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NORM STRUCTURES OF A FUZZY NORMED SPACE

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Abstract: In this paper, the problem of norm structures in the fuzzy normed space is studied. By using the "cut" method and introducing K function, we discuss the norm structures existing in the fuzzy normed space with continuous t-norm in a broader sense, and give the relations between these norm structures. The obtained results generalize the existing conclusions and provide a new way for the further research of the fuzzy normed space.

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1 Introduction

Inspired by the notion of probabilistic metric spaces, Kramosil and Michalek [1] in 1975 introduced the notion of fuzzy metric, a fuzzy set in the Cartesian product$X \times X \times (-\infty + \infty)$ satisfying certain conditions. Later, George and Veeramani [2] used the concept of continuous t-norm to modify this definition of fuzzy metric space and showed that every fuzzy metric space generates a Hausdorff first countable topology. In 1994, Cheng and Mordeson [3] introduced an idea of a fuzzy norm on a linear space in such a manner that the corresponding fuzzy metric is of Kramosil and Michalek type [1]. Following Cheng and Mordeson, Bag and Samanta [4] introduced a similar definition of fuzzy norm. The novelty of this definition is the validity of a decomposition theorem for a special type of fuzzy norm into a family of crisp norms. This concept was used in fuzzy functional analysis and its applications [5–8]. In [9], Sadeqi and Kia proved that a separating family of seminorms introduces a fuzzy norm in general but it is not true in classical analysis. In [10], Alegre and Romaguera also dealt with fuzzy normed spaces in the sense of [3], characterized fuzzy norms in terms of a non-decreasing and separating family of seminorms, and generalized the classical Hahn-Banach extension theorem for normed spaces. In addition to this, Bag and Samanta established some principles of functional analysis in fuzzy settings, which represent a foundation for the development of fuzzy functional analysis. Some other conclusions can be found in [11–16].

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In this paper, we introduce the concept of r-norm and show some norm structures in a fuzzy normed space. Moreover, we investigate the relationships between the normed topology and the topology induced by these norm structures. The structure of the paper is as follows. In the next section, we give the preliminaries on a fuzzy normed space. In Section 3, we show our main results.

In this paper, R is the set of all real numbers, R^+ is the set of all positive real numbers, X is a real linear space.

2 Preliminaries

In this section, we first recall some basic concepts of fuzzy normed spaces.

Definition 2.1 (see [17]) A binary operation $*$: $[0,1]\times[0,1]\to[0,1]$ is a continuous t-norm, if $*$ satisfies the following conditions: $\forall a,b,c,d\in[0,1]$,

(1) commutative $a*b=b*a$;

(2) associative $(a*b)*c=a*(b*c)$;

(3) $a*b\le c*d$ whenever $a\le c$ and $b\le d$;

(4) $a*1=a$;

(5) $*$ is continuous.

Lemma 2.1 (see [17]) Let $*$: $[0,1]\times[0,1]\to[0,1]$ be a continuous t-norm.

(1) If $1>r_1>r_2>0$, then there exists $r_3\in(0,1)$ such that $r_1*r_3\ge r_2$;

(2) If $r_4\in(0,1)$, then there exists $r_5\in(0,1)$ such that $r_5*r_5\ge r_4$.

We can strengthen Lemma 2.1 to the following form.

Lemma 2.2 Let $*$: $[0,1]\times[0,1]\to[0,1]$ be a continuous t-norm.

(1) If $1>r_1>r_2>0$, then there exists $r_3\in(0,1)$ such that $r_1*r_3>r_2$;

(2) If $r_4\in(0,1)$, then there exists $r_5\in(r_4,1)$, such that $r_5*r_5>r_4$.

Proof (1) Let $1>r_1>r_2>0$. Take $r\in(r_2,r_1)$. From Lemma 2.1 (1), there exists $r_3\in(0,1)$ such that $r_1*r_3\ge r$. Thus $r_1*r_3>r_2$.

(2) Let $r_4\in(0,1)$. Take $r\in(r_4,1)$. From Lemma 2.1 (2), there exists $r_5\in(0,1)$ such that $r_5*r_5\ge r$. Thus $r_5*r_5>r_4$.

