Vol. 38 (2018)
No. 2

数 学 杂 志
J. of Math. (PRC)

PERIODIC SOLUTIONS OF A NONHOMOGENEOUS ITERATIVE FUNCTIONAL DIFFERENTIAL EQUATION

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Abstract: In this paper, we use Krasnoselskii's fixed point theorem to study the existence and uniqueness of periodic solutions of a nonhomogeneous iterative functional differential equation $x'(t) = c_1x(t) + c_2x^{[2]}(t) + F(t)$, which develops the theory about the periodic solutions of iterative functional differential equation.

Keywords: iterative functional differential equation; periodic solutions; fixed point theorem

2010 MR Subject Classification: 39B12; 39B82

Document code: A **Article ID:** 0255-7797(2018)02-0191-09

1 Introduction

Recently, iterative functional differential equations of the form

$$x'(t) = H(x^{[0]}(t), x^{[1]}(t), x^{[2]}(t), \cdots, x^{[n]}(t))$$

appeared in several papers, here $x^{[0]}(t) = t, x^{[1]}(t) = x(t), x^{[2]}(t) = x(x(t)), \cdots, x^{[n]}(t) = x(x^{n-1}(t))$. In [1], Cooke pointed out that it is highly desirable to establish the existence and stability properties of periodic solutions for equations of the form

$$x'(t) + ax(t - h(t, x(t))) = F(t)$$

in which the lag $h(t, x(t))$ implicitly involves $x(t)$. Stephan [2] studied the existence of periodic solutions of equation

$$x'(t) + ax(t - r + \mu h(t, x(t))) = F(t).$$

Eder [3] considered the iterative functional differential equation

$$x'(t) = x^{[2]}(t)$$

* **Received date:** 2016-03-06 **Accepted date:** 2016-08-23

Foundation item: Supported by National Natural Science Foundation of China (11326120; 11501069); Foundation of Chongqing Municipal Education Commission (KJ1400528; KJ1600320).

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and obtained that every solution either vanishes identically or is strictly monotonic. Feckan [4] studied the equation

$$x'(t) = f(x^{[2]}(t))$$

and obtained an existence theorem for solutions satisfying $x(0) = 0$. Later, Wang and Si [5] studied

$$x'(x^{[r]}(t)) = c_0 z + c_1 x(t) + c_2 x^{[2]}(t) + \cdots + x^{[n]}(t),$$

and showed the existence theorem of analytic solutions. In particularly, Si and Cheng [6] discussed the smooth solutions of equation of

$$x'(t) = c_1 x(t) + c_2 x^{[2]}(t) + \cdots + c_n x^{[n]}(t) + F(t).$$

Some various properties of solutions for several iterative functional differential equations, we refer the interested reader to [7–10].

Since Burton [11] applied Krasnoselskii's fixed theorem to prove the existence of periodic solutions, which was extensively used in proving stability, periodic of solutions and boundedness of solutions in functional differential (difference) equations. 2005, Raffoul [12] used fixed point theorem to show a nonlinear neutral system

$$\frac{d}{dt}[x(t) - ax(t-\tau)] = r(t)x(t) - f(t, x(t-\tau))$$

has a periodic solution. In [13], Guo and Yu discussed the existence and multiplicity of periodic of the second order difference equation. Some other works can also be found in [14–16].

In this paper, we consider the existence of periodic solutions of equation

$$x'(t) = c_1 x(t) + c_2 x^{[2]}(t) + F(t), \tag{1.1}$$

where $c_1 > 0$. For convenience, we will make use of $C(\mathbb{R}, \mathbb{R})$ to denote the set of all real valued continuous functions map $\mathbb{R}$ into $\mathbb{R}$.

