Vol. 38 (2018)
No. 5

数 学 杂 志
J. of Math. (PRC)

STABILITY ANALYSIS FOR ONE CLASS OF COUPLED SYSTEM OF FRACTIONAL-ORDER DIFFERENTIAL EQUATIONS ON NETWORK

GAO Yang, ZHAO Wei

(*Department of Teaching Education, Daqing Normal University, Daqing 163712, China*)

Abstract: In this paper, the stability problem of the new coupled model constructed by two fractional-order differential equations for every vertex is studied. By using the method of constructing Lyapunov functions based on graph-theoretical approach for coupled systems, sufficient conditions that the coexistence equilibrium of the coupling model is globally Mittag-Leffler stable in R^{2n} are derived. An example is given to illustrate the applications of main results.

Keywords: Mittag-Leffler stable; coupled model; global stability; Caputo derivative

2010 MR Subject Classification: 93B05; 35B37; 34D23; 92D30

Document code: A **Article ID:** 0255-7797(2018)05-0843-08

1 Introduction

The global-stability problem of equilibria was investigated for coupled systems of differential equations on networks for many years [1–6]. For example, Li and Shuai developed a systematic approach that allowed one to construct global Lyapunov functions for large-scale coupled systems from building blocks of individual vertex systems by using results from graph theory. The approach was applied to several classes of coupled systems in engineering, ecology and epidemiology. Although there exist many results about stability of coupled systems on networks (CSNs), most efforts have been devoted to CSNs whose nodes are constructed by integer-order differential equations. In fact, it is more valuable and practical to investigate coupled system of fractional-order differential equations on network. Recently, Li [7] investigated the global Mittag–Leffler stability of the following coupled system of fractional-order differential equations on network (CSFDEN)

$$\begin{cases} {}_{t_0}D_t^{\alpha} x_i = -\alpha_i x_i(t) + f_i(x_i(t)) + \sum\limits_{j=1}^{n} \beta_{ij}^{x}(x_j(t) - x_i(t)), \\ x_i(t_0) = x_{it_0}, i = 1, \cdots, n, \end{cases} \tag{1.1}$$

where D denoted Caputo fractional derivative, $\alpha \in (0, 1)$. t_0 was the inial time, n $(n \geq 2)$ denoted the number of vertices in the network. $(x(t))^T = (x_1(t), x_2(t), \cdots, x_n(t))^T$ denoted

* **Received date:** 2016-12-09 **Accepted date:** 2017-08-11

Foundation item: Supported by the Natural Science Foundation of Heilongjiang Province for Youths (QC2015066); the Natural Science Foundation of Daqing Normal University for Doctor (12ZR09)

Biography: Gao Yang (1979–), male, born at Siping, Jilin, associate professor, major in control theory and application.

the state variable of the system where $x_i(t) \in R$. α_i was positive constant. Constant β_{ij}^x represented the influence of vertex j on vertex i with $\beta_{ii}^x = 0, \beta_{ij}^x = -\beta_{ji}^x$, if $i \neq j$. Function f_i was Lipschitz continuous. Several sufficient conditions were obtained to ensure the Mittag-Leffler stability of CSFDEN by using graph theory and the Lyapunov method.

Furthermore, Li [8] investigated a coupled system of fractional-order differential equations on network with feedback controls (CSFDENFCs). By using the contraction mapping principle, Lyapunov method, graph theoretic approach and inequality techniques, some sufficient conditions were derived to ensure the existence, uniqueness and global Mittag–Leffler stability of the equilibrium point of CSFDENFCs.

As far as we know, most of researchers are interested in CSNs constructed by only one fractional-order differential equation for every vertex. To the best of authors' knowledge, there are less results about CSNs constructed by two or many fractional-order differential equations for every vertex. In this paper, the coupled model (1.1) is generalized to the more complicated model. The vertex's dynamical character is presented by the two-dimensional system. The coupled relationship is constructed by two components of the vertex. The coupled system of fractional differential equations on network is studied. Sufficient conditions that the coexistence equilibrium of the coupling model is globally Mittag-Leffler stable in R^{2n} are derived by using the method of constructing Lyapunov functions based on graph-theoretical approach for coupled systems.

