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HECKE-TYPE DOUBLE SUMS AS MOCK THETA FUNCTIONS

YANG Ji-zhen[1], SONG Han-fei[2], ZHANG Zhi-zheng[1,3]

(1. Department of Mathematics, Luoyang Normal University, Luoyang 471934, China)

(2. Department of Mathematics, Shanghai Normal University, Shanghai 200234, China)

(3. College of Mathematics and Information Science, Henan Normal University, Xinxiang 453001, China)

Abstract: The main purpose of this paper is to present Hecke-type double sums associated with mock theta functions. By using the method of Bailey pairs, we obtain three known identities for the seventh and tenth order mock theta functions and two new Hecke-type double sums as third order mock theta functions.

Keywords: mock theta functions; Hecke-type double sums; Bailey pairs; q-series

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1 Introduction

In his last letter, written in January 1920 to Hardy, Ramanujan [1] defined four third order mock theta functions, ten fifth order mock theta functions in two groups each having five functions and three seventh order mock theta functions, among them one third order mock theta function is defined as

$$f(q) := \sum_{n\geq 0} \frac{q^{n^2}}{(-q)_n^2}.$$

Here we first introduce the standard notation and terminology for q-series [2]: the q-shifted factorials of complex variable x with the base q are given by

$$(x;q)_\infty = (x)_\infty := \prod_{n\geq 0}(1-xq^n), \qquad (x;q)_n = (x)_n := \frac{(x;q)_\infty}{(xq^n;q)_\infty},$$

for all integers n.

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Biography: Yang Jizhen(1984–), female, born at Luoyang, Henan, associate professor, major in combinatorics, E-mail: yangjizhen116@163.com

In [3], Watson studied the third order mock theta functions and introduced the three one shown below, which were later found in the Lost Notebook [1].

$$\omega(q) := \sum_{n\geq 0} \frac{q^{2n(n+1)}}{(q;q^2)_{n+1}^2}, \quad \nu(q) := \sum_{n\geq 0} \frac{q^{n(n+1)}}{(-q;q^2)_{n+1}}, \quad \rho(q) := \sum_{n\geq 0} \frac{(q;q^2)_{n+1} q^{2n(n+1)}}{(q^3;q^6)_{n+1}}.$$

In recent years, the Hecke-type double sums in terms of mock theta functions have made several appearances in the literature. In [4], Andrews presented Hecke-type double sums as the fifth and seventh order mock theta functions by utilizing the Bailey pairs. The seventh order mock theta functions are defined as

$$\mathcal{F}_0(q) := \sum_{n\geq 0} \frac{q^{n^2}}{(q^{n+1})_n}, \qquad \mathcal{F}_1(q) := \sum_{n\geq 1} \frac{q^{n^2}}{(q^n)_n}, \qquad \mathcal{F}_2(q) := \sum_{n\geq 0} \frac{q^{n(n+1)}}{(q^{n+1})_{n+1}}.$$

With the aid of the Bailey pairs from [4], Choi [5, 6] established the Hecke-type double sums associated with tenth order mock theta functions, two of them are defined as

$$\phi(q) := \sum_{n\geq 0} \frac{q^{\binom{n+1}{2}}}{(q;q^2)_{n+1}}, \qquad \psi(q) := \sum_{n\geq 0} \frac{q^{\binom{n+2}{2}}}{(q;q^2)_{n+1}}.$$

In [7], Hickerson and Mortenson used the Appell–Lerch sums and Hecke-type double sums to facilitate the study of mock theta functions. They rewrote the respective Hecke-type double sums from [5, 8].

$$\mathcal{F}_2(q) = \frac{1}{J_1} f_{3,4,3}(q^3, q^3, q), \tag{1.1}$$

$$\phi(q) = \frac{1}{J_{1,2}} f_{2,3,2}(q^2, q^2, q), \tag{1.2}$$

$$\psi(q) = -\frac{q^2}{J_{1,2}} f_{2,3,2}(q^4, q^4, q), \tag{1.3}$$

where let $x, y \in \mathbb{C}^* = \mathbb{C} - \{0\}$, a and m be integers with m positive,

$$J_{a,m} := j(q^a; q^m), \qquad \overline{J}_{a,m} := j(-q^a; q^m), \qquad J_m := J_{m,3m},$$

$$j(z;q) := (z, q/z, q)_\infty = \sum_n (-1)^n z^n q^{\binom{n}{2}},$$

$$f_{a,b,c}(x,y,q) := \Big(\sum_{r,s\geq 0} - \sum_{r,s<0} \Big) (-1)^{r+s} x^r y^s q^{a\binom{r}{2} + brs + c\binom{s}{2}}.$$

The first object of this paper is to prove the above Hecke-type double sums associated with mock theta functions by utilizing another Bailey pair.

