Vol. 37 (2017)
No. 3

数 学 杂 志
J. of Math. (PRC)

ϕ^*-ANALYTIC VECTOR FIELDS IN ALMOST CONTACT MANIFOLDS

CHEN Xiao-min

(*College of Science, China University of Petroleum-Beijing, Beijing 102249, China*)

Abstract: In this article, we introduce the conception of ϕ^*-analytic vector field in almost contact manifold (M, ϕ, ξ, η, g) and study its properties. Making use of the properties of almost contact manifold, we prove that in a contact metric manifold the ϕ^*-analytic vector field v is Killing, and that ϕv must not be ϕ^*-analytic unless zero vector field. Particularly, if M is normal, we get that v is collinear to ξ with constant length, and for the case of three dimensional contact metric manifold it is proved that there does not exist a non-zero ϕ^*-analytic vector field.

Keywords: ϕ^*-analytic vector field; Killing vector field; almost contact structure; contact manifold; Sasakian manifold

2010 MR Subject Classification: 53D10; 53D15; 53C25

Document code: A **Article ID:** 0255-7797(2017)03-0558-09

1 Introduction

Tachiana [10, 11] introduced the conception of contravariant almost analytic vector field in a certain almost Hermitian manifold based on the fact that on a compact Kähler manifold, the inner product of a contravariant vector field and a covariant vector field is constant. Later, this conception was generalized to the general almost complex manifolds by several mathematicians [8, 15]. Precisely, in an almost complex manifold M with almost complex structure J a contravariant almost analytic vector field v is defined by $\mathcal{L}_v J = 0$, where $\mathcal{L}_v$ denotes the Lie derivative along v. Further, Sawaki and Tamakatsu [9] in 1967 defined and studied an extended contravariant almost analytic vector v in an almost complex manifold, namely, in a local orthonormal frame it satisfies $\mathcal{L}_v J^i_j + \lambda J^r_j N^i_{rl} v^l = 0$, here λ is C^∞ scalar function and N^i_{rl} is Nijenhuis tensor. In addition, Tamakatsu gave a decomposition of this kind of vector fields [12].

As an odd dimensional analogue of contravariant almost analytic vector fields in an almost complex manifold, Sato [7] defined a called contravariant C-analytic vector v in a Sasakian manifold with $(1,1)$-tensor ϕ by $(\mathcal{L}_v \phi)\phi = 0$. Following this definition, Eum and

* **Received date:** 2016-05-16 **Accepted date:** 2016-07-22

Foundation item: Supported by the Science Foundation of China University of Petroleum-Beijing (2462015YQ0604) and partially by the Personnel Training and Academic Development Fund (2462015QZDX02).

Biography: Chen Xiaomin (1983–), male, born at Ganzhou, Jiangxi, lectuer, major in symplectic geometry.

Kim [5] provided a definition of contravariant C^*-analytic vector field v in a cosymplectic manifold defined by

$$(\mathcal{L}_v\phi)\phi = 0 \quad \text{and} \quad \mathcal{L}_v\xi = 0.$$

They obtained an unique decomposition of this vector field in compact cosymplectic η-Einstein manifolds. This result is the analogue of a contravariant analytic vector in a compact Kähler space [14] and Sasakian manifold [7], respectively.

Besides, in [4] Deshmukh defined a called ϕ-analytic vector field and studied the case where the induced structure vector ξ in real hypersurfaces of complex projective space $CP^{\frac{n+1}{2}}$ is ϕ-analytic.

For an almost contact manifold with almost contact structure (ϕ, ξ, η), we may consider a vector field v such that it leaves ϕ invariant, i.e., $\mathcal{L}_v\phi = 0$. By a simple calculation we have $\mathcal{L}_v\xi = \sigma\xi$, where σ is a smooth function. Particularly, Ghosh and Sharma proved that σ is constant in a contact metric manifold [6, Lemma 1].

Motivated by the above background, in the present paper we define a vector field called ϕ^*-analytic vector field, which leaves the $(1,1)$-tensor ϕ invariant and satisfies $\sigma = 0$ in an almost contact metric manifold (see Def.2.1). In Section 2, we will give some basic conceptions and properties, and the main results and proofs are showed in Section 3.