Remark 1.1 The generalization of model (1.1) is important and meanful. Because a lot of ecological model can be seen as high-dimensional coupled system. Every node is constructed by two or many differential equations in integer-order systems. For example, predator-prey models with patches and dispersal are studied by a lot of researchers [1–6].

This paper is organized as follows. Preliminary results are introduced in Section 2. In Section 3, main results are obtained. In the sequel, an example is presented in Section 4. Finally, the conclusions and outlooks are drawn in Section 5.

2 Preliminaries

In this section, we will list some definitions and theorems which will be used in the later sections.

A directed graph or digraph $G = (V, E)$ contains a set $V = \{1, 2, \cdots, n\}$ of vertices and a set E of arcs (i, j) leading from initial vertex i to terminal vertex j. A subgraph H of G is said to be spanning if H and G have the same vertex set. A digraph G is weighted if each arc (j, i) is assigned a positive weight. $a_{ij} > 0$ if and only if there exists an arc from vertex j to i in G.

The weight $w(H)$ of a subgraph H is the product of the weights on all its arcs. A directed path P in G is a subgraph with distinct vertices $i_1, i_2, \cdots, i_m$ such that its set of arcs is $\{(i_k, i_{k+1}) : k = 1, 2, \cdots, m\}$. If $i_m = i_1$, we call P a directed cycle.

A connected subgraph T is a tree if it contains no cycles, directed or undirected.

A tree T is rooted at vertex i, called the root, if i is not a terminal vertex of any arcs, and each of the remaining vertices is a terminal vertex of exactly one arc. A subgraph Q is unicyclic if it is a disjoint union of rooted trees whose roots form a directed cycle.

Given a weighted digraph G with n vertices, the weight matrix $A = (a_{ij})_{n\times n}$ can be defined by their entry a_{ij} equals the weight of arc (j,i) if it exists, and 0 otherwise. For our purpose, we denote a weighted digraph as (G,A). A digraph G is strongly connected if, for any pair of distinct vertices, there exists a directed path from one to the other. A weighted digraph (G,A) is strongly connected if and only if the weight matrix A is irreducible.

The Laplacian matrix of (G,A) is denoted by L. Let c_i denote the cofactor of the i-th diagonal element of L. The following results are listed.

Lemma 2.1 [6] Assume $n \geq 2$. Then

$$c_i = \sum_{\mathbf{T}\in\mathbf{T}_i} w(\mathbf{T}),$$

where T_i is the set of all spanning trees $\mathbf{T}$ of (G,A) that are rooted at vertex i, and $w(T)$ is the weight of T. In particular, if (G,A) is strongly connected, then $c_i > 0$ for $1 \leq i \leq n$.

Lemma 2.2 [6] Assume $n \geq 2$. Let c_i be given in Lemma 2.1. Then the following identity holds

$$\sum_{i,j=1}^{n} c_i a_{ij} F_{ij}(x_i, x_j) = \sum_{Q\in \mathrm{Q}} w(Q) \sum_{(s,r)\in E(C_{\mathrm{Q}})} F_{rs}(x_r, x_s),$$

here $F_{ij}(x_i, x_j), 1 \leq i,j \leq n$, are arbitrary functions, Q is the set of all spanning unicyclic graphs of (G,A), $w(Q)$ is the weight of Q, and C_{Q} denotes the directed cycle of Q.

