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THE SPREADING SPEED FOR A NONLOCAL DIFFUSIVE PREDATOR-PREY MODEL WITH ONE PREDATOR AND TWO PREYS

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Abstract: In this paper, we investigate a nonlocal diffusive predator-prey model with one predator and two preys. Our main concern is the invasion process of the predator into the habitat of two aboriginal preys. By using comparison principle and semigroup theory of reaction diffusion equation, we confirm the asymptotic spreading speed of the predator.

Keywords: predator-prey system; spreading speed; nonlocal diffusion; comparison principle; Invasion

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1 Introduction

In recent years, the reaction-diffusion equation with nonlocal diffusion effect has gradually become a hot research area in the field of materials science and biological mathematics[1–3]. The introduction of nonlocal diffusion terms in mathematical modeling can, in many cases, better describe some natural phenomena. Although many scholars[4, 5] have studied the traveling wave solutions of reaction-diffusion equations with nonlocal diffusion effect, there is little research on invading phenomenon in reaction-diffusion systems.

Recently, Ducrot et.al[6] have investigated a two-species predator-prey model with nonlocal dispersal as follows

$$\begin{cases} \frac{\partial U(x,t)}{\partial t} = d_1\mathcal{N}_1[U(\cdot,t)](x) + r_1U(x,t)(1-U(x,t)) - aU(x,t)V(x,t), x\in R, t\geq 0, \\ \frac{\partial V(x,t)}{\partial t} = d_2\mathcal{N}_2[V(\cdot,t)](x) + bU(x,t)V(x,t) - r_2V(x,t)(1+V(x,t)), x\in R, t\geq 0, \end{cases} \tag{1.1}$$

in which the terms $d_1\mathcal{N}_1$ and $d_2\mathcal{N}_2$ describe the spatial dispersal of the prey and the predator, other coefficients are nonnegative constants. $\mathcal{N}_i$ is the linear nonlocal diffusion operator defined by

$$\mathcal{N}_i[\varphi](x) := (J_i * \varphi)(x) - \varphi(x) = \int_R J_i(x-y)\varphi(y)dy - \varphi(x), \quad i=1,2,$$

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wherein $J_i, i = 1, 2$ are probability kernel functions and the symbol $*$ denotes the concolution product relative to the space variable, $x \in R$ (We also refer readers to specific definitions by [7] while be given later). Under certain conditions, they obtained the spreading speed c_1 of predator invading native prey habitat as follow

$$c_1 := \inf_{0<\lambda<\bar{\lambda}_2} \frac{d_2[\int_R J_2(y)e^{\lambda y}dy - 1] + b - r_2}{\lambda}.$$

In fact, due to the diversity and complexity of ecosystems, it is more practical to study the interaction between multiple species. However, when there are more than two species, it becomes more difficult to study the ecosystem. Chin-Chin Wu [8] has studied the following three-component reaction diffusion system

$$\begin{cases} \frac{\partial u(x,t)}{\partial t} = d_1\Delta u(x,t) + r_1 u(x,t)[1 - u(x,t) - a_1 v(x,t) - b_1 w(x,t)], x \in R, t \geq 0, \\ \frac{\partial v(x,t)}{\partial t} = d_2\Delta v(x,t) + r_2 v(x,t))[1 - a_2 u(x,t) - v(x,t) - b_2 w(x,t)], x \in R, t \geq 0, \\ \frac{\partial w(x,t)}{\partial t} = d_3\Delta w(x,t) + r_3 w(x,t)[-1 + b_1 u(x,t) + b_2 v(x,t) - w(x,t)], x \in R, t \geq 0, \end{cases} \tag{1.2}$$

where $d_i, r_i, a_i, b_i, i = 1, 2, 3$ are positive constants. When the above system satisfies a series of conditions such as $a_1, a_2 < 1$ and $b_1 = b_2 = b > 1$, the author has characterized the asymptotic spreading speed c_2 of the predator with

$$c_2 := 2\sqrt{d_3 r_3(-1 + b(\frac{2 - a_1 - a_2}{1 - a_1 a_2}))}.$$

Remark 1.1 In the system (1.2), the author assumes that the predator has the same predation rate to the two kinds of preys and it is also the conversion rate of the predator's absorption. These hypothetical conditions make the predator-prey model too idealistic. Moreover, the author does not consider the spatial nonlocal diffusion. For the system (1.1), the authors consider the spatial nonlocal diffusion effect, but the model only has two species. So it is reasonable for us to consider the model of three species interaction. To the author's knowledge, the asymptotic spreading speed of three species model with nonlocal diffusion effect is still a problem to be solved.