In this paper, the definition of a fuzzy normed linear space in [4] is changed accordingly, and the following definition is given.

Definition 2.2 Let X be a linear space, $*$ be a continuous t-norm, N be a fuzzy subset of $X\times(0,+\infty)$. N is called a fuzzy norm on X if the following conditions are satisfied: for all $x,y\in X$,

(1) $\forall t>0$, $N(x,t)>0$;

(2) ($\forall t>0$, $N(x,t)=1$) iff $x=\theta$, where θ is the zero element of X;

(3) $\forall t>0$, $N(\lambda x,t)=N\left(x,\frac{t}{|\lambda|}\right)$ if $\lambda\ne 0$;

(4) $\forall t,s>0$, $N(x,t)*N(y,s)\le N(x+y,t+s)$;

(5) $N(x,\cdot):(0+\infty)\to(0,1]$ is continuous.

The 3-tuple $(X,N,*)$ is said to be a fuzzy normed linear space. Obviously, if N is a fuzzy norm, then $N(x,\cdot)$ is non-decreasing for all $x\in X$.

Remark 2.1 If (1) and (5) are replaced by

(N1) $\forall t \in R$ with $t \leq 0$, $N(x,t) = 0$;

(N5) $N(x,\cdot)$ is a non-decreasing function of R and $\lim\limits_{t\to\infty} N(x,t) = 1$; respectively, then N is a fuzzy norm in the sense of [4].

Example 2.1 Let X be a linear normed space, $N : X \times (0,+\infty) \to (0,1]$ is defined by $N(x,t) = \frac{t}{t+\|x\|}$, then N is a fuzzy norm on X.

In the rest of this paper, we always suppose the function $K : [0,1] \to [0,1]$ satisfies the following conditions: $K(0) = 0$, $K(t) \not\equiv 0$, K is increasing and continuous at 0.

Theorem 2.1 Let $(X,N,*)$ be a fuzzy normed linear space. For $x \in X$, $r \in (0,1)$, $t > 0$, we define

$$B_N(x,r,t) = \{y \in X : N(x-y,t) + K(r) > 1\}.$$

Then $\{B_N(x,r,t) : r \in (0,1), t > 0\}$ is a base of neighborhoods at x.

Proof (1) $\forall x \in X$, $t > 0$, $r \in (0,1)$, $x \in B_N(x,r,t)$.

(2) $\forall x \in X$, $0 < r_1, r_2 < 1$, $t_1, t_2 > 0$, there exists $r_3 = \min\{r_1, r_2\}$, $t_3 = \{t_1, t_2\}$, from the non-decreasing of $N(x,\cdot)$, we have $B_N(x,r_3,t_3) \subseteq B_N(x,r_2,t_2)$, $B_N(x,r_3,t_3) \subseteq B_N(x,r_1,t_1)$, so $B_N(x,r_3,t_3) \subseteq B_N(x,r_2,t_2) \cap B_N(x,r_1,t_1)$.

(3) For any $B_N(x,r,t)$, from Lemma 2.2, we have $0 < r_1 < r$ such that $(1-K(r_1)) * (1-K(r_1)) > 1-K(r)$. Let $y \in B_N(x,r_1,t) \subseteq B_N(x,r,t)$, we know $N(x-y,t) + K(r_1) > 1$. Since $N(x-y,\cdot)$ is continuous, we can take $\delta > 0$ such that $N(x-y,t-\delta) + K(r_1) > 1$. Therefore, for any $z \in B_N(y,r_1,\delta)$, we have $N(y-z,\delta) + K(r_1) > 1$ and

$$N(x-z,t) + K(r) \geq N(x-y,t-\delta) * N(y-z,\delta) + K(r)$$
$$\geq (1-K(r_1)) * (1-K(r_1)) + K(r) > 1-K(r) + K(r) = 1.$$

Thus, $z \in B_N(x,r,t)$. From the arbitrariness of z, we know $B_N(y,r_1,\delta) \subseteq B_N(x,r,t)$. From (1)–(3), we can conclude that $\{B_N(x,r,t) : x \in X,\ r \in (0,1),\ t > 0\}$ forms a base of neighborhoods at $x \in X$.

