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NOTES ON STRONGLY SEPARABLE EXTENSIONS

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Abstract: Let R and S be rings and $\varphi : R \to S$ a strongly separable extension. In this paper, we study the relationship of (Gorenstein) global dimensions and representation type between R and S. By using the homological methods, we proved that (1) R and S have the same left global dimension, left weak global dimension, left Gorenstein global dimension; (2) Assume that R and S are Artin algebras, then R is CM-finite (resp., CM-free, of finite representation type) if and only if so is S, which generalize some known results.

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1 Introduction

For any ring extension $\varphi : R \to S$, we can always consider the triple of functors $\Gamma = (S \otimes_R -, \varphi_*, \mathrm{Hom}_R(_RS, -))$, where $S \otimes_R -$ and $\mathrm{Hom}_R(_RS, -)$ are, respectively, the left and the right adjoint of the restriction of scalars functor $\varphi_* : S$-Mod$\to R$-Mod. The functor φ_* is termed a quasi-Frobenius (Frobenius) functor if $S \otimes_R -$ and $\mathrm{Hom}_R(_RS, -)$ are similar (naturally isomorphic). It was shown in [8] (see [22]) that φ_* is a quasi-Frobenius (Frobenius) functor if and only if φ is a quasi-Frobenius (Frobenius) extension in the sense of [21] (see [18]), i.e., S is finitely generated and projective as a left R-module and the (S, R)-bimodules S and $\mathrm{Hom}_R(_RS,_R R)$ are similar (i.e., S is finitely generated and projective as a left R-module and $S \cong \mathrm{Hom}_R(_RS,_R R)$ as (S, R)-bimodules). It was known [8] that if φ is a quasi-Frobenius extension, then the functors $S \otimes_R -$, φ_* and $\mathrm{Hom}_R(_RS, -)$ are exact and preserve all limits and colimits as well as injective and projective objects. Separable extensions were studied in [10, 14, 16, 17, 19, 23, 27, 28] among others. The ring extension φ is separable [25] if the natural multiplication map $S \otimes_R S \to S$ is a split epimorphism as S-bimodules; φ is separable if and only if φ_* is a separable functor in the sense of [23]; φ is split [25] if φ is a split monomorphism of R-bimodules; φ is split if and only if $S \otimes_R -$ is a separable functor; φ is called strongly separable [10] if φ is a separable, split and quasi-Frobenius extension.

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But φ is called strongly separable in [19] if φ is a separable, split and Frobenius extension. And hence the notion of strongly separable extensions in [10] is weaker than that of [19].

Gorenstein homological algebra was initiated by Auslander and Bridger in [1, 2], where they introduced the G-dimension of any finitely generated module over a two-sided Noetherian ring. Over a general ring, Enochs and Jenda [12] introduced Gorenstein projective modules, which is a generalization of finitely generated modules of G-dimension zero. And to complete the analogy with the classical homological algebra, Gorenstein injective and flat modules were introduced by Enochs et al. in [12, 13]. The Gorenstein homological dimensions are similar to (and refinements of) the classical homological dimensions. In 2004, Holm [15] generalized several well-known results on Gorenstein dimensions over Noetherian rings to arbitrary rings, and then the Gorenstein homological dimensions theory witnessed a new impetus. Let R be a ring and M a left R-module. We use $Gpd(M)$, $Gid(M)$ and $Gfd(M)$ to denote, respectively, the Gorenstein projective, injective, and flat dimensions of M. In [5], several classical results on global homological dimensions were extended to global Gorenstein homological dimensions. Namely, it was proved in [5] that for a ring R, $\sup\{Gpd(M)|M$ is a left R-module$\}$=$\sup\{Gid(M)|M$ is a left R-module$\}$. The common value of the terms of this equality is called, the left Gorenstein global dimension of R, and denoted by $l.Ggl\dim(R)$. Also, the left Gorenstein weak global dimension of a ring R, $l.wGg\dim(R)$=$\sup\{Gfd(M)|M$ is a left R-module$\}$, is investigated.