In this paper, we consider the following three-component reaction-diffusion system with nonlocal dispersal

$$\begin{cases} \frac{\partial u_1(x,t)}{\partial t} = d_1\mathcal{N}_1[u_1(\cdot,t)](x) + r_1 u_1(x,t)[1 - u_1(x,t) - a_1 u_2(x,t) - b_1 u_3(x,t)], x \in R, t \geq 0, \\ \frac{\partial u_2(x,t)}{\partial t} = d_2\mathcal{N}_2[u_2(\cdot,t)](x) + r_2 u_2(x,t)[1 - a_2 u_1(x,t) - u_2(x,t) - b_2 u_3(x,t)], x \in R, t \geq 0, \\ \frac{\partial u_3(x,t)}{\partial t} = d_3\mathcal{N}_3[u_3(\cdot,t)](x) + r_3 u_3(x,t)[-1 + a_3 u_1(x,t) + b_3 u_2(x,t) - u_3(x,t)], x \in R, t \geq 0, \end{cases} \tag{1.3}$$

$$\mathcal{N}_i[\varphi](x) := (J_i * \varphi)(x) - \varphi(x) = \int_R J_i(x-y)\varphi(y)dy - \varphi(x), \quad i = 1, 2, 3,$$

where $d_i, r_i, a_i, b_i, i = 1, 2, 3$ are positive constants, u_1, u_2 and u_3 represent the population density of preys and predator at space x and time t, respectively. The parameters r_1 and r_2 represent the growth rates of u_1 and u_2, r_3 is the death rate of u_3, $a_1$and a_2 represent the interspecific competition coefficient of two kinds of preys, r_1b_1 and r_2b_2 are the predation rates, and r_3a_3, r_3b_3 are the conversion rates.

It's noting that system (1.3) represents a model of a three-species system in which two preys compete with each other and are preyed upon by a predator. Such systems occur frequently in nature. For example, one population could be the predator such as lady beetles, and the second and third could be prey species such as English grain aphid and the oat-bird cherry aphid[9].

In this paper we study a three-species predator-prey model with nonlocal diffusion. Our main concern is the invasion process of the predator into the habitat of two aborigine preys. Under certain conditions, we are able to characterize the asymptotic spreading speed by the use of comparison principle and semigroup theory. Moverover, we can get the same results as system (1.2) by degenerating our system.

2 The Spreading Speed of the Predator

For the sake of describing the process of species invasion, we need to introduce some measures such as the asymptotic spreading speed.

Definition 2.1 [10] Let $z(x,t)$ be nonnegative for $x \in R, t > 0$. Then s^* is called the spreading speed of $z(x,t)$ when

$$\lim_{t\longrightarrow\infty}\sup\sup_{|x|>(s^*+\epsilon)t} z(x,t) = 0, \qquad \lim_{t\longrightarrow\infty}\inf\inf_{|x|<(s^*-\epsilon)t} z(x,t) > 0, \qquad \forall\epsilon \in (0,s^*).$$

In this paper, we assume that $J_i, i = 1,2,3$ satisfies the following definition.

Definition 2.2 [11] Let $\overline{\lambda} \in (0,\infty]$ be given. We say that the kernel function $J : R \longrightarrow R$ belongs to the class $\tau(\overline{\lambda})$ if it satisfies the following properties:

(J1) The kernel J is nonnegative and continuous in R;

(J2) For all $x \in R$ it holds that

$$\int_R J(y)dy = 1 \quad and \quad J(y) = J(-y);$$

(J3) it holds that $\int_R J(y)e^{\lambda y}dy < \infty$ for any $\lambda \in (0,\overline{\lambda})$ and

$$\int_R J(y)e^{\lambda y}dy \longrightarrow \infty \quad as \quad \lambda \longrightarrow \overline{\lambda}.$$

Let $d > 0, r > 0, s > 0, \overline{\lambda} \in (0,\infty]$ and $J \in \tau(\overline{\lambda})$. First, we consider the following nonlocal logistic equation

$$\begin{cases} \frac{\partial z(x,t)}{\partial t} = d\mathcal{N}[z(\cdot,t)](x) + rz(x,t)[s - z(x,t)], \\ z(x,0) = z(x), \end{cases} \tag{2.1}$$

where $\mathcal{N}[w] =: J * w - w$ and $0 < z(x) \leq s$ is a bounded and continuous function with nonempty support, for the scalar nonlocal equation enjoys the following comparison principle.