Theorem 1.1 Identities (1.1)–(1.3) are true.

Recently, Cui and Gu [9] established certain new mock theta functions and expressed classical mock theta functions in terms of Hecke-type double sums as corollaries, such as

$$\nu(q) = \frac{1}{J_{2,4}} f_{1,2,1}(iq^{\frac{3}{2}}, -iq^{\frac{3}{2}}, q).$$

The second object of this paper is to show the following Hecke-type double sums as the third order mock theta functions $f(q)$ and $\omega(q)$ similar to $\nu(q)$.

Theorem 1.2 We have

$$f(q)=\frac{1}{\overline{J}_{1,4}}f_{1,2,1}(-q,-q,q), \tag{1.4}$$

$$\omega(q)=\frac{1}{J_{1,2}}f_{1,2,1}(q^3,q^3,q^2). \tag{1.5}$$

In the next section, we list some useful results on q-series. We prove Theorems 1.1 and 1.2 in Sections 3 and 4, respectively.

2 Preliminaries

For brevity, in this paper, we also employ the usual notation

$$(x_1,\cdots,x_r)_n=(x_1,\cdots,x_r;q)_n=(x_1)_n\cdots(x_r)_n,$$
$$(x_1,\cdots,x_r)_\infty=(x_1,\cdots,x_r;q)_\infty=(x_1)_\infty\cdots(x_r)_\infty.$$

The pair of sequences (α_n,β_n) is called a Bailey pair with respect to a, namely

$$\beta_n=\sum_{r=0}^{n}\frac{\alpha_r}{(q)_{n-r}(aq)_{n+r}},$$

for all $n\geq 0$.

The following identities from [10] will be used frequently.

$$\sum_{n\geq 0}(\rho,\sigma)_n(aq/\rho\sigma)^n\beta_n=\frac{(aq/\rho,aq/\sigma)_\infty}{(aq,aq/\rho\sigma)_\infty}\sum_{n\geq 0}\frac{(\rho,\sigma)_n(aq/\rho\sigma)^n}{(aq/\rho,aq/\sigma)_n}\alpha_n, \tag{2.1}$$

and

$$\sum_{n\geq 0}(\rho,\sigma)_n(a/\rho\sigma)^n\beta_n=\frac{(aq/\rho,aq/\sigma)_\infty}{(aq,aq/\rho\sigma)_\infty}\sum_{n\geq 0}\frac{(\rho,\sigma)_n(a/\rho\sigma)^n}{(aq/\rho,aq/\sigma)_n}\left(\frac{\rho\sigma(1+aq^{2n})-aq^n(\rho+\sigma)}{\rho\sigma-a}\right)\alpha_n. \tag{2.2}$$

The identity (2.1) is known as Bailey's lemma. And the equality (2.2) is also a useful tool. In [11], Zhang and Song used (2.2) to derive a q-series expansion formula and obtained some Hecke-type identities as special cases.

Next we shall introduce two Bailey pairs. One is a Bailey pair relative to 1 due to Andrews [4].

$$\begin{cases}\alpha_{2n}=-(1-q^{4n})q^{3n^2-2n}\displaystyle\sum_{j=-n}^{n-1}q^{-j^2-j},\\ \alpha_{2n+1}=(1-q^{4n+2})q^{3n^2+n}\displaystyle\sum_{j=-n}^{n}q^{-j^2},\end{cases}\qquad \beta_n=\begin{cases}0, & n=0,\\ \frac{1}{(q^n)_n}, & n>0,\end{cases} \tag{2.3}$$

The other is a Bailey pair relative to q^2 given by Srivastava [12].