For $T > 0$, we define

$$\mathcal{P}_T = \Big\{ x \in C(\mathbb{R}, \mathbb{R}) : x(t+T) = x(t),\ \forall t \in \mathbb{R} \Big\},$$

then $\mathcal{P}_T$ is a Banach space with the norm

$$\|x\| = \max_{t\in[0,T]} |x(t)| = \max_{t\in\mathbb{R}} |x(t)|.$$

For $P, L \geq 0$, we define the set

$$\mathcal{P}_T(P, L) = \Big\{ x \in \mathcal{P}_T : \|x\| \leq P,\ |x(t_2) - x(t_1)| \leq L|t_2 - t_1|,\ \forall t_1, t_2 \in \mathbb{R} \Big\},$$

which is a closed convex and bounded subset of $\mathcal{P}_T$, and we wish to find T-periodic functions $x \in \mathcal{P}_T(P, L)$ satisfies (1.1).

2 Periodic Solutions of (1.1)

In this section, the existence of periodic solutions of equation (1.1) will be proved. Now let us state the Krasnoselskii's fixed point theorem, it will be used to prove our main theorem.

Theorem 2.1 (see [17]) Let Ω be a closed convex nonempty subset of a Banach space $(\mathbb{B}, \|\cdot\|)$. Suppose that A and B map Ω into $\mathbb{B}$ such that

(i) A is compact and continuous,

(ii) B is a contraction mapping,

(iii) $x, y \in \Omega$, implies $Ax + By \in \Omega$,

then there exists $z \in \Omega$ with $z = Az + Bz$.

We begin with the following lemma.

Lemma 2.2 For any $\varphi, \psi \in \mathcal{P}_T(P, L)$,

$$||\varphi^{[n]} - \psi^{[n]}|| \leq \sum_{j=0}^{n-1} L^j \|\varphi - \psi\|, \ n = 1, 2, \cdots. \tag{2.1}$$

The result can be obtained by the definition of $\mathcal{P}_T(P, L)$.

Lemma 2.3 Suppose $c_1 \neq 0$. If $x \in \mathcal{P}_T$, then $x(t)$ is a solution of equation (1.1) if and only if

$$x(t) = c_2 \int_t^{t+T} x^{[2]}(s) G(t, s) ds + \int_t^{t+T} F(s) G(t, s) ds, \tag{2.2}$$

where

$$G(t, s) = \frac{e^{c_1(t-s)}}{e^{-c_1 T} - 1}. \tag{2.3}$$

Proof Let $x(t) \in \mathcal{P}_T(P, L)$ be a solution of (1.1), multiply both sides of the resulting equation with $e^{-c_1 t}$ and integrate from t to $t + T$ to obtain

$$x(t+T)e^{-c_1(t+T)} - x(t)e^{-c_1 t} = c_2 \int_t^{t+T} x^{[2]}(s) e^{-c_1 s} ds + \int_t^{t+T} F(s) e^{-c_1 s} ds.$$

Using the fact $x(t + T) = x(t)$, the above expression can be put in the form

$$x(t) = c_2 \int_t^{t+T} x^{[2]}(s) \frac{e^{c_1(t-s)}}{e^{-c_1 T} - 1} ds + \int_t^{t+T} F(s) \frac{e^{c_1(t-s)}}{e^{-c_1 T} - 1} ds.$$

This completes the proof.

It is clear that $G(t, s) = G(t+T, s+T)$ for all $(t, s) \in \mathbb{R}^2$, and for $s \in [t, t+T]$, we have

$$m = \frac{e^{-|c_1|T}}{|e^{-c_1 T} - 1|} \leq |G(t, s)| \leq \frac{e^{|c_1|T}}{|e^{-c_1 T} - 1|} = M. \tag{2.4}$$

Now we need to construct two mappings to satisfy Theorem 2.1. Set the map $A, B : \mathcal{P}_T(P, L) \to \mathcal{P}_T$ as the follwoing,

$$(Ax)(t) = c_2 \int_t^{t+T} x^{[2]}(s)G(t,s)ds,\ t \in \mathbb{R}, \tag{2.5}$$

$$(Bx)(t) = \int_t^{t+T} F(s)G(t,s)ds,\ t \in \mathbb{R}, \tag{2.6}$$

where $F \in \mathcal{P}_T(P, L)$, $G(t,s)$ defined as (2.3).