Based on the $\{B_N(x,r,t) : x \in X,\ r \in (0,1),\ t > 0\}$, we have a topology τ_N which is said to be the topology generated by the fuzzy norm N. It is easy to see the topology τ_N is the first countable. In fact, the countable collection of balls $\{B_N(x, 1/n, 1/n) : x \in X,\ n = 2,3,...\}$ forms a base of neighborhoods at $x \in X$.

3 Main Results

In this section, we shall introduce norm structures in a fuzzy normed space, and investigate the relationships between the fuzzy topology τ_N and the topologies induced by these norm structures. For simplicity, we always suppose a fuzzy normed space $(X,N,*)$ satisfies the regular condition: $\forall x \in X$, there exists $t > 0$ such that $N(x,t) = 1$.

Theorem 3.1 Let $(X,N,*)$ be a fuzzy normed space. Define a function $\|\cdot\|_0$ on X as follows

$$\|x\|_0 = \inf\{t > 0, N(x,t) = 1\}, \forall x \in X, \tag{3.1}$$

then $\|\cdot\|_0$ is a norm on X.

Proof Obviously, for all $x \in X$, $\|x\|_0 \geq 0$, $\|x\|_0 = 0$ if and only if $x = \theta$. Now, for any $x, y \in X$, $\varepsilon > 0$, from the definition of $\|\cdot\|_0$, we get that

$$N\left(x, \|x\|_0 + \frac{\varepsilon}{2}\right) = 1, N\left(y, \|y\|_0 + \frac{\varepsilon}{2}\right) = 1.$$

Therefore

$$N\left(x+y, \|x\|_0+\|y\|_0 + \varepsilon\right) \geq N\left(x, \|x\|_0 + \frac{\varepsilon}{2}\right) * N\left(y, \|y\|_0 + \frac{\varepsilon}{2}\right) = 1,$$

and hence $\|x+y\|_0 \leq \|x\|_0 + \|y\|_0+\varepsilon$. From the arbitrariness of $\varepsilon > 0$, we have

$$\|x+y\|_0 \leq \|x\|_0 + \|y\|_0.$$

Additionally, $\forall \alpha \in R\setminus\{0\}$, we have

$$\|\alpha x\|_0 = \inf\{t > 0, N(\alpha x, t) = 1\} = |\alpha| \inf\left\{\frac{t}{|\alpha|} > 0, N\left(x, \frac{t}{|\alpha|}\right) = 1\right\} = |\alpha| \, \|x\|_0.$$

Obviously, $\|\alpha x\|_0 = |\alpha| \, \|x\|_0$ if $\alpha = 0$. Thus $\|\cdot\|_0$ is a norm on X.

Theorem 3.2 $(X, N, *)$ is a fuzzy normed space, $\|\cdot\|_0$ is the norm defined by (3.1). Let $r \in (0, 1]$, $x \in X$,

$$\|x\|_r = \inf\{t > 0 : N(x, t) + K(r) \geq 1\}. \tag{3.2}$$

Then, for a fixed point $x \in X$, $\|x\|_r$ is a decreasing function with respect to $r \in (0, 1]$, and

$$\|x\|_0 = \lim_{r\to 0^+} \|x\|_r = \sup_{r\in(0,1]} \|x\|_r. \tag{3.3}$$

Proof Given $x \in X$, $r_1, r_2 \in (0, 1)$ with $r_1 \geq r_2$, we have

$$\{t > 0, N(x, t) + K(r_1) \geq 1\} \supseteq \{t > 0, N(x, t) + K(r_2) \geq 1\},$$

hence $\|x\|_{r_1} \leq \|x\|_{r_2}$, which means $\|x\|_r$ is decreasing with respect to $r \in (0, 1]$. So $\lim\limits_{r\to 0^+} \|x\|_r$ exists and

$$\lim_{r\to 0^+} \|x\|_r = \sup_{r\in(0,1]} \|x\|_r.$$

Moreover, for any $\varepsilon > 0$, $x \in X$, $N(x, \|x\|_0 + \varepsilon) = 1$. From (3.2), we obtain