2 Definitions, Examples and Basic Properties

Let M^{2n+1} be a $(2n+1)$-dimensional Riemannian manifold. An almost contact structure on M is a triple (ϕ, ξ, η), where ϕ is a $(1,1)$-tensor field, ξ a unit vector field, η a one-form dual to ξ satisfying

$$\phi^2 = -I + \eta \otimes \xi, \quad \eta \circ \phi = 0, \quad \phi \circ \xi = 0. \tag{2.1}$$

A smooth manifold with such a structure is called an almost contact manifold. It is well-known that there exists a Riemannian metric g such that

$$g(\phi X, \phi Y) = g(X, Y) - \eta(X)\eta(Y) \tag{2.2}$$

for any $X, Y \in \mathfrak{X}(M)$. It is easy to get from (2.1) and (2.2) that

$$g(\phi X, Y) = -g(X, \phi Y), \quad g(X, \xi) = \eta(X). \tag{2.3}$$

Due to (2.3), we can decompose the tangent bundle of an almost contact manifold as the orthonormal sum of the codimension 1 bundle $\mathcal{D} = \ker\eta$ and the 1-dimensional foliation defined by ξ. An almost contact structure (ϕ, ξ, η) is said to be normal if the Nijenhuis torsion

$$N_\phi(X, Y) = \phi^2[X, Y] + [\phi X, \phi Y] - \phi[\phi X, Y] - \phi[X, \phi Y] + 2d\eta(X, Y)\xi \tag{2.4}$$

vanishes for any vector fields X, Y on M.

A contact metric manifold is an almost contact manifold (M,ϕ,ξ,η,g) such that the metric g satisfies

$$d\eta(X,Y)=g(X,\phi Y), \tag{2.5}$$

$\forall X,Y\in\mathfrak{X}(M)$. Moreover, it is well-known that there exists a $(1,1)$-tensor field h on a contact manifold defined by $h=\frac{1}{2}\mathcal{L}_\xi\phi$ and it satisfies

$$h\xi=0,\ \ \phi h=-h\phi, \tag{2.6}$$

$$\nabla_X\xi=-\phi X-AX, \tag{2.7}$$

where $A=\phi h$.

From (2.5), we have $\xi\lrcorner d\eta=0$, which means that ξ is a geodesic vector field (see [2, Lemma 6.3.3]). If the structure vector field ξ is Killing, then M is said to be K-contact. It is easy to see that M is K-contact if and only if $h=0$.

Also, if the Nijenhuis tensor of contact manifold $N_\phi=0$ then it is said to be Sasakian, and the following formula is also well-known for a Sasakian manifold (see [1])

$$(\nabla_X\phi)Y=g(X,Y)\xi-\eta(Y)X. \tag{2.8}$$

Moreover, we know that a Sasakian manifold is automatically K-contact and in general the converse is not true, but a three dimensional K-contact manifold is always Sasakian.

Definition 2.1 A vector field v in an almost contact manifold (M,ϕ,ξ,η,g) is called ϕ^*-analytic vector field if it satisfies

$$\mathcal{L}_v\xi=0\quad\text{and}\quad\mathcal{L}_v\phi=0. \tag{2.9}$$

Two equations of (2.9) are respectively equivalent to $[v,\xi]=0$ and $[v,\phi X]=\phi[v,X]$, $\forall X\in\mathfrak{X}(M)$, where $[\cdot,\cdot]$ denotes by the Lie bracket.

Next, we will give an example of ϕ^*-analytic vector field in almost contact manifold.

Example 2.2 Consider the $(2n+1)$-dimensional Euclidean space $\mathbb{R}^{2n+1}$ equipped with the Cartesian coordinates $(x_1,\cdots,x_n,y_1,\cdots,y_n,z)$. Define the almost contact structure (ϕ,ξ,η,g) by

$$\begin{aligned}&\phi(\frac{\partial}{\partial x_i})=-\frac{\partial}{\partial y_i},\quad \phi(\frac{\partial}{\partial y_i})=\frac{\partial}{\partial x_i}+y_i\frac{\partial}{\partial z},\quad \phi(\frac{\partial}{\partial z})=0,\\ &\xi=2\frac{\partial}{\partial z},\quad \eta=\frac{1}{2}(dz-\sum_{i=1}^n y_i dx_i),\quad g=\eta\otimes\eta+\frac{1}{4}\sum_{i=1}^n\big((dx_i)^2+(dy_i)^2\big).\end{aligned}$$