Lemma 2.1 [12] Let $z(x,t)$ be a solution of (2.1), $z(\cdot,t), t>0$ and $0<z(x)\leq s$ are continuous and bounded for $x\in R$, $t>0$. Assume that $0<w(x,0)\leq s$ and $w(x,t)$ are continuous and bounded for $x\in R$, $t>0$, if they satisfy

$$\begin{cases} \frac{\partial w(x,t)}{\partial t} \geq (\leq) d\mathcal{N}[w(\cdot,t)](x) + rw(x,t)[s-w(x,t)], x\in R, t>0, \\ w(x,0)\geq(\leq) z(x), x\in R, \end{cases}$$

then $w(x,t)\geq(\leq) z(x,t)$, $x\in R$, $t>0$.

Lemma 2.2 [12] Let $z(x,t)$ be a solution of (2.1), $z(\cdot,t)$ is continuous and bounded for all $t>0$ for a given $0<z(x)\leq s$. If $z(x)$ has a nonempty compact support. Then we have

$$\lim_{t\longrightarrow\infty}\sup\sup_{|x|>(c_3+\epsilon)t} z(x,t)=0, \qquad \lim_{t\longrightarrow\infty}\inf\inf_{|x|<(c_3-\epsilon)t} z(x,t)=s, \qquad \forall\epsilon\in(0,c_3),$$

where

$$c_3 := \inf_{0<\lambda<\bar{\lambda}} \frac{d[\int_R J(y)e^{\lambda y}dy - 1] + rs}{\lambda}.$$

In this paper, due to the weak competition between two preys, we consider the following initial conditions of the system (1.3)

$$(u_1(x,0),u_2(x,0),u_3(x,0)) = (u_1^*, u_2^*, v(x)) =: (\frac{1-a_2}{1-a_1a_2}, \frac{1-a_1}{1-a_1a_2}, v(x)) \quad x\in R, \tag{2.2}$$

where $v(x)$ is a nonnegative continuous function with nonempty compact support. Throughout of this paper, we always assume that $0<v(x)\leq \alpha := a_3u_1^* + b_3u_2^* - 1$.

Because of the nonnegativity of u_1, u_2 and u_3, we can easily get $u_1(x,t)\leq 1$, $u_2(x,t)\leq 1$ for all $(x,t)\in R\times[0,\infty)$ by the comparison principle. Hence $u_3(x,t)$ satisfies the equation

$$\frac{\partial u_3(x,t)}{\partial t} \leq d_3\mathcal{N}_3[u_3(\cdot,t)](x) + r_3u_3(x,t)[-1+a_3+b_3-u_3(x,t)].$$

By Lemma 2.1, we have $u_3(x,t)\leq -1+a_3+b_3$.

Further by a comparison, we obtain

$$u_1(x,t) \geq 1-a_1-b_1(-1+a_3+b_3) =: \underline{u}_1, \tag{2.3}$$

$$u_2(x,t) \geq 1-a_2-b_2(-1+a_3+b_3) =: \underline{u}_2. \tag{2.4}$$

Since the asymptotic spreading involves long time behavior, we have

$$u_3 \geq a_3\underline{u}_1 + b_3\underline{u}_2 - 1 =: \underline{u}_3. \tag{2.5}$$

Next, we discuss the spreading speed of the predator $u_3(x,t)$ in system (1.3) with following conditions

$$\underline{u}_i > 0, \quad i=1,2,3, \quad 1-a_1\underline{u}_2 - b_1\underline{u}_3 \leq u_1^*, \quad 1-a_2\underline{u}_1 - b_2\underline{u}_3 \leq u_2^*. \tag{2.6}$$

We now state our main result on the spreading speed of the predator as follows.