$$\|x\|_r \leq \|x\|_0 + \varepsilon \text{ for any } r \in (0, 1].$$

From the arbitrariness of $\varepsilon > 0$, we know

$$\sup_{r\in(0,1]} \|x\|_r \leq \|x\|_0. \tag{3.4}$$

On the other hand, for any $\varepsilon > 0$, we know $t > \|x\|_0 - \varepsilon$ whenever $N(x, t) = 1$ from the definition of $\|\cdot\|_0$. That is, $N(x, t) < 1$ on $(0, \|x\|_0 - \varepsilon]$. Recalling that K is increasing and continuous at 0, there exists $r_0 = r_0(\varepsilon)$ such that $N(x,t) + K(r_0) < 1$ on $(0, \|x\|_0 - \varepsilon]$. That is, $t > \|x\|_0 - \varepsilon$ whenever $N(x,t) + K(r_0) \geq 1$. Then we have

$$\|x\|_0 - \varepsilon \leq \inf \{t > 0 : N(x,t) + K(r_0) \geq 1\} \leq \sup_{r\in(0,1]} \inf \{t > 0 : N(x,t) + K(r) \geq 1\},$$

which together with the arbitrariness of ε implies that

$$\|x\|_0 \leq \sup_{r\in(0,1]} \|x\|_r. \tag{3.5}$$

Inequalities (3.4) and (3.5) imply the equation (3.3).

Remark 3.1 In [4], $\|x\|_r$ was defined as $\inf \{t > 0 : N(x,t) \geq r\}$. Therefore, the definition (3.2) of $\|x\|_r$ is a generalization of that in [4]. We call $\|\cdot\|_r$ the r-norm in a fuzzy normed space $(X, N, *)$.

Lemma 3.1 Let $(X, N, *)$ be a fuzzy normed space, $x \in X$. If $N(x, \cdot)$ is strictly increasing, then

$$\|x\|_r = \inf \{t > 0 : N(x,t) + K(r) > 1\} \text{ for any } r \in (0,1). \tag{3.6}$$

Proof For any $r \in (0,1)$, let $t_0 = \inf \{t > 0 : N(x,t) + K(r) > 1\}$. From (3.2), we get $\|x\|_r \leq t_0$. If $\|x\|_r < t_0$, then there is $0 < t_2 < t_0$ such that $N(x, t_2) + K(r) \geq 1$. Since $N(x, \cdot)$ is strictly increasing, then $N(x, t_0) + K(r) > 1$. From the continuity of $N(x, \cdot)$, there is $\delta > 0$ such that $N(x, t_0 - \delta) + K(r) > 1$ which conflicts with the definition of t_0. Thus, $\|x\|_r = t_0 = \inf \{t > 0 : N(x,t) + K(r) > 1\}$.

Lemma 3.2 $(X, N, *)$ is a fuzzy normed space. $N(x, \cdot)$ is strictly increasing for the fixed point $x \in X$. Let $t > 0$ and $r \in (0,1)$. Then $\|x - y\|_r < t$ if and only if $N(x - y, t) + K(r) > 1$, that is $B_N(x, r, t) = N_r(x, t)$, where

$$N_r(x,t) = \{y \in X : \|x - y\|_r < t\}. \tag{3.7}$$

Proof Suppose $\|x - y\|_r < t$. From Lemma 3.1, there exists $0 < t_0 < t$ such that $N(x - y, t_0) + K(r) > 1$. Hence, $N(x - y, t) + K(r) > N(x - y, t_0) + K(r) > 1$. So $B_N(x, r, t) \supseteq N_r(x, t)$.

Now, we suppose $N(x - y, t) + K(r) > 1$. Since $N(x - y, \cdot)$ is continuous, there exists $0 < t_1 < t$ such that $N(x - y, t_1) + K(r) > 1$. From (3.6), we know $\|x - y\|_r \leq t_1 < t$. So $B_N(x, r, t) \subseteq N_r(x, t)$. This completes the proof.