The ϕ^*-analytic vector field v can be written as $v=\sum\limits_{i=1}^n(V^i\frac{\partial}{\partial x_i}+\overline{V}^i\frac{\partial}{\partial y_i})+V^z\frac{\partial}{\partial z}$. By (2.9), we derive that V^i and $\overline{V}^i$ do not depend on z and that the following PDEs hold

$$\begin{aligned}&\frac{\partial V^i}{\partial x_j}=\frac{\partial\overline{V}^i}{\partial y_j},\quad \frac{\partial V^i}{\partial y_j}=-\frac{\partial\overline{V}^i}{\partial x_j},\quad y_i\frac{\partial V^i}{\partial y_j}=\frac{\partial V^z}{\partial y_j},\\ &\overline{V}^j=y_j\frac{\partial V^z}{\partial z}-y_i\frac{\partial\overline{V}^i}{\partial y_j},\quad \frac{\partial V^z}{\partial z}=0.\end{aligned} \tag{2.10}$$

It is clear that there exists a non-zero solution $V^i = c$, $\overline{V}^i = 0$, $V^z = H(x_1, \cdots, x_n)$, where H is a smooth function on $\mathbb{R}^{2n+1}$ and c is a non-zero constant. Hence we see that $c\sum_{i=1}^{n}\frac{\partial}{\partial x_i} + H(x_1, \cdots, x_n)\frac{\partial}{\partial z}$ is a ϕ^*-analytic vector field in $(\mathbb{R}^{2n+1}, \phi, \xi, \eta, g)$.

For any an almost contact manifold, we have the following:

Proposition 2.3 For a ϕ^*-analytic vector field v in almost contact manifold (M, ϕ, ξ, η, g), the following identity holds: $g(\nabla_X v, \xi) + g(\nabla_\xi v, X) = 0$, $\forall X \in \mathfrak{X}(M)$.

Proof From the definition of the ϕ^*-analytic vector field we know $[\xi, v] = 0$, i.e., $\nabla_v \xi = \nabla_\xi v$. Using (2.1), a straightforward computation yields

$$[\phi^2 X, v] = \nabla_{\phi^2 X} v - \nabla_v(\phi^2 X) = -\nabla_X v + \nabla_v X - \eta(\nabla_v X)\xi - g(\nabla_\xi v, X)\xi.$$

On the other hand,

$$\phi[\phi X, v] = \phi^2(\nabla_X v - \nabla_v X) = -\nabla_X v + \nabla_v X + \eta(\nabla_X v)\xi - \eta(\nabla_v X)\xi.$$

Then by replacing X by ϕX in (2.9) and comparing the previous two equations, we complete the proof.

Write $f = g(\xi, v)$, which is a smooth function, then the following corollaries are easy to obtain from Proposition 2.3.

Corollary 2.4 If (M, ϕ, ξ, η, g) is an almost contact manifold with ϕ^*-analytic vector field v, then $v(f) = 0$. In particular, if $v \in \mathcal{D}$ the integral curves of ξ are geodesics.

Proof The first assertion is obvious by making use of Proposition 2.3 with $X = v$. If $v \in \mathcal{D}$, then from Proposition 2.3 with $X = \xi$, we have $g(v, \nabla_\xi \xi) = 0$, i.e., $\nabla_\xi \xi \in \mathrm{Span}\{\xi\}$, so $\nabla_\xi \xi = 0$.

Corollary 2.5 Let v be a ϕ^*-analytic vector in almost contact manifold (M, ϕ, ξ, η, g). If η is closed, then f is constant.

Proof Applying Proposition 2.3, for any vector field X, we have

$$d\eta(X, v) = X(\eta(v)) - v(\eta(X)) - \eta([X, v]) = g(v, \nabla_X \xi) - g(X, \nabla_\xi v) = X(f).$$

Corollary 2.6 Let (M, ϕ, η, ξ, g) be a contact metric manifold with ϕ^*-analytic field v. Then $\nabla_v \xi - \phi \nabla_{\phi v} \xi = -2\phi v$ and $\xi(f) = 0$.