According to the Definition 2.2, we know there exists $\lambda_3 \in (0, \infty]$ such that $J_3 \in \tau(\overline{\lambda_3})$. Then we define the quantity

$$c^* := \inf_{0<\lambda<\bar{\lambda}_3} \frac{d_3[\int_R J_3(y)e^{\lambda y}dy - 1] + r_3\alpha}{\lambda}.$$

Theorem 2.1 Assume that conditions of (2.6) are enforced. Let $(u_1(x,t), u_2(x,t), u_3(x,t))$, $x \in R$, $t > 0$ be a solution of (1.3) with initial data (2.2). As long as $v(x)$ is a non-zero compactly supported continuous function with $0 \le v(x) \le \alpha$, then the density of the predator $u_3(x,t)$ satisfies

$$\lim_{t\longrightarrow\infty} \sup_{|x|>ct} u_3(x,t) = 0 \quad , c > c^*, \tag{2.7}$$

$$\lim_{t\longrightarrow\infty} \inf \inf_{|x|<ct} u_3(x,t) > 0, c \in (0, c^*). \tag{2.8}$$

Now we first prove (2.7). On the base of the results of (2.3) – (2.6), we have

$$0 \le u_1(x,t) \le u_1^*, \quad 0 \le u_2(x,t) \le u_2^*.$$

Further, $u_3(x,t)$ satisfies

$$\frac{\partial u_3(x,t)}{\partial t} \le d_3\mathcal{N}_3[u_3(\cdot,t)](x) + r_3u_3(x,t)[\alpha - u_3(x,t)], \quad x \in R, t > 0.$$

By Lemma 2.1 we obtain $u_3(x,t) \le \widetilde{u}_3(x,t), (x,t) \in R\times[0,\infty)$, where $\widetilde{u}_3(x,t)$ is the solution to

$$\begin{cases} \frac{\partial \widetilde{u}_3(x,t)}{\partial t} = d_3\mathcal{N}_3[\widetilde{u}_3(\cdot,t)](x) + r_3\widetilde{u}_3(x,t)[\alpha - \widetilde{u}_3(x,t)], x \in R, t \ge 0, \\ \widetilde{u}_3(x,0) = v(x), x \in R. \end{cases}$$

It follows from the classical result of Lemma 2.2 that

$$0 \le \lim_{t\longrightarrow\infty} \sup_{|x|>ct} u_3(x,t) \le \lim_{t\longrightarrow\infty} \sup_{|x|>ct} \widetilde{u}_3(x,t) = 0, \quad c > c^*.$$

This proves (2.7) and then we will prove (2.8). From the first formula of system (1.3) and $u_2(x,t) \le u_2^*$, $u_1(x,t)$ satisfies

$$\frac{\partial u_1(x,t)}{\partial t} \ge d_1\mathcal{N}_1[u_1(\cdot,t)](x) + r_1\underline{u}_1[u_1^* - u_1(x,t)] - r_1b_1u_3(x,t), \quad x \in R, t > 0.$$

Set $\overline{u}_1(x,t) = u_1^* - u_1(x,t)$, $x \in R$, $t > 0$, then previous formula can be rewritten as

$$\frac{\partial \overline{u}_1(x,t)}{\partial t} \le d_1\mathcal{N}[\overline{u}_1(\cdot,t)](x) + r_1b_1u_3(x,t) - r_1\underline{u}_1\overline{u}_1(x,t), \quad x \in R, t > 0.$$

Next, by using $\overline{u}_1(x,t) = 0$ at $t = 0$ and the theory of semigroup, we have

$$\overline{u}_1(x,t) \le r_1b_1 \int_0^t e^{-r_1\underline{u}_1(t-s)}\{exp((t-s)d_1\mathcal{N}_1)[u_3(\cdot,s)]\}(x)ds, \quad x \in R, t > 0. \tag{2.9}$$

Set $\overline{u}_2(x,t) = u_2^* - u_2(x,t)$, similarly, we have

$$\overline{u}_2(x,t) \le r_2 b_2 \int_0^t e^{-r_2 \underline{u}_2 (t-s)} \{exp((t-s)d_2\mathcal{N}_2)[u_3(\cdot,s)]\}(x)ds, \quad x \in R, t > 0. \tag{2.10}$$