Lemma 2.4 Operator A is continuous and compact on $\mathcal{P}_T(P, L)$.

Proof Take $\varphi, \psi \in \mathcal{P}_T(P, L), t \in \mathbb{R}$, use (2.1) and (2.4),

$$\begin{aligned} |(A\varphi)(t) - (A\psi)(t)| &\leq |c_2| \int_t^{t+T} |\varphi^{[2]}(s) - \psi^{[2]}(s)||G(t,s)|ds \\ &\leq |c_2| MT(1+L)\|\varphi - \psi\|. \end{aligned}$$

This proves A is continuous.

Now we show that A is a compact map. It is easy to see that $\mathcal{P}_T(P, L)$ is uniformly bounded and equicontinuous on $\mathbb{R}$, thus by Arzela-Ascoli theorem, it is a compact set. Since A is continuous, it maps compact sets into compact sets, therefore A is compact. This completes the proof.

Lemma 2.5 Operator B is a contraction mapping on $\mathcal{P}_T(P, L)$.

Proof Take $\varphi, \psi \in \mathcal{P}_T(P, L)$,

$$\|B\varphi - B\psi\| = \max_{t\in[0,T]} \Big| \int_t^{t+T} F(s)G(t,s)ds - \int_t^{t+T} F(s)G(t,s)ds \Big| = 0 \leq \eta\|\varphi - \psi\|$$

for any $0 \leq \eta < 1$, hence B defines a contraction mapping.

Theorem 2.6 Suppose $F \in \mathcal{P}_T(P, L)$ is given, $c_1 > 0$ and the following inequalities are held

$$(1 + |c_2|)MT \leq 1, \quad 2P(1 + |c_2|) \leq L, \tag{2.7}$$

then eq. (1.1) has a periodic solution in $\mathcal{P}_T(P, L)$.

Proof For any $\varphi, \psi \in \mathcal{P}_T(P, L)$, by (2.4) and (2.7),

$$\begin{aligned} \Big|(A\varphi)(t) + (B\psi)(t)\Big| &\leq |c_2| \Big| \int_t^{t+T} \varphi^{[2]}(s)G(t,s)ds \Big| + \Big| \int_t^{t+T} F(s)G(t,s)ds \Big| \\ &\leq (1 + |c_2|)MTP \\ &\leq P. \end{aligned} \tag{2.8}$$