Proof Using (2.7) for $X = v$ and $X = \phi v$, respectively, we have

$$\nabla_v \xi = -\phi v - Av, \ \ \nabla_{\phi v} \xi = v - f\xi - A\phi v.$$

Therefore it completes the proof the first assertion in view of (2.6) and the above two equations.

On other hand, in terms of Proposition 2.3 with $X = \xi$, we have $g(\nabla_\xi v, \xi) = 0$, that is, $\xi(f) = 0$.

3 Main Results and Proofs

In this section, we shall suppose that (M, ϕ, ξ, η, g) is always a contact metric manifold.

Theorem 3.1 A ϕ^*-analytic vector field v in a contact metric manifold M is Killing.

Proof In view of Proposition 2.3, for any $X \in \mathfrak{X}(M)$, we have

$$(v \lrcorner d\eta)(X) = d\eta(v, X) = -X(f),$$

which means that $v \lrcorner d\eta = -df$. Thus $\mathcal{L}_v d\eta = d(v \lrcorner d\eta) = 0$. Since for a contact metric manifold the associated metric g may write as $d\eta \circ (\phi \otimes \mathbb{I}) + \eta \otimes \eta$ by (2.5), where $\mathbb{I}$ is the identical map on TM, we get $\mathcal{L}_v g = d\eta \circ (\mathcal{L}_v \phi \otimes \mathbb{I})$. Therefore we complete the proof from (2.9).

Remark 3.2 In fact, a Killing vector field is not necessary ϕ^*-analytic, however, in particular, if the structure vector field ξ is Killing then the converse of Theorem 3.1 is valid (see [2, Proposition 6.6.12]).

From [13, Theorem 3.4], we have

Corollary 3.3 The ϕ^*-analytic vector field v in contact metric manifold satisfies $\Delta v + Qv = 0$, here Δ and Q denote by the Laplace operator and Ricci operator, respectively.

Next we will prove the following conclusion.

Theorem 3.4 Let (M, ϕ, η, ξ, g) be a contact metric manifold with ϕ^*-analytic field v. Then ϕv must be not a ϕ^*-analytic vector field unless zero vector.

To prove this theorem we need the following two lemmas.

Lemma 3.5 Under the assumption of Theorem 3.4, if ϕv is also a ϕ^*-analytic vector field, then $\nabla_\xi v = -\phi v$.

Proof Since $v \lrcorner d\eta = -df$, using (2.5) we have $\phi v = Df$, where D denotes by the gradient operator. Thus if ϕv is ϕ^*-analytic, $[\phi X, Df] = \phi[X, Df]$ for any $X \in \mathfrak{X}(M)$, which implies $[\xi, Df] = 0$. It reduces that $hv = 0$ from the definition of h. Hence we complete the proof in terms of (2.7).

Lemma 3.6 Under the assumption of Theorem 3.4, if ϕv is also a ϕ^*-analytic vector field then

$$(\nabla_X \phi)v = g(v, X)\xi - fX.$$

Proof For any contact metric manifold the following identity (see [2, Lemma 7.3.2]) holds

$$2g((\nabla_X \phi)Y, Z) = g(N_\phi(Y, Z), \phi X) + 2d\eta(\phi Y, X)\eta(Z) - 2d\eta(\phi Z, X)\eta(Y)$$

for any $X, Y, Z \in \mathfrak{X}(M)$.

Since v and ϕv are ϕ^*-analytic vector fields, we obtain

$$\begin{aligned} 2g((\nabla_X \phi)v, Y) &= g(N_\phi(v, Y), \phi X) + 2d\eta(\phi v, X)\eta(Y) - 2d\eta(\phi Y, X)\eta(v) \\ &= g((\mathcal{L}_{\phi v}\phi)Y + \phi(\mathcal{L}_v \phi)Y, \phi X) - 2g(\phi^2 v, X)\eta(Y) + 2g(\phi^2 Y, X)\eta(v) \\ &= -2g(\phi^2 v, X)\eta(Y) + 2g(\phi^2 Y, X)\eta(v). \end{aligned}$$