Recall that an Artin algebra R is said to be of finite representation type if there exist only finitely many isomorphism classes of finitely generated indecomposable R-modules. It is well known that determining the representation type of algebras is fundamental and important in representation theory of Artin algebras. As an analogy of Artin algebras of finite representation type, recall that an Artin algebra R is called CM-finite if there exist only finitely many isomorphism classes of finitely generated indecomposable Gorenstein projective R-modules. This notion was introduced by Beligiannis in [3]. Since then CM-finite Artin algebras have attracted considerable attentions [3, 4]. Recall from [9] that an Artin algebra R is called CM-free if any finitely generated Gorenstein projective R-module is projective. Note that CM-free algebras are an extreme case of CM-finite algebras.

In this paper, we study the invariant properties under strongly separable extensions, such as the Gorenstein (weak) global dimensions, the representation type and CM-finite type of Artin algebras. In Section 2, we show that if $\varphi : R \to S$ is a strongly separable extension, then R and S have the same left global dimension, left weak global dimension, left Gorenstein global dimension. Moreover, if S is right coherent, then they have the same left Gorenstein weak global dimension. Finally, it is proved that if φ is a strongly separable extension of Artin algebras, then R is CM-finite (resp., CM-free, of finite representation type) if and only if so is S.

Throughout this paper, all rings are associative with identity and all modules are unitary. Let R be a ring, R-Mod denotes the category of all left R-modules. We write ${}_RM$ (M_R) to indicate a left (right) R-module. For an R-module M, we use $pd(M)$ and $fd(M)$

to denote, respectively, projective, flat dimension of M. $lD(R)$ and $wD(R)$, stand for the left global dimension, the weak global dimension of a ring R, respectively. add$M$ denotes the class of all direct summands of finite direct sums of copies of M. Let M, N be two left R-modules. If M is a direct summand of N, then we denote it by $M|N$. For unexplained concepts and notations, we refer the reader to [11, 20, 25, 26].

2 Main Results

Definition 2.1 [10] A ring extension $\varphi : R \to S$ is called strongly separable if φ is a separable, split and quasi-Frobenius extension.

Example 2.2 [10, 19] (1) Let G be a group with a subgroup H of finite index, say n, if R is a ring with $nR = R$, then RG is a strongly separable extension of RH.

(2) Let R be any ring, then the ring $M_n(R)$ of $n \times n$ matrices over R is a strongly separable extension of R.

(3) Suppose H is a finite dimensional, semisimple, cosemisimple Hopf algebra over a field K. If A is an H-Galois extension of S, then A is a strongly separable extension of S.

For a ring extension $\varphi : R \to S$, we consider the following conditions

(C1) ${}_SY|_S(S \otimes_R Y)$ for every $Y \in S$-Mod.

(C2) R is an R-bimodule direct summand of S.

(C3) $\mathrm{Hom}_R({}_RS, -)$ preserve all projective objects, $- \otimes_R S$ and φ_* preserve all injective objects.

(C4) ${}_RS$ and S_R are finitely generated projective.

By [7, Lemma 3.1], we know that if $\varphi : R \to S$ is separable, then φ satisfies (C1). And hence any strongly separable extension $\varphi : R \to S$ satisfies the conditions (C1)–(C4), there are some other examples satisfy the above conditions. For example,

(1) the extension $\varphi : R \to A \otimes_k F = S$, where R is a finite-dimensional algebra over a field k, and F a finite separable field extension of k by [6, p.6, Lemma 1] and [24, p.275, Lemma 2.3].

(2) the extension $\varphi : R \to R * G = S$, where R is any ring and G is a finite group such that $|G|^{-1} \in R$.

Lemma 2.3 Let $\varphi : R \to S$ be a ring extension with (C1) and (C4). If M is a left S-module, then we have

1. M is a projective left R-module if and only if M is a projective left S-module;
2. M is a flat left R-module if and only if M is a flat left S-module

Proof (1) If M is a projective left R-module, then $S \otimes_R M$ is a projective S-module and hence M is a projective left S-module since ${}_SM|_S(S \otimes_R M)$. Conversely if M is a projective left S-module, then M is a projective left R-module by (C4).

(2) If M is a flat left S-module, then it is a flat left R-module since S is projective as a left R-module. Conversely, assume M is a flat left R-module, then $S \otimes_R M$ is a flat left S-module. But ${}_SM|_S(S \otimes_R M)$ and hence M is a flat left S-module.