It follows from the third formula of system (1.3) that

$$\frac{\partial u_3(x,t)}{\partial t} = d_3\mathcal{N}_3[u_3(\cdot,t)](x) + r_3 u_3(x,t)[\alpha - a_3\overline{u}_1(x,t) - b_3\overline{u}_2(x,t) - u_3(x,t)], \quad x \in R, t \ge 0. \tag{2.11}$$

For any given constant $\epsilon \in (0, c^*)$, we can choose constant $\delta \in (0,\alpha)$ small enough such that

$$\inf_{0<\lambda<\bar{\lambda}_3} \frac{d_3[\int_R J_3(y)e^{\lambda y}dy - 1] + (\alpha-\delta)r_3}{\lambda} > c^* - \epsilon.$$

We first assume that there exist $M_1 > 0, M_2 > 0$ and $\tau_1 > 0, \tau_2 > 0$ large enough such that

$$a_3\overline{u}_1(x,t) \le \frac{\delta}{2} + M_1 u_3(x,t) \quad x \in R, t \ge \tau_1, \tag{2.12}$$

$$b_3\overline{u}_2(x,t) \le \frac{\delta}{2} + M_2 u_3(x,t) \quad x \in R, t \ge \tau_2. \tag{2.13}$$

Then, combining (2.12), (2.13) with the third formula, we obtain

$$\frac{\partial u_3(x,t)}{\partial t} \ge d_3\mathcal{N}_3[u_3(\cdot,t)](x) + r_3 u_3(x,t)[\alpha-\delta-(M_1+M_2)u_3(x,t)], x \in R, t \ge \tau := \max\{\tau_1,\tau_2\}.$$

By the comparison principle, we have $u_3(x,t+\tau) \ge \underline{U}_3(x,t), x \in R, t \ge 0$, where $\underline{U}_3(x,t)$ is the solution to

$$\begin{cases} \frac{\partial \underline{U}_3(x,t)}{\partial t} = d_3\mathcal{N}_3[\underline{U}_3(\cdot,t)](x) + r_3\underline{U}_3(x,t)[\alpha-\delta-(M_1+M_2)\underline{U}_3(x,t)], \\ \underline{U}_3(x,0) = v(x). \end{cases}$$

Finally, combining Lemma 2.2 and Lemma 2.3, we obtain

$$\lim_{t\longrightarrow\infty} \inf \inf_{|x|<ct} u_3(x,t) \ge \lim_{t\longrightarrow\infty} \inf \inf_{|x|<ct} \underline{U}_3(x,t) = \frac{\alpha-\delta}{M_1+M_2+1} > 0, c \in (0, c^*-\epsilon).$$

Hence, this completes the proof of (2.8) since ϵ is arbitrary.

To complete the proof of Theorem 1.1, it remains to derive (2.12) and (2.13). In the following, we only prove the formula (2.12). In order to achieve our goal, we need to know more detailed properties about the strongly positive semigroup $T_i(t) := exp(td_i\mathcal{N}_i)$.

Let $Z(x,t)$ be a solution of the following problem

$$\begin{cases} \frac{\partial Z(x,t)}{\partial t} = d_i\{[J_i Z(\cdot,t)](x) - Z(x,t)\}, \\ Z(x,0) = \delta_0(x), \end{cases}$$

wherein $\delta_0(x)$ denotes the Dirac mass at $x=0$. According to[12], $Z(x,t)$ can be decomposed into the following forms

$$Z(x,t) = e^{-d_1 t}\delta_0(x) + K(x,t), \quad x \in R, t \ge 0.$$

where $K(x,t)$ is a nonnegative smooth function and $\int_R K(x,t)dx \le 2$, $t \ge 0$.

For any bounded and continuous function $0 \le \phi(x)$, the semigroup $T_i(t)$, $t \ge 0$ can be expressed as

$$T_i(t)[\phi](x) = e^{-d_i t}\phi(x) + \int_R K(x-y,t)\phi(y)dy, \quad t \ge 0.$$

Next we start to prove the formula of (2.12). By (2.9), we have

$$\begin{aligned} a_3\overline{u}_1(x,t) \le& r_1b_1a_3[\int_0^t \{e^{-(d_1+r_1\underline{u}_1)(t-s)}u_3(x,s) + \int_R e^{-r_1\underline{u}_1(t-s)}K(x-y,t-s)u_3(y,s)dy\}ds] \\ =:& V_1(x,t)+V_2(x,t) \quad x \in R, t>0. \end{aligned} \tag{2.14}$$