Hence

$$(\nabla_X \phi)v = -g(\phi^2 v, X)\xi + \eta(v)\phi^2 X = g(v, X)\xi - fX.$$

Proof of Theorem 3.4 We assume that ϕv is also a ϕ^*-analytic vector field, then ϕv is also Killing because of Theorem 3.1, namely, for any $X, Y \in \mathfrak{X}(M)$, we have

$$g(\nabla_X(\phi v), Y) + g(\nabla_Y(\phi v), X) = 0.$$

Making use of Lemma 3.6, we obtain

$$g(v, X)\eta(Y) + g(v, Y)\eta(X) - 2fg(X, Y) + g(\phi\nabla_X v, Y) - g(\nabla_Y v, \phi X) = 0.$$

Hence using Theorem 3.1 gives

$$g(v, X)\xi + \eta(X)v - 2fX + \phi\nabla_X v + \nabla_{\phi X} v = 0. \tag{3.1}$$

By replacing ϕX by X in (3.1) and using Lemma 3.5, we get

$$g(v, \phi X)\xi - 2f\phi X + \phi\nabla_{\phi X} v - \nabla_X v - \eta(X)\phi v = 0. \tag{3.2}$$

Operating ϕ onto (3.2) and using Proposition 2.3, we have

$$2fX - 4f\eta(X)\xi - \nabla_{\phi X} v + g(v, X)\xi - \phi\nabla_X v + \eta(X)v = 0.$$

Thus by comparing with (3.1) it yields that $2f\eta(X)\xi - g(v, X)\xi - \eta(X)v = 0$ for any $X \in \mathfrak{X}(M)$, which implies $\phi^2 v = 0$ by taking $X = \xi$ in the above formula. So ϕv must be identically zero. We complete the proof.

For the case where M is a normal contact metric manifold, i.e., a Sasakian manifold, we have

Theorem 3.7 Let v be a ϕ^*-analytic vector field in a Sasakian manifold. Then $f = g(v, \xi)$ is constant and v is collinear to ξ with constant length.

Proof In the proof of Theorem 3.1, we have known $v \lrcorner d\eta = -df$. Since M is Sasakian, $N_\phi = 0$. Thus it follows from (2.4) and (2.9) that $N_\phi(X, v) = 2d\eta(X, v)\xi = 0$ for every field X, that means that $df = 0$.

On the other hand, from (2.5) and Proposition 2.3, we know that $\phi v = Df$. Hence $v = -\phi Df + f\xi = f\xi$. We complete the proof of theorem.

At last we consider that M is a three dimensional contact metric manifold. Let U be the open subset where the tensor $h \neq 0$ and U' be the open subset such that h is identically zero. Thus $U \cup U'$ is open dense in M. Assume that M is non-K-contact, then U is non-empty and there exits a local orthonormal frame field $\mathcal{E} = \{e_1, e_2 = \phi e_1, \xi\}$ such that $he_1 = \mu e_1$ and $he_2 = -\mu e_2$, where μ is a positive non-vanishing smooth function of M. With respect to the frame field, we have

Lemma 3.8 [3] Let $(M^3, \phi, \eta, \xi, g)$ be a contact metric manifold. Then with respect to $\mathcal{E}$ the Levi-Civita connection ∇ is given by

$$\begin{aligned}
&\nabla_{e_1} e_1 = be_2,\ \nabla_{e_1} e_2 = -be_1 + (1+\mu)\xi,\ \nabla_{e_1}\xi = -(1+\mu)e_2,\\
&\nabla_{e_2} e_1 = -ce_2 + (\mu - 1)\xi,\ \nabla_{e_2} e_2 = ce_1,\ \nabla_{e_2}\xi = (1-\mu)e_1,\\
&\nabla_\xi e_1 = -ae_2,\ \nabla_\xi e_2 = ae_1,\ \nabla_\xi \xi = 0,
\end{aligned}$$

where a, b and c are smooth functions.

Theorem 3.9 There does not exist a non-zero ϕ^*-analytic vector field in a three dimensional contact metric manifold.