Lemma 2.4 Let $\varphi : R \to S$ be a ring extension with (C1) and (C4). If M is a left S-module, then we have

1. $pd({}_SM) = pd({}_RM) = pd({}_S(S \otimes_R M))$;
2. $fd({}_SM) = fd({}_RM) = fd({}_S(S \otimes_R M))$.

Proof (1) By Lemma 2.3 (1) we have $pd({}_SM) \geq pd({}_RM)$. By (C1), we get $pd({}_S(S \otimes_R M)) \geq pd({}_SM)$. If $pd({}_RM) = n < \infty$, then there exists a projective resolution of ${}_RM$:

$$0 \to G_n \to \cdots \to G_0 \to_R M \to 0.$$

By (C4), we get a projective resolution of the S-module $S \otimes_R M$:

$$0 \to S \otimes_R G_n \to \cdots \to S \otimes_R G_0 \to S \otimes_R M \to 0,$$

this implies $pd({}_RM) \geq pd({}_S(S \otimes_R M))$, and hence $pd({}_SM) = pd({}_RM) = pd({}_S(S \otimes_R M))$.

(2) By Lemma 2.3 (2), we have $fd({}_SM) \geq fd({}_RM)$. By (C1), we get $fd({}_SM) \leq fd({}_S(S \otimes_R M))$. By (C4), we find that $fd({}_RM) \geq fd({}_S(S \otimes_R M))$, it follows that $fd({}_SM) = fd({}_RM) = fd({}_S(S \otimes_R M))$.

The following theorem extends [19, Theorem 4.2].

Theorem 2.5 Let $\varphi : R \to S$ be a ring extension with (C1), (C2) and (C4). Then $lD(R) = lD(S)$ and $wD(R) = wD(S)$.

Proof By Lemma 2.4, we have $lD(R) \geq lD(S)$ and $wD(R) \geq wD(S)$. We now prove $lD(R) \leq lD(S)$. For any left R-module M, we have ${}_RM|_R(S \otimes_R M)$ by (C2), and hence $pd({}_RM) \leq pd({}_R(S \otimes_R M)) = pd({}_S(S \otimes_R M)) \leq lD(S)$ by Lemma 2.4, that is, $lD(R) \leq lD(S)$. Similarly we have $wD(R) \leq wD(S)$.

By [7, Lemma 3.1], we get the following.

Corollary 2.6 Let $\varphi : R \to S$ a separable split ring extension with (C4), then $lD(R) = lD(S)$ and $wD(R) = wD(S)$.

The following lemma is very useful for us.

Lemma 2.7 Let $\varphi : R \to S$ be a ring extension with (C3) and (C4).

1. If $M \in R$-Mod is Gorenstein projective (resp. Gorenstein flat), then $S \otimes_R M$ is Gorenstein projective (resp. Gorenstein flat).

2. If $M \in S$-Mod is Gorenstein projective (resp. Gorenstein flat), then ${}_RM$ is Gorenstein projective (resp. Gorenstein flat).

Proof (1) If $M \in R$-Mod is Gorenstein projective, then we have an exact sequence $\xi = \cdots F^{-1} \to F^0 \to F^1 \to \cdots$ of projective R-modules with $M = \ker(F^0 \to F^1)$ and such that it remains exact whenever $\mathrm{Hom}_R(-, P)$ is applied for every projective R-module P. Since S is a flat right R-module, we get that $S \otimes_R \xi$ is exact and $S \otimes_R M = \ker(S \otimes_R F^0 \to S \otimes_R F^1)$. We also get that $S \otimes_R F^i$ is projective for every i. Let us suppose finally that $P \in S$-Mod is projective. Notice that P is also a projective R-module by (C4) and $\mathrm{Hom}_S(S \otimes_R F^i, P) \cong \mathrm{Hom}_R(F^i, \mathrm{Hom}_S(S, P)) \cong \mathrm{Hom}_R(F^i, P)$. Thus $S \otimes_R M$ is Gorenstein projective.