If (G,A) is balanced, then

$$\sum_{i,j=1}^{n} c_i a_{ij} F_{ij}(x_i, x_j) = \frac{1}{2}\sum_{Q\in \mathrm{Q}} w(Q) \sum_{(j,i)\in E(C_{\mathrm{Q}})} [F_{ij}(x_i, x_j) + F_{ji}(x_j, x_i)].$$

Definition 2.3 [9] The Caputo fractional derivative of order $\alpha \in (n-1, n)$ for a continuous function $f : R^+ \to R$ is given by

$${}_{t_0}D_t^{\alpha} f(t) = \frac{1}{\Gamma(n-\alpha)} \int_{t_0}^{t} \frac{f^{(n)}(s)}{(t-s)^{\alpha+1-n}} ds.$$

3 Main Results

A coupled system of fractional differential equations on network is constructed as follows

$$\begin{cases} {}_{t_0}D_t^{\alpha} x_i = -\alpha_i x_i(t) + \theta_i y_i(t) + f_i(x_i(t)) + \sum\limits_{j=1}^{n} \beta_{ij}^{x}(x_j(t) - x_i(t)), \\ {}_{t_0}D_t^{\alpha} y_i = -\beta_i y_i(t) - \varepsilon_i x_i(t) + g_i(y_i(t)) + \sum\limits_{j=1}^{n} \beta_{ij}^{y}(y_j(t) - y_i(t)), \\ x_i(t_0) = x_{it_0}, y_i(t_0) = y_{it_0}, i = 1, \cdots, n, \end{cases} \tag{3.1}$$

here D denotes Caputo fractional derivative, $\alpha \in (0,1)$, t_0 is the inial time, n $(n \geq 2)$ denotes the number of vertices in the network. $z(t) = (x(t), y(t))^T = (x_1(t), x_2(t), \cdots, x_n(t),$

$y_1(t), y_2(t), \cdots, y_n(t))^T$ denotes the state variable of the system where $x_i(t) \in R$ and $y_i(t) \in R$. $\alpha_i, \beta_i, \theta_i = \varepsilon_i = l$ are all positive constants. Constant β_{ij}^x represents the influence of x_j on x_i with $\beta_{ii}^x = 0, \beta_{ij}^x = -\beta_{ji}^x$, if $i \neq j$. Constant β_{ij}^y represents the influence of y_j on y_i with $\beta_{ii}^y = 0, \beta_{ij}^y = -\beta_{ji}^y$, if $i \neq j$. The following assumptions are given for system (3.1).

(H1) Function f_i, g_i are Lipschtiz-continuous on R with Lipschitz constant $L_i^x > 0, L_i^y > 0$, respectively, i.e.,

$$|f_i(u) - f_i(v)| \leq L_i^x |u - v|, |g_i(u) - g_i(v)| \leq L_i^y |u - v|$$

for all $u, v \in R$.

(H2) There exists a constant λ such that

$$\lambda = \min\{2(\alpha_i + \sum_{j=1}^{n} \beta_{ij}^x - L_i^x), 2(\beta_i + \sum_{j=1}^{n} \beta_{ij}^y - L_i^y)| \quad i = 1, 2, \cdots, n\} > 0.$$

A mathematical description of a network is a directed graph consisting of vertices and directed arcs connecting them. At each vertex, the local dynamics are given by a system of differential equations called the vertex system. The directed arcs indicate inter-connections and interactions among vertex systems.

Let β_{ij} represent the influence of vertex j on vertex i with

$$\beta_{ij} = \begin{cases} \beta_{ij}^x, & \text{if } |\beta_{ij}^x| \geq |\beta_{ij}^y|, \\ \beta_{ij}^y, & \text{if } |\beta_{ij}^x| < |\beta_{ij}^y|. \end{cases}$$

Let $A = (|\beta_{ij}|)_{n\times n}, A^x = (|\beta_{ij}^x|)_{n\times n}, A^y = (|\beta_{ij}^y|)_{n\times n}$.

A digraph (G, A) with n vertices for system (3.1) can be constructed as follows. Each vertex represents a patch and $(j, i) \in E(G)$ if and only if $\beta_{ij}^x \neq 0$ or $\beta_{ij}^y \neq 0$. Here $E(G)$ denotes the set of arcs (i, j) leading from inial vertex i to terminal vertex j. At each vertex of G, the vertex dynamics are described by the following system (3.2),

$$\begin{cases} {}_{t_0}D_t^{\alpha} x_i = -\alpha_i x_i(t) + \theta_i y_i(t) + f_i(x_i(t)), \\ {}_{t_0}D_t^{\alpha} y_i = -\beta_i y_i(t) - \varepsilon_i x_i(t) + g_i(y_i(t)). \end{cases} \tag{3.2}$$

The coupling among system (3.1) is provided by the network. The G is strongly connected if and only if the matrix $A = (|\beta_{ij}|)_{n\times n}$ is irreducible.