Theorem 3.3 $(X, N, *)$ is a fuzzy normed space, $N(x, \cdot)$ is strictly increasing for the fixed point $x \in X$. Then $\|x\|_r = \|x\|_0$ for $r \in (0,1]$ if and only if N satisfies the following condition: for all $t > 0$,

$$N(x,t) = 1 \text{ whenever } N(x,t) + K(r) > 1. \tag{3.8}$$

Proof The sufficiency is obvious.

To prove the necessity, we suppose that $\|x\|_r = \|x\|_0$ and $N(x,t) + K(r) > 1$. From the definition of $\|x\|_r$, $\|x\|_r \le t$, that is $\|x\|_0 \le t$. For any $\varepsilon > 0$, from the definition of $\|\cdot\|_0$, we get $t' > 0$ such that $t + \varepsilon > t'$ and $N(x,t') = 1$. Therefore, $N(x, t+\varepsilon) = 1$, which together with the continuity of $N(x,\cdot)$ implies that $N(x,t) = 1$.

Theorem 3.4 If $(X, N, *)$ is a fuzzy normed space, then

(i) the topology τ_0 generated by $\|\cdot\|_0$ is stronger than the topology τ_N;

(ii) $\tau_0 = \tau_N$ if and only if N satisfies the following condition: for each $x \in X$, $t > 0$ and $y \in N_0(x,t)$, there exist $r' \in (0,1)$, $t' > 0$ and $0 < s' < t$ such that

$$N(x - z, s') = 1 \text{ whenever } N(z - y, t') + K(r') > 1 \text{ for any } z \in X, \tag{3.9}$$

where

$$N_0(x,t) = \{y \in X : \|x - y\|_0 < t\}. \tag{3.10}$$

Proof (i) To prove $\tau_0 \supseteq \tau_N$, it is sufficient to prove $\{x_n\}$ is convergent to x_0 with respect to τ_N whenever $\{x_n\}$ is convergent to x_0 with respect to τ_0. In fact, if $\{x_n\}$ is convergent to x_0 with respect to τ_0, then for any $\varepsilon > 0$, there exists N such that $\sup\limits_{r \in (0,1]} \|x_n - x_0\|_r < \varepsilon$ for all $n \ge N$. Therefore, for any $r \in (0,1]$, we have $\|x_n - x_0\|_r < \varepsilon$ for all $n \ge N$. From Lemma 3.2, we have $N(x_n - x_0, \varepsilon) + K(r) > 1$ for all $n \ge N$. That is, $x_n \in B_N(x_0, r, \varepsilon)$ for all $n \ge N$. Thus, $\{x_n\}$ is convergent to x_0 with respect to τ_N.

(ii) **Necessity** We suppose that $\tau_0 = \tau_N$. Then, for each $x \in X$ and $t > 0$, $N_0(x,t) \in \tau_0 \subseteq \tau_N$. Hence, for each $y \in N_0(x,t)$, there exist $r' \in (0,1)$ and $t' > 0$ such that $B_N(y, r', t') \subseteq N_0(x,t)$, that is, $\|x - z\|_0 < t$ whenever $N(y - z, t') + K(r') > 1$ for any $z \in X$. Obviously, $\|x - z\|_0 < t$ is equivalent to that there exists $0 < s' < t$ such that $N(x - z, s') = 1$.

Sufficiency From (i), we only have to prove $\tau_0 \subseteq \tau_N$. To do that, it is sufficient to prove $N_0(x,t) \in \tau_N$ for each $x \in X$, $t > 0$. In fact, for any $y \in N_0(x,t)$, by the supposition, there exist $r' \in (0,1)$, $t' > 0$ and $0 < s' < t$ such that (3.9) holds, Which means that $B_N(y, r', t') \subseteq N_0(x,t)$. Thus $N_0(x,t) \in \tau_N$.

Corollary 3.1 $(X, N, *)$ is a fuzzy normed space. $N(x,\cdot)$ is strictly increasing for the fixed point $x \in X$. If N satisfies the following condition: there exists $r' \in (0,1)$ such that for any $t > 0$, $x \in X$,

$$N(x,t) = 1 \text{ whenever } N(x,t) + K(r') > 1. \tag{3.11}$$

Then $\tau_0 = \tau_N$.