Set $\beta_1 = d_1 + r_1\underline{u}_1, \beta_2 = r_1\underline{u}_1$, then $V_1(x,t), V_2(x,t), x \in R, t > 0$ can be rewritten as

$$\begin{aligned} V_1(x,t) =& r_1b_1a_3\int_0^t e^{-\beta_1(t-s)}u_3(x,s)ds, \\ V_2(x,t) =& r_1b_1a_3\int_0^t\int_R e^{-\beta_2(t-s)}K(x-y,t-s)u_3(y,s)dyds. \end{aligned} \tag{2.15}$$

For the fixed positive constant δ, there is a large enough constant τ_1 such that

$$r_1b_1a_3\alpha\int_0^{t-\tau_1} e^{-\beta_1(t-s)}ds \le \frac{\delta}{8}, \quad 2r_1b_1a_3\alpha\int_0^{t-\tau_1} e^{-\beta_1(t-s)}ds \le \frac{\delta}{8}. \tag{2.16}$$

For all $x \in R, t \ge \tau_1$,if $a_3\overline{u}_1(x,t) \le \frac{\delta}{2}$, then it is obvious that the formula (2.12) is tenable. Otherwise, there is a point (x_0,t_0) with $t_0 \ge \tau_1$ such that $a_3\overline{u}_1(x_0,t_0) > \frac{\delta}{2}$.

In the circumstances, we deduce from (2.14) $-$ (2.16) that

$$a_3\overline{u}_1(x_0,t_0) \le \frac{\delta}{4}+r_1b_1a_3[\int_{t_0-\tau_1}^{t_0}\{e^{-\beta_1(t-s)}u_3(x_0,s)+\int_R e^{-\beta_2(t_0-s)}K(x_0-y,t-s)u_3(y,s)dy\}ds]. \tag{2.17}$$

Further

$$r_1b_1a_3\int_0^{\tau_1} e^{-\beta_1 l}u_3(x_0,t_0-l)dl + r_1b_1a_3\int_0^{\tau_1}\int_R e^{-\beta_2 l}K(y,l)u_3(x_0-y,t_0-l)dydl \ge \frac{\delta}{4},$$

where $l = t_0 - s$. Next, we choose $R > 0$ such that

$$r_1b_1a_3\int_0^{\tau_1} e^{-\beta_1 l}u_3(x_0,t_0-l)dl + r_1b_1a_3\int_0^{\tau_1}\int_{-R}^{R} e^{-\beta_2 l}K(y,l)u_3(x_0-y,t_0-l)dydl \ge \frac{\delta}{8}.$$

Note that R is independent of (x_0,t_0). Then we choose $\eta > 0$ small enough such that

$$r_1b_1a_3\alpha\int_0^{\eta} e^{-\beta_1 l}dl + r_1b_1a_3\alpha\int_0^{\eta}\int_{-R}^{R} e^{-\beta_2 l}K(y,l)dydl \le \frac{\delta}{16}. \tag{2.18}$$

It follows from (2.17) that

$$r_1b_1a_3\int_\eta^{\tau_1} e^{-\beta_1 l}u_3(x_0,t_0-l)dl + r_1b_1a_3\int_\eta^{\tau_1}\int_{-R}^{R} e^{-\beta_2 l}K(y,l)u_3(x_0-y,t_0-l)dydl \ge \frac{\delta}{16}.$$

Then choose $\theta > 0$ such that

$$\frac{\delta}{16} = \theta[r_1 b_1 a_3 \int_{\eta}^{\tau_1} e^{-\beta_1 l} dl + r_1 b_1 a_3 \int_{\eta}^{\tau_1} \int_{-R}^{R} e^{-\beta_2 l} K(y,l) dy dl],$$

there exist $l_0 \in [t_0 - \tau_1, t_0 - \eta]$ and $y_0 \in [x_0 - R, x_0 + R]$ such that $u_3(y_0, l_0) \geq \theta$. Moreover, since the function $u_3(x,t)$is uniformly continuous on $R \times [0, \infty)$, there exists $\rho > 0$ independent of (y_0, l_0) such that $u_3(y, l_0) \geq \frac{\theta}{2}$, $\forall y \in [y_0 - \rho, y_0 + \rho]$.