Without loss of generality, we assume $t_2 \geq t_1$, by (2.7),

$$\begin{aligned}
& \Big|\Big((A\varphi)(t_2)+(B\psi)(t_2)\Big)-\Big((A\varphi)(t_1)+(B\psi)(t_1)\Big)\Big| \\
\leq\ & |c_2|\Big|\int_{t_2}^{t_2+T}\varphi^{[2]}(s)G(t_2,s)ds-\int_{t_1}^{t_1+T}\varphi^{[2]}(s)G(t_1,s)ds\Big| \\
& +\Big|\int_{t_2}^{t_2+T}F(s)G(t_2,s)ds-\int_{t_1}^{t_1+T}F(s)G(t_1,s)ds\Big| \\
\leq\ & \frac{|c_2|}{|e^{-c_1T}-1|}\Big|e^{c_1t_2}\int_{t_2}^{t_2+T}\varphi^{[2]}(s)e^{-c_1s}ds-e^{c_1t_1}\int_{t_1}^{t_1+T}\varphi^{[2]}(s)e^{-c_1s}ds\Big| \\
& +\frac{1}{|e^{-c_1T}-1|}\Big|e^{c_1t_2}\int_{t_2}^{t_2+T}F(s)e^{-c_1s}ds-e^{c_1t_1}\int_{t_1}^{t_1+T}F(s)e^{-c_1s}ds\Big| \\
\leq\ & \frac{|c_2|}{|e^{-c_1T}-1|}\Big|\Big(e^{c_1t_2}-e^{c_1t_1}\Big)\int_{t_2}^{t_2+T}\varphi^{[2]}(s)e^{-c_1s}ds\Big| \\
& +\frac{|c_2|}{|e^{-c_1T}-1|}\Big|e^{c_1t_1}\Big(\int_{t_2}^{t_2+T}\varphi^{[2]}(s)e^{-c_1s}ds-\int_{t_1}^{t_1+T}\varphi^{[2]}(s)e^{-c_1s}ds\Big)\Big| \\
& +\frac{1}{|e^{-c_1T}-1|}\Big|\Big(e^{c_1t_2}-e^{c_1t_1}\Big)\int_{t_2}^{t_2+T}F(s)e^{-c_1s}ds\Big| \\
& +\frac{1}{|e^{-c_1T}-1|}\Big|e^{c_1t_1}\Big(\int_{t_2}^{t_2+T}F(s)e^{-c_1s}ds-\int_{t_1}^{t_1+T}F(s)e^{-c_1s}ds\Big)\Big| \\
\leq\ & \frac{|c_2|}{|e^{-c_1T}-1|}\Big|\Big(e^{c_1t_2}-e^{c_1t_1}\Big)\int_{t_2}^{t_2+T}\varphi^{[2]}(s)e^{-c_1s}ds\Big| \\
& +\frac{|c_2|}{|e^{-c_1T}-1|}\Big|e^{c_1t_1}\Big|\Bigg|\int_{t_2}^{t_1}\varphi^{[2]}(s)e^{-c_1s}ds+\int_{t_1+T}^{t_2+T}\varphi^{[2]}(s)e^{-c_1s}ds\Big)\Bigg| \\
& +\frac{1}{|e^{-c_1T}-1|}\Big|\Big(e^{c_1t_2}-e^{c_1t_1}\Big)\int_{t_2}^{t_2+T}F(s)e^{-c_1s}ds\Big| \\
& +\frac{1}{|e^{-c_1T}-1|}\Big|e^{c_1t_1}\Big|\Bigg|\int_{t_2}^{t_1}F(s)e^{-c_1s}ds+\int_{t_1+T}^{t_2+T}F(s)e^{-c_1s}ds\Bigg| \\
\leq\ & \frac{P|c_2|}{|e^{-c_1T}-1|}\Big|e^{c_1(\xi-t_2)}\Big||e^{-c_1T}-1||t_2-t_1| \\
& +\frac{P|c_2|}{|c_1||e^{-c_1T}-1|}\Big|c_1e^{c_1(t_1-\xi)}\Big||e^{-c_1T}-1||t_2-t_1| \\
& +\frac{P}{|e^{-c_1T}-1|}\Big|e^{c_1(\xi-t_2)}\Big||e^{-c_1T}-1||t_2-t_1| \\
& +\frac{P}{|e^{-c_1T}-1|}\Big|e^{c_1(t_1-\xi)}\Big||e^{-c_1T}-1||t_2-t_1| \\
\leq\ & 2P(1+|c_2|)|t_2-t_1| \\
\leq\ & L|t_2-t_1|, \qquad (2.9)
\end{aligned}$$

where $t_1 \leq \xi \leq t_2$.

This shows that $(A\varphi)(t) + (B\psi)(t) \in \mathcal{P}_T(P, L)$. By Lemma 2.4 and Lemma 2.5, we see that all the conditions of Krasnoselskii's theorem are satisfied on the set $\mathcal{P}_T(P, L)$. Thus there exists a fixed point x in $\mathcal{P}_T(P, L)$ such that

$$\begin{aligned} x(t) &= (Ax)(t) + (Bx)(t) \\ &= c_2 \int_t^{t+T} x^{[2]}(s)G(t, s)ds + \int_t^{t+T} F(s)G(t, s)ds. \end{aligned} \tag{2.10}$$

Differential both sides of (2.10) and from Lemma 2.3, we can find (1.1) has a T-periodic solution. This completes the proof.