$$\begin{cases} \alpha_{2n} = \dfrac{(1-q^{4n+2})}{(1-q)(1-q^2)} q^{3n^2+n} \displaystyle\sum_{j=-n}^{n} q^{-j^2}, \\ \alpha_{2n+1} = -\dfrac{(1-q^{4n+4})}{(1-q)(1-q^2)} q^{3n^2+4n+1} \displaystyle\sum_{j=-n-1}^{n} q^{-j^2-j}, \end{cases} \qquad \beta_n = \frac{1}{(q^{n+1})_{n+1}}. \tag{2.4}$$

We point out that the Bailey pair (2.4) can be also derived by combining [13, Eq. (6.3)] and [13, Eq. (3.12)]. Utilizing (2.4), Andrews [13] showed that [13, Eq. (6.9)] is equivalent to the equality [4, Eq. (7.23)] for seventh order mock theta function $\mathcal{F}_1(q)$.

3 Proof of Theorem 1.1

Proof For (1.1), substituting the Bailey pair (2.4) into (2.2) with $\rho, \sigma \to \infty$, we obtain

$$\begin{aligned} LHS =& \sum_{n\geq 0} q^{n^2+n}\beta_n = \sum_{n\geq 0} \frac{q^{n^2+n}}{(q^{n+1})_{n+1}}, \\ RHS =& \frac{1}{(q^3)_\infty} \sum_{n\geq 0} q^{n^2+n}(1+q^{2n+2})\alpha_n \\ =& \frac{1}{(q^3)_\infty} \left(\sum_{n\geq 0} q^{4n^2+2n}(1+q^{4n+2})\alpha_{2n} + \sum_{n\geq 0} q^{4n^2+6n+2}(1+q^{4n+4})\alpha_{2n+1} \right) \\ =& \frac{1}{(q)_\infty} \left(\sum_{n\geq 0} q^{7n^2+3n}(1-q^{8n+4}) \sum_{j=-n}^{n} q^{-j^2} - \sum_{n\geq 0} q^{7n^2+10n+3}(1-q^{8n+8}) \sum_{j=-n-1}^{n} q^{-j^2-j} \right) \\ =& \frac{1}{(q)_\infty} \left(\sum_{n\geq 0} q^{7n^2+3n} \sum_{j=-n}^{n} q^{-j^2} - \sum_{n\geq 0} q^{7n^2+11n+4} \sum_{j=-n}^{n} q^{-j^2} \right. \\ & \left. - \sum_{n\geq 0} q^{7n^2+10n+3} \sum_{j=-n-1}^{n} q^{-j^2-j} + \sum_{n\geq 0} q^{7n^2+18n+11} \sum_{j=-n-1}^{n} q^{-j^2-j} \right). \end{aligned} \tag{3.1}$$

We replace n with $-n-1$ in the second sum and n with $-n-2$ in the fourth sum, and let $n = (r+s)/2$, $j = (r-s)/2$ in the first two sums and $n = (r+s-1)/2$, $j = (r-s-1)/2$ in the latter two sums to arrive at

$$\begin{aligned} RHS = \mathcal{F}_2(q) =& \frac{1}{(q)_\infty} \left(\left\{ \sum_{\substack{r,s\geq 0 \\ r\equiv s \pmod 2}} - \sum_{\substack{r,s<0 \\ r\equiv s \pmod 2}} \right\} q^{\frac{3}{2}r^2+4rs+\frac{3}{2}s^2+\frac{3}{2}r+\frac{3}{2}s} \right. \\ & \left. - \left\{ \sum_{\substack{r,s\geq 0 \\ r\not\equiv s \pmod 2}} - \sum_{\substack{r,s<0 \\ r\not\equiv s \pmod 2}} \right\} q^{\frac{3}{2}r^2+4rs+\frac{3}{2}s^2+\frac{3}{2}r+\frac{3}{2}s} \right) \end{aligned}$$

$$
\begin{aligned}
&= \frac{1}{(q)_\infty}\Big(\sum_{r,s\geq 0}-\sum_{r,s<0}\Big)(-1)^{r+s}q^{\frac{3}{2}r^2+4rs+\frac{3}{2}s^2+\frac{3}{2}r+\frac{3}{2}s}\\
&= \frac{1}{(q)_\infty}f_{3,4,3}(q^3,q^3,q).
\end{aligned}
$$

It is evident that the identity (3.1) appears in [4, Eq. (7.24)]. The fact implies that the same identity can be obtained by the different Bailey pairs. A similar situation will arise in the following steps to prove (1.2) and (1.3).