If $M \in R$-Mod is Gorenstein flat, then we have an exact sequence $\xi = \cdots F^{-1} \to F^0 \to F^1 \to \cdots$ of flat R-modules with $M = \ker(F^0 \to F^1)$ and such that it remains exact whenever $E \otimes_R -$ is applied for every injective right R-module E. Since S is a flat right R-module, we get that $S \otimes_R \xi$ is exact and $S \otimes_R M = \ker(S \otimes_R F^0 \to S \otimes_R F^1)$. We also get that $S \otimes_R F^i$ is flat for every i. Let us suppose finally that $E \in S$-Mod is injective. Notice that E is also an injective R-module by (C3) and $E \otimes_S S \otimes_R F^i \cong E \otimes_R F^i$. Thus $S \otimes_R M$ is Gorenstein flat.

(2) If $M \in S$-Mod is Gorenstein projective, then we have an exact sequence $\xi = \cdots F^{-1} \to F^0 \to F^1 \to \cdots$ of projective S-modules with $M = \ker(F^0 \to F^1)$ and such that it remains exact whenever $\mathrm{Hom}_S(-, P)$ is applied for every projective S-module P. Every F^i is a projective left R-module by (C4). Let us suppose finally that $P \in R$-Mod is projective. Notice that $\mathrm{Hom}_R({}_RS, P)$ is also a projective S-module by (C3) and $\mathrm{Hom}_S(F^i, \mathrm{Hom}_R({}_RS, P)) \cong \mathrm{Hom}_R(S \otimes_S F^i, P) \cong \mathrm{Hom}_R(F^i, P)$. Thus M is a Gorenstein projective R-module.

If $M \in S$-Mod is Gorenstein flat, then we have an exact sequence $\xi = \cdots F^{-1} \to F^0 \to F^1 \to \cdots$ of flat S-modules with $M = \ker(F^0 \to F^1)$ and such that it remains exact whenever $E \otimes_S -$ is applied for every injective right S-module E. Every F^i is a flat left R-module by (C4). Let us suppose finally that $E \in R$-Mod is injective. Notice that $E \otimes_R S$ is also an injective S-module by (C3) and $E \otimes_R S \otimes_S F^i \cong E \otimes_R F^i$. Thus M is a Gorenstein flat left R-module.

Lemma 2.8 Let $\varphi : R \to S$ be a ring extension with (C1), (C2) and (C4). Then R is right coherent if and only if S is right coherent.

Proof Let R be right coherent and $\{M_i | i \in I\}$ a family of flat left S-modules. Then every M_i is a flat left R-module by (C4) and $\prod M_i$ is a flat left R-module. And hence $S \otimes_R \prod M_i$ is a flat left S-module. Since ${}_S\prod M_i |_S (S \otimes_R \prod M_i)$, $\prod M_i$ is a flat left S-module which implies that S is right coherent. Conversely, we suppose that S is right coherent. Let $\{M_i | i \in I\}$ be a family of flat left R-modules. It is easy to see that $\prod (S \otimes_R M_i)$ is a flat left S-module. By (C4), it is also a flat left R-module. Note that $\prod (S \otimes_R M_i) \cong S \otimes_R \prod M_i$ since S_R is finitely presented. ${}_R(\prod M_i)|_R(S \otimes_R \prod M_i)$ by (C2). Thus $\prod M_i$ is a flat left R-module, and so R is right coherent.

Corollary 2.9 Let $\varphi : R \to S$ be strongly separable, then R is right coherent if and only if S is right coherent.

We now give a Gorenstein version of Lemma 2.3.

Lemma 2.10 Let $\varphi : R \to S$ be a ring extension with (C1), (C3) and (C4) and M be a left S-modules.

1. M is a Gorenstein projective left R-module if and only if M is a Gorenstein projective left S-module.

2. If S is right coherent, then M is a Gorenstein flat left R-module if and only if M is a Gorenstein flat left S-module.

Proof (1) Assume that ${}_RM$ is Gorenstein projective, then $S \otimes_R M$ is a Gorenstein projective S-module by Lemma 2.7. And hence ${}_SM$ is Gorenstein projective by [15, Theorem 2.5] and (C1). Conversely, it is immediate by Lemma 2.7. The corresponding proof for Gorenstein flat modules are analogous to that of (1).

Theorem 2.11 Let $\varphi : R \to S$ be a ring extension. Then

1. If φ satisfies (C1), (C3) and (C4), then $l.Ggl\dim(S) \leq l.Ggl\dim(R)$. Moreover, if φ also satisfies (C2), then $l.Ggl\dim(S) = l.Ggl\dim(R)$. In particular, if $\varphi : R \to S$ is a strongly separable extension, then $l.Ggl\dim(S) = l.Ggl\dim(R)$.