3 Uniqueness and Stability

In this section, uniqueness and stability of (1.1) will be proved.

Theorem 3.1 In addition to the assumption of Theorem 2.6, suppose that

$$|c_2|MT(1 + L) < 1, \tag{3.1}$$

then (1.1) has a unique solution in $\mathcal{P}_T(P, L)$.

Proof Define an operator H from $\mathcal{P}_T(P, L)$ into $\mathcal{P}_T$,

$$(Hx)(t) = (Ax)(t) + (Bx)(t) = c_2 \int_t^{t+T} x^{[2]}(s)G(t, s)ds + \int_t^{t+T} F(s)G(t, s)ds, \tag{3.2}$$

where $G(t, s)$ defined as (2.3). Denote $\varphi, \psi \in \mathcal{P}_T(P, L)$ are two different T-periodic solutions of (1.1),

$$\begin{aligned} |\varphi(t) - \psi(t)| &= \Big|(H\varphi)(t) - (H\psi)(t)\Big| \\ &\le |c_2| \int_t^{t+T} \Big|\varphi^{[2]}(s) - \psi^{[2]}(s)\Big| |G(t, s)|ds \\ &\le |c_2|MT(1 + L)\|\varphi - \psi\| \\ &= \Gamma\|\varphi - \psi\|, \end{aligned}$$

where $\Gamma = |c_2|MT(1 + L)$, thus

$$\|\varphi - \psi\| \le \Gamma\|\varphi - \psi\|.$$

From (3.1), we know $\Gamma < 1$ and the fixed point φ must be unique.

Theorem 3.2 The unique solution obtained in Theorem 3.1 depends continuously on the given functions F and c_i $(i = 1, 2)$.

Proof Under the assumptions of Theorem 3.1, for any two functions $F_i(x)$ in $\mathcal{P}_T(P, L)$ are given, λ_i and $\mu_i,\ i = 1, 2$ are constants satisfy (2.7). Then there are two unique corresponding functions $\varphi(t)$ and $\psi(t)$ in $\mathcal{P}_T(P, L)$ such that

$$\varphi(t) = \lambda_2 \int_t^{t+T} \varphi^{[2]}(s)G_1(t, s)ds + \int_t^{t+T} F_1(s)G_1(t, s)ds$$

and

$$\psi(t)=\mu_2\int_t^{t+T}\psi^{[2]}(s)G_2(t,s)ds+\int_t^{t+T}F_2(s)G_2(t,s)ds,$$

where

$$G_1(t,s)=\frac{e^{\lambda_1(t-s)}}{e^{-\lambda_1T}-1},\ \ G_2(t,s)=\frac{e^{\mu_1(t-s)}}{e^{-\mu_1T}-1}.$$