For (1.2), applying the Bailey pair (2.4) to (2.2) with $\rho=-q,\sigma\to\infty$, we have

$$
\begin{aligned}
LHS &= \sum_{n\geq 0}(-q)_n q^{\binom{n+1}{2}}\beta_n = \sum_{n\geq 0}\frac{(-q)_n q^{\binom{n+1}{2}}}{(q^{n+1})_{n+1}} = \phi(q),\\
RHS &= \frac{(-q)_\infty}{(q^3)_\infty}\sum_{n\geq 0}\frac{q^{\binom{n+1}{2}}(1+q^{n+1}+q^{2n+2})}{1+q^{n+1}}\alpha_n\\
&= \frac{(-q)_\infty}{(q^3)_\infty}\left(\sum_{n\geq 0}\frac{q^{2n^2+n}(1+q^{2n+1}+q^{4n+2})}{1+q^{2n+1}}\alpha_{2n}+\sum_{n\geq 0}\frac{q^{2n^2+3n+1}(1+q^{2n+2}+q^{4n+4})}{1+q^{2n+2}}\alpha_{2n+1}\right)\\
&= \frac{(-q)_\infty}{(q)_\infty}\left(\sum_{n\geq 0}q^{5n^2+2n}(1-q^{6n+3})\sum_{j=-n}^{n}q^{-j^2}-\sum_{n\geq 0}q^{5n^2+7n+2}(1-q^{6n+6})\sum_{j=-n-1}^{n}q^{-j^2-j}\right).
\end{aligned}
$$

And after simplifying similar to the first identity (3.1), we deduce (1.2).

For (1.3), inserting the Bailey pair (2.4) into (2.1) with $\rho=-q,\sigma\to\infty$, we conclude

$$
\begin{aligned}
LHS &= \sum_{n\geq 0}(-q)_n q^{n(n+3)/2}\beta_n = \sum_{n\geq 0}\frac{(-q)_n q^{n(n+3)/2}}{(q^{n+1})_{n+1}},\\
RHS &= \frac{(-q)_\infty}{(q^3)_\infty}\sum_{n\geq 0}\frac{q^{n(n+3)/2}}{1+q^{n+1}}\alpha_n\\
&= \frac{(-q)_\infty}{(q^3)_\infty}\left(\sum_{n\geq 0}\frac{q^{2n^2+3n}}{1+q^{2n+1}}\alpha_{2n}+\sum_{n\geq 0}\frac{q^{2n^2+5n+2}}{1+q^{2n+2}}\alpha_{2n+1}\right)\\
&= \frac{(-q)_\infty}{(q)_\infty}\left(\sum_{n\geq 0}q^{5n^2+4n}(1-q^{2n+1})\sum_{j=-n}^{n}q^{-j^2}-\sum_{n\geq 0}q^{5n^2+9n+3}(1-q^{2n+2})\sum_{j=-n-1}^{n}q^{-j^2-j}\right).
\end{aligned}
$$

Multiplying both sides of the above equality by q, we get

$$
\begin{aligned}
\psi(q) &= \sum_{n\geq 0}\frac{(-q)_n q^{\binom{n+2}{2}}}{(q^{n+1})_{n+1}}\\
&= \frac{(-q)_\infty}{(q)_\infty}\left(\sum_{n\geq 0}q^{5n^2+4n+1}(1-q^{2n+1})\sum_{j=-n}^{n}q^{-j^2}-\sum_{n\geq 0}q^{5n^2+9n+4}(1-q^{2n+2})\sum_{j=-n-1}^{n}q^{-j^2-j}\right),
\end{aligned}
$$

and by a straightforward calculation, we conclude (1.3). This completes the proof.