In this section, the coupled system of fractional differential equations on network is studied. By using the method of constructing Lyapunov functions based on graph-theoretical approach for coupled systems, sufficient conditions that the coexistence equilibrium of the coupling model (3.1) is globally Mittag-Leffler stable in R^{2n} are derived.

We obtain main theorem as follows.

Theorem 3.1 Assume the following conditions hold

1. diagraph (G, A) is balanced;
2. $A^x = (|\beta_{ij}^x|)_{n\times n}, A^y = (|\beta_{ij}^y|)_{n\times n}$ are irreducible;

3. condition (H1) and (H2) hold;

4. there exists constant $p \geq 0$ such that $\beta_{ij}^y = p\beta_{ij}^x$ for $i, j = 1, 2, \cdots, n$.

Then system (3.1) is globally Mittag-Leffler stable.

Proof Let $E^* = (x^*, y^*)^T = (x_1^*, x_2^*, \cdots, x_n^*, y_1^*, y_2^*, \cdots, y_n^*)^T$ be an equilibrium of (3.1). Assume that $e_i^x(t) = x_i(t) - x_i^*, e_i^y(t) = y_i(t) - y_i^*$ $(i = 1, 2, \cdots, n)$. After calculating, we obtain that

$$\begin{aligned}
{}_{t_0}D_t^\alpha e_i^x(t) &= -\alpha_i e_i^x(t) + \theta_i e_i^y(t) + f_i(x_i^* + e_i^x(t)) - f_i(x_i^*) \\
&\quad + \sum_{j=1}^n \beta_{ij}^x (x_j^* + e_j^x(t) - x_i^* - e_i^x(t)) - \sum_{j=1}^n \beta_{ij}^x (x_j^* - x_i^*). \\
{}_{t_0}D_t^\alpha e_i^y(t) &= -\beta_i e_i^y(t) - \varepsilon_i e_i^x(t) + g_i(y_i^* + e_i^y(t)) - g_i(y_i^*) \\
&\quad + \sum_{j=1}^n \beta_{ij}^y (y_j^* + e_j^y(t) - y_i^* - e_i^y(t)) - \sum_{j=1}^n \beta_{ij}^y (y_j^* - y_i^*).
\end{aligned}$$

Let $e(t) = (e_1^x(t), e_1^y(t), e_2^x(t), e_2^y(t), \cdots, e_n^x(t), e_n^y(t))$ and

$$V_i(e_i^x(t), e_i^y(t)) = \frac{1}{2}[\varepsilon_i (e_i^x(t))^2 + \theta_i (e_i^x(t))^2].$$

Two case will be discussed about p.

Case I $0 \leq p \leq 1$.

Case II $p > 1$.

For Case I, It is easy to obtain that $|\beta_{ij}^y| \leq |\beta_{ij}^x|$. Therefore, $A = A^x$. From the condition of theorem, we obtain A^x is irreducible. Furthermore, (G, A) is strongly connected. Let c_i denote the cofactor of the ith diagonal element of Laplacian matrix of (G, A). Then we have $c_i > 0$. Let

$$V(t, e(t)) = \sum_{i=1}^n c_i V_i(e_i^x(t), e_i^y(t)).$$

Calculating the fractional-order derivative of $V(t, e(t))$ along the solution of system (3.1), we have