We have

$$\begin{aligned}
\|\varphi-\psi\| \ \le\ & \max_{t\in[0,T]}\left|\lambda_2\int_t^{t+T}\varphi^{[2]}(s)\frac{e^{\lambda_1(t-s)}}{e^{-\lambda_1T}-1}ds-\mu_2\int_t^{t+T}\psi^{[2]}(s)\frac{e^{\mu_1(t-s)}}{e^{-\mu_1T}-1}ds\right|\\
&+\max_{t\in[0,T]}\left|\int_t^{t+T}F_1(s)\frac{e^{\lambda_1(t-s)}}{e^{-\lambda_1T}-1}ds-\int_t^{t+T}F_2(s)\frac{e^{\mu_1(t-s)}}{e^{-\mu_1T}-1}ds\right|\\
\le\ & \max_{t\in[0,T]}|\lambda_2-\mu_2|\left|\int_t^{t+T}\varphi^{[2]}(s)\frac{e^{\lambda_1(t-s)}}{e^{-\lambda_1T}-1}ds\right|\\
&+|\mu_2|\max_{t\in[0,T]}\int_t^{t+T}|\varphi^{[2]}(s)-\psi^{[2]}(s)|\left|\frac{e^{\mu_1(t-s)}}{e^{-\mu_1T}-1}\right|ds\\
&+|\mu_2|\max_{t\in[0,T]}\left|\int_t^{t+T}|\varphi^{[2]}(s)|\left(\frac{e^{\lambda_1(t-s)}}{e^{-\lambda_1T}-1}-\frac{e^{\mu_1(t-s)}}{e^{-\mu_1T}-1}\right)ds\right|\\
&+\max_{t\in[0,T]}\int_t^{t+T}|F_1(s)-F_2(s)|\left|\frac{e^{\lambda_1(t-s)}}{e^{-\lambda_1T}-1}\right|ds\\
&+\max_{t\in[0,T]}\left|\int_t^{t+T}|F_2(s)|\left(\frac{e^{\lambda_1(t-s)}}{e^{-\lambda_1T}-1}-\frac{e^{\mu_1(t-s)}}{e^{-\mu_1T}-1}\right)ds\right|\\
\le\ & TPM_1|\lambda_2-\mu_2|+|\mu_2|TM_2(1+L)\|\varphi-\psi\|+TM_1\|F_1-F_2\|\\
&+\frac{P(1+|\mu_2|)}{\lambda_1\mu_1}|\lambda_1-\mu_1|, \qquad (3.3)
\end{aligned}$$

where

$$M_1=\frac{e^{|\lambda_1|T}}{|e^{-\lambda_1T}-1|},\ \ M_2=\frac{e^{|\mu_1|T}}{|e^{-\mu_1T}-1|},$$

thus

$$\begin{aligned}
(1-|\mu_2|TM_2(1+L))\|\varphi-\psi\| \ \le\ & \frac{P(1+|\mu_2|)}{\lambda_1\mu_1}|\lambda_1-\mu_1|+TPM_1|\lambda_2-\mu_2|\\
&+TM_1\|F_1-F_2\|.
\end{aligned}$$

From (3.1),

$$\delta=1-|\mu_2|TM_2(1+L)>0$$

and

$$\|\varphi-\psi\|\le\frac{T}{\delta}\left(\frac{P(1+|\mu_2|)}{\lambda_1\mu_1}|\lambda_1-\mu_1|+TPM_1|\lambda_2-\mu_2|+TM_1\|F_1-F_2\|\right).$$

This completes the proof.

Example 1 Now we will show that the conditions in Theorem 2.6 do not self-contradict. Consider the following equation

$$x'(t) = 5x(t) + \frac{1}{10}x(x(t)) + \frac{1}{10}\sin 20\pi t, \tag{3.4}$$

where

$$c_1 = 5,\ c_2 = \frac{1}{10},\ F(t) = \frac{1}{10}\sin 20\pi t,\ T = \frac{1}{10}.$$

A simple calculation yields $4.19 < M = \frac{e}{e^{\frac{1}{2}}-1} < 4.2$ and $(1+|c_2|)MT < 0.47 < 1$. Let $P = 1,\ L = 8,\ 2P(1+|c_2|) = 2.2 < 8$, then (2.7) is satisfied. By Theorem 2.6, equation (3.4) has a $\frac{1}{10}$-periodic solution x such that $\|x\| \le 1$, and

$$|x(t_2) - x(t_1)| \le 8|t_2 - t_1|,\ \forall t_1, t_2 \in \mathbb{R}.$$

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一类非齐次迭代泛函微分方程的周期解

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摘要: 本文利用Krasnoselskii 不动点定理考虑了一类非齐次迭代泛函微分方程$x'(t) = c_1x(t) + c_2x^{[2]}(t) + F(t)$ 周期解的存在唯一性问题, 推广了迭代泛函微分方程周期解的相关理论.

关键词: 迭代泛函微分方程; 周期解; 不动点定理

MR(2010)**主题分类号:** 39B12; 39B82 **中图分类号:** O193