4 Proof of Theorem 1.2

Proof Inserting the Bailey pair (2.3) into (2.1) with $\rho=\sqrt{q}$, $\sigma=-\sqrt{q}$, it follows that

$$\begin{aligned}
2LHS =&2\sum_{n\geq 1}(-1)^n(q;q^2)_n\beta_n = 2\sum_{n\geq 1}\frac{(-1)^n(q;q^2)_n}{(q^n)_n} := 2M_1(q),\\
2RHS =&\frac{(q;q^2)_\infty}{(q^2;q^2)_\infty}\sum_{n\geq 0}(-1)^n\alpha_n\\
=&\frac{(q;q^2)_\infty}{(q^2;q^2)_\infty}\left(\sum_{n\geq 0}\alpha_{2n}-\sum_{n\geq 0}\alpha_{2n+1}\right)\\
=&\frac{(q;q^2)_\infty}{(q^2;q^2)_\infty}\Bigg(-\sum_{n\geq 0}q^{3n^2-2n}\sum_{j=-n}^{n-1}q^{-j^2-j}+\sum_{n\geq 0}q^{3n^2+2n}\sum_{j=-n}^{n-1}q^{-j^2-j}\\
&-\sum_{n\geq 0}q^{3n^2+n}\sum_{j=-n}^{n}q^{-j^2}+\sum_{n\geq 0}q^{3n^2+5n+2}\sum_{j=-n}^{n}q^{-j^2}\Bigg).
\end{aligned}$$

We replace n with $-n$ in the second sum and n with $-n-1$ in the fourth sum, and then let $n=(r+s+1)/2$, $j=(r-s-1)/2$ in the first two sums and $n=(r+s)/2$, $j=(r-s)/2$ in the latter two sums to get

$$\begin{aligned}
2RHS =&\frac{(q;q^2)_\infty}{(q^2;q^2)_\infty}\Bigg(-\Bigg\{\sum_{\substack{r,s\geq 0\\ r\not\equiv s(mod2)}}-\sum_{\substack{r,s<0\\ r\not\equiv s(mod2)}}\Bigg\}q^{\frac12 r^2+2rs+\frac12 s^2+\frac12 r+\frac12 s}\\
&-\Bigg\{\sum_{\substack{r,s\geq 0\\ r\equiv s(mod2)}}-\sum_{\substack{r,s<0\\ r\equiv s(mod2)}}\Bigg\}q^{\frac12 r^2+2rs+\frac12 s^2+\frac12 r+\frac12 s}\Bigg)\\
=&-\frac{(q;q^2)_\infty}{(q^2;q^2)_\infty}\Big(\sum_{r,s\geq 0}-\sum_{r,s<0}\Big)q^{\frac12 r^2+2rs+\frac12 s^2+\frac12 r+\frac12 s}\\
=&-\frac{(q;q^2)_\infty}{(q^2;q^2)_\infty}f_{1,2,1}(-q,-q,q).
\end{aligned}$$

It is easy to see that

$$\sum_{n\geq 1}\frac{(-1)^n(q;q^2)_n}{(q^n)_n}=\sum_{n\geq 0}\frac{(-1)^{n+1}}{(-q)_n},$$

and owing to the fact from [14, Eq. (13)]

$$f(q)=2\sum_{n\geq 0}\frac{(-1)^n}{(-q)_n},$$

we have

$$f(q) = -2M_1(q),$$

which yields (1.4).

Substituting the Bailey pair (2.3) into (2.1) with $q \to q^2$, $\rho = -1$ and $\sigma = -q$ to give

$$\begin{aligned}
LHS =& \sum_{n\geq 1}(-1,-q;q^2)_n q^n \beta_n = 2\sum_{n\geq 1}\frac{(-q^2;q^2)_{n-1}(-q;q^2)_n q^n}{(q^{2n};q^2)_n} := 2M_2(q),\\
RHS =& \frac{2(-q)_\infty}{(q)_\infty}\sum_{n\geq 0}\frac{q^n}{1+q^{2n}}\alpha_n\\
=& \frac{2(-q)_\infty}{(q)_\infty}\left(\sum_{n\geq 0}\frac{q^{2n}}{1+q^{4n}}\alpha_{2n} + \sum_{n\geq 0}\frac{q^{2n+1}}{1+q^{4n+2}}\alpha_{2n+1}\right)\\
=& \frac{2(-q)_\infty}{(q)_\infty}\left(-\sum_{n\geq 0}q^{6n^2-2n}\sum_{j=-n}^{n-1}q^{-2j^2-2j} + \sum_{n\geq 0}q^{6n^2+2n}\sum_{j=-n}^{n-1}q^{-2j^2-2j}\right.\\
&\left. + \sum_{n\geq 0}q^{6n^2+4n+1}\sum_{j=-n}^{n}q^{-2j^2} - \sum_{n\geq 0}q^{6n^2+8n+3}\sum_{j=-n}^{n}q^{-2j^2}\right).
\end{aligned}$$