$$\begin{aligned}
&{}_{t_0}D_t^\alpha V(t, e(t)) = \frac{1}{2}\sum_{i=1}^n c_i \ {}_{t_0}D_t^\alpha [\varepsilon_i (e_i^x(t))^2 + \theta_i (e_i^y(t))^2] \\
\leq\ & \sum_{i=1}^n [c_i \varepsilon_i e_i^x(t) {}_{t_0}D_t^\alpha e_i^x(t) + c_i \theta_i e_i^y(t) {}_{t_0}D_t^\alpha e_i^y(t)] \\
\leq\ & \sum_{i=1}^n c_i e_i^x(t) 2(-\alpha_i - \sum_{j=1}^n \beta_{ij}^x + L_i^x) \varepsilon_i e_i^x(t) + \sum_{i=1}^n c_i e_i^y(t) 2(-\beta_i - \sum_{j=1}^n \beta_{ij}^y + L_i^y) \theta_i e_i^y(t) \\
& + c_i \varepsilon_i \theta_i e_i^y(t) e_i^x(t) - c_i \theta_i \varepsilon_i e_i^x(t) e_i^y(t) + \sum_{i=1}^n c_i \varepsilon_i a_{ij} F_{ij}(t, e_i^x, e_j^x) + \sum_{i=1}^n p c_i \theta_i a_{ij} F_{ij}(t, e_i^y, e_j^y),
\end{aligned}$$

here $a_{ij} = |\beta_{ij}| = |\beta_{ij}^x|$ and $F_{ij}(t, x, y) = \operatorname{sgn}(\beta_{ij}) xy$. Using the (G, A)'s balanced and strongly connected character, we obtain that

$$\begin{aligned}\sum_{i=1}^{n} c_i\varepsilon_i a_{ij}F_{ij}(t,e_i^x,e_j^x) &= \frac{1}{2}\varepsilon_i\sum_{Q\in\mathrm{Q}} w(Q)\sum_{(j,i)\in E(C_{\mathrm{Q}})}[F_{ij}(t,e_i^x,e_j^x)+F_{ji}(t,e_i^x,e_j^x)]\\ &= \frac{1}{2}\varepsilon_i\sum_{Q\in\mathrm{Q}} w(Q)\sum_{(j,i)\in E(C_{\mathrm{Q}})}[\mathrm{sgn}(\beta_{ij})e_i^x e_j^x+\mathrm{sgn}(\beta_{ji})e_j^x e_i^x]\\ &= \frac{1}{2}\varepsilon_i\sum_{Q\in\mathrm{Q}} w(Q)\sum_{(j,i)\in E(C_{\mathrm{Q}})}[\mathrm{sgn}(\beta_{ij})e_i^x e_j^x-\mathrm{sgn}(\beta_{ij})e_i^x e_j^x]\\ &= 0.\end{aligned}$$

Furthermore, we obtain that

$$\begin{aligned}\sum_{i=1}^{n} pc_i\theta_i a_{ij}F_{ij}(t,e_i^y,e_j^y) &= \frac{1}{2}p\theta_i\sum_{Q\in\mathrm{Q}} w(Q)\sum_{(j,i)\in E(C_{\mathrm{Q}})}[F_{ij}(t,e_i^y,e_j^y)+F_{ji}(t,e_i^y,e_j^y)]\\ &= \frac{1}{2}p\theta_i\sum_{Q\in\mathrm{Q}} w(Q)\sum_{(j,i)\in E(C_{\mathrm{Q}})}[\mathrm{sgn}(\beta_{ij})e_i^y e_j^y+\mathrm{sgn}(\beta_{ji})e_j^y e_i^y]\\ &= \frac{1}{2}p\theta_i\sum_{Q\in\mathrm{Q}} w(Q)\sum_{(j,i)\in E(C_{\mathrm{Q}})}[\mathrm{sgn}(\beta_{ij})e_i^y e_j^y-\mathrm{sgn}(\beta_{ij})e_i^y e_j^y]\\ &= 0.\end{aligned}$$

In the sequel, we have

$${}_{t_0}D_t^{\alpha}V(t,e(t))\le-\lambda V(t,e(t)).$$

Let ${}_{t_0}D_t^{\alpha}V(t,e(t))+M(t)=-\lambda V(t,e(t))$. Using Laplace transform for the equation above [10, 11], we have

$$s^{\alpha}w(s)-w(0)s^{\alpha-1}+M(s)=-\beta w(s),$$

where $w(s)$, $M(s)$ are the Laplace transform of $V(t,e(t))$ and $M(t)$, respectively. Using the inverse Laplace transform for the formula above, we have

$$V(t,e(t))\le V(0,e(0))E_{\alpha}(-\beta t^{\alpha}).$$

By the definition of $V(t,e(t))$, we obtain that system (3.1) is globally Mittag-Leffler stable.

