Vol. 36 (2016)
No. 5

数 学 杂 志
J. of Math. (PRC)

STABILITY AND HOPF BIFURCATION FOR A DIFFERENTIAL ALGEBRAIC PREDATOR-PREY SYSTEM WITH NONLINEAR HARVESTING AND GESTATIONAL TIME DELAY

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Abstract: In this paper, we investigate the stability and Hopf bifurcation for a new differential algebraic predator-prey system which combined with nonlinear harvesting in prey and gestational time delay of predator. Using bifurcation theorem and stability theorem, through considering gestational time delay of predator as bifurcation parameter, we obtain the interrelated stability criterion and the related conditions of producing Hopf bifurcation at the positive equilibrium point of the proposed system, which popularize the conclusions of the general differential algebraic predator-prey system which combined with linear harvesting and time delay.

Keywords: biological economic; differential algebraic system; nonlinear harvesting; gestational time delay; Hopf bifurcation; stability

2010 MR Subject Classification: 34C23; 34C25; 34D20; 34K15; 92B05

Document code: A **Article ID:** 0255-7797(2016)05-0993-12

1 Introduction

In biology dynamics, the interactions between species can be summed up in the power system, power system can be divided into the predator-prey system, competition and mutualism system, such as the predator-prey system has been highly valued in ecological research at this years. In today's economic and social life, it is necessary to consider thing's economic benefit, therefore on the basis of the biological population theory, considering the economic benefit of ecological economics come into being. Thus, the research of ecological economics which is based on the biological population theory makes meaningful for protecting the balance of ecological and reflecting its economic.

Mathematical biology's application has an immense impact towards the development of commonly used biological resources like fishery. Recently, scientists and researchers gave emphasis on the interaction between mathematics and biology which initiate a new research area. Some fundamental issues in biology appear to require new thoughts quantitatively or

* **Received date:** 2014-04-14 **Accepted date:** 2014-06-23

Foundation item: Supported by the Project of Higher School Outstanding Youth Scientific and Technological Innovation Team of Hubei Province (T201412).

Biography: Li Meng (1988–), male, born at Wuhan, Hubei, master, major in differential equations and control theory.

analytically. Most of our biological theories evolve rapidly, therefore it is necessary to develop some useful mathematical models to describe the consequences of these biological systems. It is observed that these newly developed mathematical models are significantly influenced through the biological theories in the past and the consequent expansion of those theories in recent time. For this purpose, differential algebraic equations can be considered as an important tool for the analysis of a biological model. In the early 1970s, Rosenbrock used the differential algebraic system method which was used to research the complex electrical network system at a first time to study the predator-prey system. Through on the basis of the original differential system, increase a algebra system to describe its economic benefit, it makes the system better realistic. Thus, differential algebraic predator-prey system has gradually become the hot issue for the scientific research personnel.

In this years, researches made a lot of contributions in the research of predator-prey system, and the research is greater. There was rapidly growing interest in the analysis and modelling of predator-prey systems [1–11]. Many authors [3–7] studied the dynamics of predator-prey models with harvesting, and obtained complex dynamic behaviors, such as Hopf bifurcation, direction of periodic solutions bifurcating from Hopf bifurcation, flip bifur cation, Bogdanov-Taken s bifurcation, limit cycle and so on. Some references [12–15] and Liu et al. [16, 17] formulated a class of differential algebraic predator-prey system, which investigated the interaction mechanism, the effect of the harvesting effort from an economic perspective and the bifurcation of the system. [18] studied the dynamic behavior of the proposed biological economic predator-prey mode, he discussed the Hopf bifurcation. Chen [19] considered normal forms for differential algebraic system.

Orosz [20] presented a formal framework for the analysis of Hopf bifurcations in delay differential equations with a single time delay. He determined closed-form linear algebraic equations and calculated the criticality of bifurcations by normal forms. Cao and Freedman [21] obtained the criterion of persistence and global attractivity for a predator-prey model with time delay due to gestation. Yafia et al. [22] considered a model with one delay and a unique non trivial equilibrium. They studied the dynamics of the model in terms of the local stability and of the description of the Hopf bifurcation at non trivial equilibrium. They proved that delay (taken as a parameter of bifurcation) crosses some critical values and determined the direction of the Hopf bifurcation and the stability or instability of the bifurcating branch of periodic solutions. Kar [23] studied a Gauss-type prey predator model with selective harvesting and introduce a time delay in the harvesting term. He concluded, in general, delay differential equations exhibit much more complicated dynamics than ordinary differential equations since time delay could cause a stable equilibrium to become unstable and cause the population to fluctuate.