Proof First, for $\mathcal{E}$, $e_3 = \xi$ is globally defined, thus we can define the global frame field, still denoted by $\{e_1, e_2, e_3 = \xi\}$, by lifting to the universal covering space $\widetilde{M}^3$ if necessary. Since $v = -\phi Df + f\xi$, with respect to the orthonormal frame field $\mathcal{E}$, v can be also written as $v = e_2(f)e_1 - e_1(f)e_2 + f\xi$. In view of Lemma 3.8 and Corollary 2.6, we compute

$$\begin{aligned}\nabla_\xi v &= \xi(e_2(f))e_1 + e_2(f)\nabla_\xi e_1 - \xi(e_1(f))e_2 + e_1(f)\nabla_\xi e_2\\ &= \xi(e_2(f))e_1 - ae_2(f)e_2 - \xi(e_1(f))e_2 + ae_1(f)e_1\\ &= \Big(\xi(e_2(f)) + ae_1(f)\Big)e_1 - \Big(ae_2(f) + \xi(e_1(f))\Big)e_2.\end{aligned} \tag{3.3}$$

Similarly, we have

$$\nabla_{e_1} v = \Big[e_1(e_2(f)) + be_1(f)\Big]e_1 + \Big[be_2(f) - e_1(e_1(f)) - f(\mu+1)\Big]e_2 - e_1(f)\mu\xi \tag{3.4}$$

and

$$\nabla_{e_2} v = \Big[e_2(e_2(f)) - ce_1(f) + f(1-\mu)\Big]e_1 - \Big[e_2(f)c + e_2(e_1(f))\Big]e_2 + e_2(f)\mu\xi. \tag{3.5}$$

By Theorem 3.1, we know

$$g(\nabla_\xi v, e_1) + g(\nabla_{e_1} v, \xi) = 0, \quad g(\nabla_\xi v, e_2) + g(\nabla_{e_2} v, \xi) = 0.$$

Thus making use of (3.3), (3.4) and (3.5), we obtain

$$\begin{cases} \xi(e_2(f)) = (\mu - a)e_1(f),\\ \xi(e_1(f)) = (\mu - a)e_2(f).\end{cases} \tag{3.6}$$

On the other hand, we notice that for a ϕ^*-analytic vector field, $\nabla_\xi v = \nabla_v \xi$, i.e.,

$$\nabla_\xi v = e_2(f)\nabla_{e_1}\xi - e_1(f)\nabla_{e_2}\xi = -e_1(f)(1-\mu)e_1 - e_2(f)(1+\mu)e_2.$$

By comparing with (3.3), we arrive at

$$\begin{cases} \xi(e_2(f)) = -(a + 1 - \mu)e_1(f),\\ \xi(e_1(f)) = (1 + \mu - a)e_2(f).\end{cases} \tag{3.7}$$

It follows from (3.6) and (3.7) that $e_1(f) = e_2(f) = 0$. Thus f is constant and $v = f\xi$.

By Theorem 3.1, v is Killing, thus we find that ξ is also Killing, namely, M is K-contact, so $h = 0$. It yields a contradiction, thus we complete the proof.

Next we apply Example 2.2 to check both Theorem 3.4 and Theorem 3.9.

Example 3.10 We consider $\mathbb{R}^{2n+1}$ equipped with contact structure (ϕ, ξ, η, g) as in Example 2.2. Let $v = \sum\limits_{i=1}^{n}(V^i \frac{\partial}{\partial x_i} + \overline{V}^i \frac{\partial}{\partial x_i}) + V^z \frac{\partial}{\partial z}$ be a ϕ^*-analytic vector field, then

$$\phi v = \sum_{i=1}^{n}(-V^i \frac{\partial}{\partial y_i} + \overline{V}^i \frac{\partial}{\partial x_i}) + \sum_{i=1}^{n} y_i \frac{\partial}{\partial z}.$$

If ϕv is also ϕ^*-analytic, we get

$$y_i \frac{\partial \overline{V}^i}{\partial y_j} = \frac{\partial}{\partial y_j} \sum_{i=1}^{n} y_i = 1 \quad \text{for all} \quad j.$$

But from the fourth equation of (2.10) we find $\overline{V}^j = 1$ for all j. It comes to a contradiction. Thus ϕv can not be ϕ^*-analytic. It is consistent with the result of Theorem 3.4.