2. If S is right coherent and φ satisfies $(C1), (C3)$ and (C4), then $l.wGgl\dim(S) \leq l.wGgl\dim(R)$. Moreover, if φ also satisfies (C2), then $l.wGgl\dim(S) = l.wGgl\dim(R)$. In particular, if S is right coherent and $\varphi : R \to S$ is a strongly separable extension, then $l.wGgl\dim(S) = l.wGgl\dim(R)$.

Proof (1) Suppose that $l.Ggl\dim(R) = m < \infty$. Let ${}_SM$ be an S-module. Then $l.Gpd({}_RM) \leq m$. So there exists a Gorenstein projective resolution of ${}_RM$:

$$0 \to G_k \to \cdots \to G_0 \to_R M \to 0, k \leq m,$$

where $G_k, \cdots, G_0$ are Gorenstein projective R-modules. Since $S \otimes_R G_i$ is a Gorenstein projective S-module for all $0 \leq i \leq k$ by Lemma 2.7, we get a Gorenstein projective resolution of the S-module $S \otimes_R M$:

$$0 \to S \otimes_R G_k \to \cdots \to S \otimes_R G_0 \to S \otimes_R M \to 0,$$

but ${}_SM|_S(S \otimes_R M)$ by (C1). So $Gpd_S({}_SM) \leq m$ by [15, Propositions 2.19]. Thus

$$l.Ggl\dim(S) \leq l.Ggl\dim(R).$$

If φ also satisfies (C2), suppose that $l.Ggl\dim(S) = n < \infty$. Let ${}_RM$ be an R-module. Then $l.Gpd_S(S \otimes_R M) \leq n$. So there exists a Gorenstein projective resolution of ${}_S(S \otimes_R M)$:

$$0 \to G_k \to \cdots \to G_0 \to_S (S \otimes_R M) \to 0, k \leq n,$$

where $G_k, \cdots, G_0$ are Gorenstein projective S-modules. Since ${}_RG_i$ is a Gorenstein projective R-module for all $0 \leq i \leq k$ by Lemma 2.7, we get a Gorenstein projective resolution of the R-module ${}_R(S \otimes_R M)$:

$$0 \to G_k \to \cdots \to G_0 \to_R (S \otimes_R M) \to 0, k \leq n,$$

but ${}_RM|_R(S \otimes_R M)$ by (C2). So $Gpd_R({}_RM) \preceq n$ by Lemma [15, Propositions 2.19]. Thus $l.Ggl\dim(R) \leq l.Ggl\dim(S)$ and hence $l.Ggl\dim(R) = l.Ggl\dim(S)$.

(2) Suppose that $l.wGgl\dim(R) = m < \infty$. Let ${}_SM$ be an S-module. Then $l.Gfd({}_RM) \leq m$. So there exists a Gorenstein flat resolution of ${}_RM$:

$$0 \to G_k \to \cdots \to G_0 \to_R M \to 0, k \leq m,$$

where $G_k, \cdots, G_0$ are Gorenstein flat R-modules. Since $S \otimes_R G_i$ is a Gorenstein flat S-module for all $0 \leq i \leq k$ by Lemma 2.7, we get a Gorenstein flat resolution of the S-module $S \otimes_R M$:

$$0 \to S \otimes_R G_k \to \cdots \to S \otimes_R G_0 \to S \otimes_R M \to 0,$$

but ${}_SM|_S(S \otimes_R M)$ by (C1). So $Gfd_S({}_SM) \preceq m$ by [15, Propositions 3.13]. Thus

$$l.wGgl\dim(S) \leq l.wGgl\dim(R).$$

If φ also satisfies (C2), suppose that $l.wGgl\dim(S) = n < \infty$. Let ${}_RM$ be an R-module. Then $l.Gfd_S(S \otimes_R M) \leq n$. So there exists a Gorenstein flat resolution of ${}_S(S \otimes_R M)$:

