & P_2G_2P_2^H \end{bmatrix} U^H \\ &= A_0 + U \begin{bmatrix} W \begin{bmatrix} 0 & 0 \\ 0 & G_1 \end{bmatrix} W^H & 0 \\ 0 & P \begin{bmatrix} 0 & 0 \\ 0 & G_2 \end{bmatrix} P^H \end{bmatrix} U^H. \end{aligned} \tag{3.5}$$

Then using the invariance of the Frobenius norm under unitary transformations, we have from (3.3), (3.4) and (3.5) that

$$\begin{aligned}
||A-\tilde{A}||^2 &= \left\|\begin{bmatrix} W_2G_1W_2^H & 0 \\ 0 & P_2G_2P_2^H \end{bmatrix} - U^H(\tilde{A}-A_0)U\right\|^2 \\
&= \left\|\begin{bmatrix} W_2G_1W_2^H & 0 \\ 0 & P_2G_2P_2^H \end{bmatrix} - \begin{bmatrix} \tilde{A}_{11}-A_{11}^{(0)} & \tilde{A}_{12} \\ \tilde{A}_{21} & \tilde{A}_{22}-A_{22}^{(0)} \end{bmatrix}\right\|^2 \\
&= \left\|W\begin{bmatrix} 0 & 0 \\ 0 & G_1 \end{bmatrix}W^H - (\tilde{A}_{11}-A_{11}^{(0)})\right\|^2 + \left\|\tilde{A}_{12}\right\|^2 \\
&\quad + \left\|P\begin{bmatrix} 0 & 0 \\ 0 & G_2 \end{bmatrix}P^H - (\tilde{A}_{22}-A_{22}^{(0)})\right\|^2 + \left\|\tilde{A}_{21}\right\|^2 \\
&= \left\|\begin{bmatrix} 0 & 0 \\ 0 & G_1 \end{bmatrix} - W^H(\tilde{A}_{11}-A_{11}^{(0)})W\right\|^2 + \left\|\tilde{A}_{12}\right\|^2 \\
&\quad + \left\|\begin{bmatrix} 0 & 0 \\ 0 & G_2 \end{bmatrix} - P^H(\tilde{A}_{22}-A_{22}^{(0)})P\right\|^2 + \left\|\tilde{A}_{21}\right\|^2 \\
&= \left\|G_1-\tilde{A}_{22}^{(1)}\right\|^2 + \left\|G_2-\tilde{A}_{22}^{(2)}\right\|^2 + \left\|\tilde{A}_{11}^{(1)}\right\|^2 + \left\|\tilde{A}_{12}^{(1)}\right\|^2 + \left\|\tilde{A}_{21}^{(1)}\right\|^2 \\
&\quad + \left\|\tilde{A}_{11}^{(2)}\right\|^2 + \left\|\tilde{A}_{12}^{(2)}\right\|^2 + \left\|\tilde{A}_{21}^{(2)}\right\|^2 + \left\|\tilde{A}_{12}\right\|^2 + \left\|\tilde{A}_{21}\right\|^2.
\end{aligned}$$

We can see that Problem II is equivalent to

$$\begin{cases} \left\|G_1-\tilde{A}_{22}^{(1)}\right\|^2 = \min\limits_{G_1\in HC^{(r-r_1)\times(r-r_1)}}, \\ \left\|G_2-\tilde{A}_{22}^{(2)}\right\|^2 = \min\limits_{G_2\in HC^{(n-r-r_2)\times(n-r-r_2)}}. \end{cases}$$

We get from Lemma 3.1 that

$$G_1 = \frac{\tilde{A}_{22}^{(1)}+(\tilde{A}_{22}^{(1)})^H}{2}, G_2 = \frac{\tilde{A}_{22}^{(2)}+(\tilde{A}_{22}^{(2)})^H}{2}.$$

By (2.10), (3.3) and (3.4) we have

$$G_1 = \frac{1}{2}W_2^H(\tilde{A}_{11}+\tilde{A}_{11}^H-2A_{11}^{(0)})W_2 = \frac{1}{2}W_2^H(\tilde{A}_{11}+\tilde{A}_{11}^H)W_2, \tag{3.6}$$

$$G_2 = \frac{1}{2}P_2^H(\tilde{A}_{22}+\tilde{A}_{22}^H-2A_{22}^{(0)})P_2 = \frac{1}{2}P_2^H(\tilde{A}_{22}+\tilde{A}_{22}^H)P_2. \tag{3.7}$$

Substituting (3.6) and (3.7) into (3.5), we have

$$\hat{A} = A_0 + U\begin{bmatrix} \frac{1}{2}W_2W_2^H(\tilde{A}_{11}+\tilde{A}_{11}^H)W_2W_2^H & 0 \\ 0 & \frac{1}{2}P_2P_2^H(\tilde{A}_{22}+\tilde{A}_{22}^H)P_2P_2^H \end{bmatrix}U^H.$$

Since $W_2W_2^H = I - X_1X_1^+$, $P_2P_2^H = I - X_2X_2^+$, we obtain that the solution to Problem II has the form as (3.2).

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Hermitian自反矩阵反问题的最小二乘解

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摘要: 本文研究了Hermitian自反矩阵反问题的最小二乘解及其最佳逼近. 利用矩阵的奇异值分解理论, 获得了最小二乘解的表达式. 同时对于最小二乘解的解集合, 得到了最佳逼近解.

关键词: Hermitian自反矩阵; 最小二乘解; 最佳逼近

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