With the similar arguments to Case I, we can prove system (3.1) is globally Mittag-Leffler stable for Case II. Then the proof is completed.

By Theorem 3.1, we obtain that the following corollary naturally.

Corollary 3.2 Consider the model

$$\begin{cases} {}_{t_0}D_t^{\alpha}x_i=-\alpha_i x_i(t)+\theta_i y_i(t)+f_i(x_i(t))+\sum\limits_{j=1}^{n}\beta_{ij}^x(x_j(t)-x_i(t)),\\ {}_{t_0}D_t^{\alpha}y_i=-\beta_i y_i(t)-\varepsilon_i x_i(t)+g_i(y_i(t)),\\ x_i(t_0)=x_{it_0},y_i(t_0)=y_{it_0},i=1,\cdots,n.\end{cases}\tag{3.3}$$

Assume that (G,A) is balanced and $A=A^x=(|\beta_{ij}^x|)_{n\times n}$ is irreducible, condition (H1) and (H2)* hold. Then system (3.3) is globally Mittag-Leffler stable. Here, condition (H2)* is denoted as follows.

(H2)* There is a constant λ such that

$$\lambda = \min\{2(\alpha_i + \sum_{j=1}^{n} \beta_{ij}^x - L_i^x)| \quad i = 1, 2, \cdots, n\} > 0.$$

4 An Example

In this section, a numerical example is presented to illustrate the Theorem 3.1.

Consider the following system of fractional equations on network

$$\begin{cases} {}_{t_0}D_t^{\alpha} x_1(t) = -\alpha_1 x_1(t) + \theta_1 y_1(t) + f_1(x_1(t)) + \sum\limits_{j=1}^{n} \beta_{1j}^x (x_j(t) - x_1(t)), \\ {}_{t_0}D_t^{\alpha} y_1(t) = -\beta_1 y_1(t) - \varepsilon_1 x_1(t) + g_1(y_1(t)) + \sum\limits_{j=1}^{n} \beta_{1j}^y (y_j(t) - y_1(t)), \\ {}_{t_0}D_t^{\alpha} x_2(t) = -\alpha_2 x_2(t) + \theta_2 y_2(t) + f_2(x_2(t)) + \sum\limits_{j=1}^{n} \beta_{2j}^x (x_j(t) - x_2(t)), \\ {}_{t_0}D_t^{\alpha} y_2(t) = -\beta_2 y_2(t) - \varepsilon_2 x_2(t) + g_2(y_2(t)) + \sum\limits_{j=1}^{n} \beta_{2j}^y (y_j(t) - y_2(t)), \end{cases} \tag{4.1}$$

where

$$\begin{aligned} &\alpha = 0.5, \alpha_1 = \alpha_2 = 5, \beta_1 = \beta_2 = 9, \theta_1 = \theta_2 = \varepsilon_1 = \varepsilon_2 = 0.5, \\ &f_1(x_1(t)) = \sin(x_1(t)), f_2(x_2(t)) = \sin(x_2(t)), \\ &g_1(y_1(t)) = \sin 2(y_1(t)), g_2(y_2(t)) = \sin 2(y_2(t)), \\ &\beta_{11}^x = \beta_{22}^x = \beta_{11}^y = \beta_{22}^y = 0, \beta_{12}^x = -\beta_{21}^x = 3, \beta_{12}^y = -\beta_{21}^y = 6. \end{aligned}$$

Therefore, we have

$$p = 2, A = \begin{pmatrix} 0 & 6 \\ 6 & 0 \end{pmatrix}, L = \begin{pmatrix} 6 & -6 \\ -6 & 6 \end{pmatrix}.$$

Then we obtain that $c_1 = c_2 = 6$. Obviously, (G, A) is strongly connected and balanced. It is easy to obtain that condition (H1), (H2) hold. According to Theorem 3.1, system (4.1) has an equilibrium point $(0, 0, 0, 0)$ which is globally Mittag-Leffler stable.