$$0 \to G_k \to \cdots \to G_0 \to_S (S \otimes_R M) \to 0, k \leq n,$$

where $G_k, \cdots, G_0$ are Gorenstein flat S-modules. Since ${}_RG_i$ is a Gorenstein flat R-module for all $0 \leq i \leq k$ by Lemma 2.7, we get a Gorenstein flat resolution of the R-module ${}_R(S \otimes_R M)$:

$$0 \to G_k \to \cdots \to G_0 \to_R (S \otimes_R M) \to 0, k \leq n,$$

but ${}_RM|_R(S \otimes_R M)$ by (C2). So $Gfd_R({}_RM) \leq n$ by Lemma and [15, Propositions 3.13]. Thus $l.wGgl\dim(R) \leq l.wGgl\dim(S)$ and hence $l.wGgl\dim(R) = l.wGgl\dim(S)$.

It is known that a ring R is quasi-Frobenius if and only if $l.Ggl\dim(R) = 0$, so we have

Corollary 2.12 [10, Proposition 5.13] Let $\varphi : R \to S$ be strongly separable, then R is quasi-Frobenius if and only if S is quasi-Frobenius.

For an Artin algebra R, let R-mod denote the category of finitely generated left R-modules. Recall that a module $M \in R$-mod is called an additive generator for R-mod if any indecomposable module in R-mod is in add$M$. Obviously, an Artin algebra R is of finite representation type if and only if R-mod has an additive generator. Let $Gp(R)$ be the full subcategory of R-mod consisting of Gorenstein projective modules. Clearly, R is CM-finite if and only if there exists a module $N \in R$-mod such that $Gp(R) = $ add$N$.

Theorem 2.13 Let $\varphi : R \to S$ be a ring extension with (C1)–(C4). If R and S are Artin algebras, then

1. R is CM-free if and only if S is CM-free.
2. R is of finite representation type if and only if S is of finite representation type.
3. R is CM-finite if and only if S is CM-finite.

Proof (1) Let R be CM-free and let $M \in S$-mod be Gorenstein projective. Then ${}_RM$ is Gorenstein projective by Lemma 2.7. So ${}_RM$ is projective and hence ${}_SM$ is also projective by Lemma 2.3 (1). Thus S is CM-free. Conversely, Let S be CM-free and $M \in R$-mod be Gorenstein projective. Then ${}_S(S \otimes_R M)$ is Gorenstein projective by Lemma 2.7. So ${}_S(S \otimes_R M)$ is projective and hence ${}_R(S \otimes_R M)$ is also projective by Lemma 2.3. Since ${}_RM|_R(S \otimes_R M)$, ${}_RM$ is projective and so R is CM-free.

(2) Let R be of finite representation type and $M \in R$-mod an additive generator for R-mod. It suffices to prove that $S \otimes_R M$ is an additive generator for S-mod. Let $T \in S$-mod be

indecomposable. Then $T \in R$-mod, $T|M^n$ for some positive integer n. So $S \otimes_R T|(S \otimes_R M)^n$. It follows from (C1) that $T \in \mathrm{add}(S \otimes_R M)$ as S-modules and $S \otimes_R M$ is an additive generator for S-mod. Conversely, if S be of finite representation type, then there exists an additive generator $N \in S$-mod for S-mod. It suffices to prove that N is an additive generator for R-mod. Let $T \in R$-mod be indecomposable. It follows from (C2) that $T|S \otimes_R T$ as R-modules. Note that $S \otimes_R T|N^n$ as S-modules for some positive integer n, so $S \otimes_R T|N^n$ as R-modules and N is an additive generator for R-mod.

(3) The proof is similar to that of (2).

Corollary 2.14 Let $\varphi : R \to S$ be a strongly separable extension of Artin algebras, then R is CM-free (resp., CM-finite, of finite representation type) if and only if so is S.

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关于强可分扩张的注记

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摘要: 设R和S是环, $\varphi: R \to S$是强可分扩张. 本文研究了(Gorenstein)整体维数和表示型在R 与S之间的关系. 利用同调方法, 证明了(1) R 与S 有相同的左整体维数, 左弱整体维数, 左Gorenstein整体维数; (2)若R和S是阿丁代数, 则R是CM-有限的(CM-自由的, 有限表示型) 当且仅当S 是CM-有限的(CM-自由的, 有限表示型), 推广了已知的结果.

关键词: 强可分扩张; 左Gorenstein整体维数; CM-有限的

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