For the case of three dimension, if v is a ϕ^*-analytic vector field, then we shall prove that v is a zero vector field. We know that the ϕ^*-analytic vector field can be written as $v = -\phi Df + f\xi$, thus a straightforward computation yields

$$\begin{cases} V = -\frac{\partial f}{\partial y}, \\ \overline{V} = \frac{\partial f}{\partial x}, \\ -y\frac{\partial f}{\partial y} = \frac{3}{2} f. \end{cases} \tag{3.8}$$

By virtue of the fourth equation of (2.10) and the third equation of (3.8), we have

$$\frac{\partial f}{\partial x} = -y \frac{\partial^2 f}{\partial x \partial y} = \frac{3}{2} \frac{\partial f}{\partial x},$$

i.e., $\overline{V} = \frac{\partial f}{\partial x} = 0$. Moreover, differentiating the third equation of (3.8) with respect to y gives

$$-\frac{\partial f}{\partial y} - y \frac{\partial^2 f}{\partial y^2} = \frac{3}{2} \frac{\partial f}{\partial y}.$$

That means that $V = -\frac{\partial f}{\partial y} = 0$ since the first equation of (3.8) and the second equation of (2.10) imply $\frac{\partial^2 f}{\partial y^2} = -\frac{\partial V}{\partial y} = \frac{\partial \overline{V}}{\partial x} = 0$. Further, we have $f = 0$ by using the third equation of (3.8) again, i.e., $v = 0$, which concides with the conclusion of Theorem 3.9.

References

[1] Blair D E. Rimemannian geometry of contact and symplectic manifolds[M]. Prog. Math., Vol. 203, Boston, MA: Birkhäuser Boston Inc., 2010.

[2] Boyer C P, Galicki K. Sasakian geometry[M]. Oxford: Oxford Univ. Press, 2008.

[3] Cho J T. Contact 3-manifolds and Ricci solions[J]. Int. J. Geom. Meth. Mod. Phys., 2012, 10: 515–532.

[4] Deshmukh S. Real hypersurfaces of a complex projective space[J]. Proc. Indian Acad. Sci. (Math. Sci.), 2011, 121: 171–179.

[5] Eum S S, Kim U K. On a decomposition of contravariantt C^*-analytic vector fields in a compact cosymplectic η-manifold[J]. J. Korean Math Soc., 1981, 18(1): 63–71.

[6] Ghosh A, Sharma R. Sasakian metric as a Ricci soliton and related results[J]. J. Geom. Phys., 2014, 75: 1–6.

[7] Sato I. On contravariant C-analytic vector fields in a compact Sasakian spaces[J]. Diff. Geom., 1972: 437–441.

[8] Sato I. Almost analytic vector fields in almost complex manifolds[J]. Tôhoku. Math. J., 1965, 17: 185–199.

[9] Swaki S, Takamatsu K. On extended almost analytic vectors and tensors in almost complex manifolds[J]. Sci. Rep. Niigata Univ. Ser. A, 1967, 4: 17–29.

[10] Tachibana S. On analytic vectors in almost Kählerian manifolds[J]. Tôhoku. Math. J., 1959, 11: 247–265.

[11] Tachibana S. On analytic vectors in certain almost Hermitian manifolds[J]. Tôhoku. Math. J., 1959, 11: 351–363.

[12] Takamatsu K. On a decomposition of an extended almost analytic vectors in a K-space with constant scalar curvature[J]. Kodai Math. Sem. Rep., 1968, 20: 186–197.

[13] Yano K. Integral formula in Riemannian geometry[M]. New York: Macrcel Dekker, 1970.

[14] Yano K. Differential geometry on complex and almost complex spaces[M]. Oxford: Pergamon Press, 1965.

[15] Yano K, Ako M. Almost analytic vectors in almost complex spaces[J]. Tôhoku. Math. J., 1961, 13: 24–45.

近切触流形的ϕ^*-解析向量场

陈小民

(中国石油大学(北京)理学院, 北京 102249)

摘要: 本文引入了近切触流形(M,ϕ,ξ,η,g)中ϕ^*-解析向量场的概念, 并研究了其性质. 利用近切触流形的性质, 证明了切触度量流形中的ϕ^*-解析向量场v是Killing 向量场且ϕv不是ϕ^*-解析的. 特别地, 如果近切触流形M是正规的, 得到v与ξ 平行且模长为常数. 另外, 证明了3维的切触度量流形不存在非零的ϕ^*-解析向量场.

关键词: ϕ^*-解析向量场; Killing向量场; 近切触结构; 切触度量流形; Sasaki流形

MR(2010)**主题分类号:** 53D10; 53D15; 53C25 **中图分类号:** O186.12