5 Conclusions and Outlooks

In this paper, the new coupled model constructed by two fractional-order differential equations for every vertex is studied. The coupled relationship is constructed by two components of the vertex. By using the method of constructing Lyapunov functions based on graph-theoretical approach for coupled systems, sufficient conditions that the coexistence equilibrium of the coupling model is globally Mittag-Leffler stable in R^{2n} are derived. Finally, an example is given to illustrate the applications of main results.

Further studies on this subject are being carried out by the presenting authors in the two aspects: one is to study the model with time delay; the other is to discuss the method to design control terms.

References

[1] Freedman H I, Takeuchi Y. Global Stability and predator dynamics in a model of prey dispersal in a patchy environment[J]. Nonl. Anal. The. Meth. Appl., 1989, 13(8): 993–1002.

[2] Kuang Y, Takeuchi Y. Predator-prey dynamics in models of prey dispersal in 2-patch environments[J]. Math. Biosci., 1994, 120(1): 77–98.

[3] Cui J G. The effect of dispersal on permanence in a predator-prey population growth model[J]. Comp. Math. Appl., 2002, 44(8-9): 1085–1097.

[4] Xu R, Chaplain M A J, and Davidson F A. Periodic solutions for a delayed predator-prey model of prey dispersal in two-patch environments[J]. Nonl. Anal.-Real World Appl., 2004, 5(1): 183–206.

[5] Zhang L, Teng Z D. Permanence for a delayed periodic predator-prey model with prey dispersal in multi-patches and predator density-independent[J]. J. Math. Anal. Appl., 2008, 338(1): 175–193.

[6] Li M Y, Shuai Z S. Global-stability problem for coupled systems of differential equations on networks[J]. J. Diff. Equ., 2010, 248(1): 1–20.

[7] Li H L, Jiang Y L, Wang Z L, Zhang L, Teng Z D. Globla Mittag-Leffler stability of coupled system of fractional-order differential equations on network[J]. Appl. Math. Comput., 2015, 270: 269–277.

[8] Li H L, Hu C, Jiang Y L, Zhang L, Teng Z D. Global Mittag-Leffler stability for a coupled system of fractional-order differential equations on network with feedback controls[J]. Neurocomputing, 2016, 214: 233–241.

[9] Li Y, Chen Y Q, Podlubny I. Mittag-Leffler stability of fractional order nonlinear dynamic systems[J]. Automatica, 2009, 45: 1965–1969.

[10] Norelys A C, Duarte-Mermoud M A, Gallegos J A. Lyapunov functions for fractional order systems[J]. Commun. Nonl. Sci. Numer. Simulat., 2014, 19: 2951–2957.

[11] Li Y, Chen Y Q, Podlubny I. Stability of fractional-order nonlinear dynamic systems: Lyapunov direct method and generalized Mittag-Leffler stability[J]. Comput. Math. Appl., 2010, 59: 1810–1821.

基于网络的一类分数阶微分方程耦合系统的稳定分析

高 扬, 赵 微

(大庆师范学院教师教育学院, 黑龙江 大庆 163712)

摘要: 本文研究顶点由两个分数阶微分方程构建的新耦合模型的稳定问题. 通过使用构建Lyapunov函数思想和耦合系统的图论, 得到新模型的平衡点Mittag-Leffler稳定的充分条件, 并且举例阐述了主要结论的应用性.

关键词: Mittag-Leffler稳定; 耦合系统; 全局稳定; Caputo导数

MR(2010)**主题分类号**: 93B05; 35B37; 34D23; 92D30 **中图分类号**: O175.1