After replacing n with $-n$ in the second sum and n with $-n-1$ in the fourth sum and letting $n = (r+s+1)/2$, $j = (r-s-1)/2$ in the first two sums and $n = (r+s)/2$, $j = (r-s)/2$ in the latter two sums, we obtain

$$\begin{aligned}
RHS =& \frac{2(-q)_\infty}{(q)_\infty}\left(-\left\{\sum_{\substack{r,s\geq 0\\ r\not\equiv s \pmod 2}} - \sum_{\substack{r,s<0\\ r\not\equiv s \pmod 2}}\right\}q^{r^2+4rs+s^2+2r+2s+1}\right.\\
&\left. + \left\{\sum_{\substack{r,s\geq 0\\ r\equiv s \pmod 2}} - \sum_{\substack{r,s<0\\ r\equiv s \pmod 2}}\right\}q^{r^2+4rs+s^2+2r+2s+1}\right)\\
=& \frac{2(-q)_\infty}{(q)_\infty}\left(\sum_{r,s\geq 0} - \sum_{r,s<0}\right)(-1)^{r+s}q^{r^2+4rs+s^2+2r+2s+1}\\
=& \frac{2q(-q)_\infty}{(q)_\infty}f_{1,2,1}(q^3,q^3,q^2).
\end{aligned}$$

It is not difficult to see that

$$\sum_{n\geq 1}\frac{(-q^2;q^2)_{n-1}(-q;q^2)_n q^n}{(q^{2n};q^2)_n} = \sum_{n\geq 0}\frac{q^{n+1}}{(q;q^2)_{n+1}}.$$

In view of [15, Eq. (26.84)]

$$\omega(q) = \sum_{n\geq 0}\frac{q^{2n(n+1)}}{(q;q^2)_{n+1}^2} = \sum_{n\geq 0}\frac{q^n}{(q;q^2)_{n+1}},$$

then we have

$$M_2(q) = q\omega(q),$$

which implies (1.5). We finish the proof.

Remark 1. Inserting the Bailey pair [4, Eqs. (7.13), (7.16), (7.17)] into (2.1) with $\rho = \sqrt{q}, \sigma = -\sqrt{q}$, we conclude

$$M_3(q) := \sum_{n\geq 0} \frac{(-1)^n (q;q^2)_n}{(q^{n+1})_n} = \frac{(q;q^2)_\infty}{2(q^2;q^2)_\infty} f_{1,2,1}(-q,-q,q).$$

Namely, $M_1(q) = -M_3(q)$.

2. Andrews, Dixit and Yee [16] established the following result by considering a new partition-theoretic interpretation of $\omega(q)$.

$$\sum_{n\geq 1} p_\omega(n) q^n = \sum_{n\geq 1} \frac{q^n}{(q^n)_{n+1}(q^{2n+2};q^2)_\infty} = q\omega(q).$$

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Mock theta函数的Hecke型双重和

杨继真[1], 宋捍飞[2], 张之正[1,3]

(1. 洛阳师范学院数学科学学院, 洛阳 河南 471934)

(2. 上海师范大学数理学院, 上海 200234)

(3. 河南师范大学数学与信息科学学院, 新乡 河南 453001)

摘 要: 本文研究了mock theta函数对应的Hecke型双重和的问题. 利用Bailey对的方法, 获得了三个与七阶和十阶mock theta函数相关的等式和两个新的与三阶mock theta函数相关的Hecke型双重和.

关 键 词: mock theta 函数; Hecke型双重和; Bailey对; q-级数

MR(2010)**主题分类号:** 33D15; 11F03 **中图分类号:** O173