We then consider the solution $w(x,t)$ to

$$\begin{cases} \frac{\partial w(x,t)}{\partial t} = d_3 \mathcal{N}_3[w(\cdot,t)](x) - r_3(1+\alpha) w(x,t), \\ z(x,0) = w(x), \end{cases}$$

where $w(x)$ is a uniformly continuous function with $w(x) \leq \frac{\rho}{2}$ such that

(i) $w(x) = \frac{\theta}{2}, \quad |x| \leq \frac{\rho}{2};$

(ii) $w(x) = 0, \quad |x| \geq \rho;$

(iii) if $x \in [\frac{\rho}{2}, \rho]$, then $w(x)$ is decreasing, if $x \in [-\rho, -\frac{\rho}{2}]$, then $w(x)$ is increasing.

By the comparison principle, we have $w(x,t) \leq u_3(y_0 + x, l_0 + t)$, $x \in R$, $t \geq 0$.

Moreover, set $w(x,t) = e^{r_3(1+\alpha)t} exp(d_3 t \mathcal{N}_3)[w](x)$, we have $w(x,t) > 0$, $x \in R$, $t \geq 0$. Thus one obtains that

$$u_3(x_0, t_0) \geq w(x_0 - y_0, t_0 - l_0) \geq \gamma := \min_{t \in [\eta, \tau_1] \quad |x| \in [-R, R]} w(x,t) > 0. \tag{2.19}$$

Further then it follows from (2.18) and (2.19) that

$$a_3 \overline{u}_1(x_0, t_0) \leq \frac{\delta}{2} + \frac{\alpha r_1 b_1 a_3 (2\beta_1 + \beta_2)}{\beta_1 \beta_2} \leq \frac{\delta}{2} + M_1 u_3(x_0, t_0),$$

by setting $M_1 = \frac{\alpha r_1 b_1 a_3 (2\beta_1 + \beta_2)}{\beta_1 \beta_2 \gamma}$. Since (x_0, t_0) is arbitrary, (2.12) is proved.

3 Example

Example 3.1 Suppose $J_3 = \frac{4}{3}(1 - x^2), | x |< 1$ and $J_3 = 0, | x |\geq 1$. If the coefficient of system (1.3) with $a_1 = a_2 = 0.2$, $a_3 = 2.4$, $b_1 = b_2 = 0.1$, $b_3 = 2.6$, $d_i = 2$ and $r_i = 0.3$, then we can obtain a three-species predator-prey competition system with nonlocal diffusion. By Theorem 2.1, we can calculate the asymptotic spreading speed c_4 of the predator with

$$c_4 := \inf_{0<\lambda<\infty} \frac{16(\lambda - 1)e^{\lambda} + 16(\lambda + 1)e^{-\lambda} - 1.05\lambda^3}{3\lambda^4} \approx 2.3820.$$

Example 3.2 Suppose $J_3 = \frac{1}{2}e^{-|x|}$, $a_1 = a_2 = 0.2$, $a_3 = b_3 = 2.5$, $b_1 = b_2 = 0.1$, $d_i = 2$ and $r_i = 0.3$, then we can obtain

$$c_5 := \inf_{0<\lambda<1} \frac{-1.05\lambda^2 - 0.95}{\lambda(\lambda - 1)(\lambda + 1)} \approx 9.7959.$$

Remark 3.1 System (1.1) is a special case of system (1.3). Assume the coefficient $a_1 = a_2 = b_2 = b_3 = 0$, $b_1 = \frac{a}{r_1}$ and $a_3 = \frac{b}{r_2}$, we can use the Theorem 2.1 to get the same result as [6].

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具有非局部扩散的三种群捕食竞争系统的渐近传播速度

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摘要: 本文研究了一类具有非局部扩散效应的三种群捕食-食饵模型. 我们主要关注的问题是当捕食者被引入两食饵物种栖息地时, 在栖息地中入侵的渐近传播速度. 通过利用反应扩散方程的比较原理和半群理论等理论, 确定了捕食者入侵的渐近传播速度.

关键词: 捕食食饵模型; 渐近传播速度; 非局部扩散; 比较原理; 入侵

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