Proof Let $x \in X$, $t > 0$ and $y \in N_0(x,t)$ arbitrarily, by the definition of $N_0(x,t)$, there exists $0 < s^* < t$ such that $N(x - y, s^*) = 1$. Take $s^* < s' < t$, $t' = s' - s^*$. By the supposition, when $N(z - y, t') + K(r') > 1$, we have $N(z - y, t') = 1$, and hence

$$N(x - z, s') \ge N(x - y, s^*) * N(y - z, t') = 1 * 1 = 1.$$

From Theorem 3.4 (ii), we know $\tau_0 = \tau_N$.

Theorem 3.5 $(X, N, *)$ is a fuzzy normed space, $N(x, \cdot)$ is strictly increasing for the fixed point $x \in X$. $\|\cdot\|_r$ is defined by (3.2) for any $r \in (0,1)$. Then $\|\cdot\|_r$ is a pseudo-norm on X if and only if N satisfies the following condition: for any $x, y \in X$, $t_1, t_2 > 0$, $N(x-z, t_1) + K(r) > 1$ and $N(z-y, t_2) + K(r) > 1$ imply that $N(x-y, t_1+t_2) + K(r) > 1$.

Proof Sufficiency It is easy to see that $\|x\|_r \geq 0$, $\|x\|_r = 0$ if $x = \theta$.

For any $\varepsilon > 0$, we obtain

$$N\left(x-z, \|x-z\|_r + \frac{\varepsilon}{2}\right) + K(r) > 1, N\left(z-y, \|z-y\|_r + \frac{\varepsilon}{2}\right) + K(r) > 1.$$

By the supposition, we obtain

$$N(x-y, \|x-z\|_r + \|z-y\|_r + \varepsilon) + K(r) > 1.$$

Therefore

$$\|x-y\|_r \leq \|x-z\|_r + \|z-y\|_r + \varepsilon.$$

From the arbitrariness of $\varepsilon > 0$, we know

$$\|x-y\|_r \leq \|x-z\|_r + \|z-y\|_r.$$

Now, we prove $\|\alpha x\|_r = |\alpha| \|x\|_r$. In fact, $\forall \alpha \in R \setminus \{0\}$, we have

$$\begin{aligned}\|\alpha x\|_r &= \inf\{t > 0 : N(\alpha x, t) + K(r) > 1\} = \inf\{t > 0 : N(x, \frac{t}{|\alpha|}) + K(r) > 1\} \\ &= \inf\{|\alpha| t' : N(x, t') + K(r) > 1\} = |\alpha| \inf\{t' : N(x, t') + K(r) > 1\} \\ &= |\alpha| \|x\|_r.\end{aligned}$$

Obviously, $\|\alpha x\|_r = |\alpha| \|x\|_r$ if $\alpha = 0$.

Necessity Suppose that $N(x-z, t_1) + K(r) > 1$ and $N(z-y, t_2) + K(r) > 1$. By the continuity of $N(x, \cdot)$, there exists $\delta > 0$ such that $N(x-z, t_1-\delta) + K(r) > 1$ and $N(z-y, t_2-\delta) + K(r) > 1$. So $\|x-z\|_r \leq t_1 - \delta$ and $\|z-y\|_r \leq t_2 - \delta$. Since $\|x-y\|_r \leq \|x-z\|_r + \|z-y\|_r$, we have $\|x-y\|_r \leq t_1 + t_2 - 2\delta < t_1 + t_2$. By (3.2), there exists $t_0 < t_1 + t_2$ such that $N(x-y, t_0) + K(r) \geq 1$. Since $N(x-y, \cdot)$ is strictly increasing, we get $N(x-y, t_1+t_2) + K(r) > 1$.

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模糊赋范空间的范数结构

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摘要: 本文研究了模糊赋范空间中的范数结构问题. 利用“切片”的方法, 通过引入K函数, 在更广泛意义下讨论了带连续 t-模的模糊范数空间中存在的范数结构, 给出了这些范数结构之间的关系. 所得结果推广了现有的结论, 并为模糊赋范空间的进一步研究提供了一种新的途径.

关键词: 模糊分析; 模糊赋范空间; 范数结构; 拓扑

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