However, to the best of our knowledge, this paper mainly studies the stability and bifurcations of a new biological economic system formulated by differential algebraic equations. Different from those presented in Zhang [24] and Liu [25, 26] which their harvesting of the system is linear, but the harvesting of our bioeconomic system is nonlinear in accordance with the actual situation of the real world. Besides, the continuous gestation delay of predation is also incorporated in our system. Our Hopf bifurcation analysis for the predator-prey system is based on the delyed differential algebraic predator-prey system, which it appears to be more complete for the real world to take account the delay of the system by using this way in our model. Then, it has taken a comprehensive analysis for the predator-prey system.

The organization of this paper is as follows. In Section 2, we introduce the building of our system. In Section 3, we discuss Hopf bifurcation of the positive equilibrium point depending on the parametermfor system (2.5) through considering delay as a bifurcation parameter. Numerical simulations inspect the effectiveness of mathematical conclusions in Section 4. Finally, this paper put across by a concise discussion and summary.

2 Mathematical Model

Our model is based on the following classical prey-dependent predator-prey system [27]:

$$\begin{cases} \frac{dx}{dt} = rx(1-\frac{x}{k}) - \frac{exy}{a+x}, \\ \frac{dy}{dt} = y\left(-d + \frac{hx}{a+x}\right), \end{cases} \tag{2.1}$$

where x and y are interpreted as the densities of prey and predator population at time t, and r, d are the intrinsic growth rate of prey and the death rate of predator in the absence of food, k is the carrying capacity of prey, separately, a, h are the capture saturation constant and the maximal predator.

Biological resources in the prey-predator system are most likely to be harvested and sold with the purpose of achieving the economic interest which motivates the introduction of harvesting in the prey-predator model. Let H represent the harvesting, then $H = qEx$ where E denotes the effort applied to harvest the prey species and q is the catchability coefficient. This function embodies unrealistic features, unbounded linear increase of H with E for a fixed x, and unbounded linear increase of H with x for a fixed E. These restrictive feature are removed in the functional form which was proposed first by Clark [28] as follows

$$H(t) = \frac{qE(t)x(t)}{m_1E(t) + m_2x(t)}.$$

Amongst the several types of harvesting Michaelis-Menten type harvesting is more realistic. This kinds of nonlinear harvesting is more realistic from biological and points of view.

Substituting these following dimensionless variables in system (2.1)

$$e = a_1, a = n, h = a_2s, d = s.$$

Then combined with nonlinear harvesting, the classical prey-dependent predator-prey model is given by

$$\begin{cases} \frac{dx}{dt} = rx(1-\frac{x}{k}) - \frac{a_1xy}{n+x} - \frac{qEx}{m_1E+m_2x}, \\ \frac{dy}{dt} = sy\left(-1 + \frac{a_2x}{n+x}\right), \end{cases} \tag{2.2}$$

where q is the catchability coefficient, E is the effort applied to harvest the prey species, and m_1, and m_2 are suitable constants. All the parameters are assumed to be positive due to biological considerations.

In 1954, Gordon studied the effect of the harvest effort on ecosystem from an economic perspective and proposed the following economic principle, which the affect of the harvesting effort on the economic system was researched from an point of economic. The equation proposed in [29] to investigate the economic interest of the yield of the harvesting effort takes the following form

$$\text{Net Economic Revenue}(NER) = \text{Total Revenue}(TR) - \text{Total Cost}(TC).$$

Referring to the predator-prey system (2.2), we get

$$NER = m, \quad TR = \frac{pqEx}{m_1E + m_2x}, \quad TC = \frac{cqE}{m_1E + m_2x}.$$

Substituting them into the economic theory equation mentioned above, then we can obtain the following algebraic equation

$$\frac{qE}{m_1E + m_2x}(px - c) = m,$$

respectively, p, c represent harvesting reward per unit harvesting effort for unit weight and harvesting cost per unit harvest effort.

And then, combined with the following biological economic algebraic equation, system (2.2) can be expressed by differential algebraic equation

$$\begin{cases} \frac{dx}{dt} = rx(1 - \frac{x}{k}) - \frac{a_1xy}{n+x} - \frac{qEx}{m_1E+m_2x}, \\ \frac{dy}{dt} = sy(-1 + \frac{a_2x}{n+x}), \\ 0 = \frac{qE}{m_1E+m_2x}(px - c) - m. \end{cases} \tag{2.3}$$

Let us now consider this harvested predator-prey system with continuous time delay due to gestation of predator. Consider the new variable z, called information variable which summarizes information about the current state of the prey biomass in predator's equation, i.e., depends on current values of state variables and also summarizes information about past values of state variables. We take up the formula

$$z(t) = \int_{-\infty}^{t} g(x(t), y(t))k(t - \tau_0)d\tau_0,$$

where $k(t-\tau_0)$ is the entire past history of prey biomass, $\tau_0 < t$ is considered as a particular time in the past and t represents the present time. Here the predator population consumes the prey population at a constant a_2, but the reproduction of predators after predating the prey population is not instantaneous thus it will be incorporated by some time lag required for gestation of predators. Let the time interval between the moments when an individual prey is killed and the corresponding biomass is added to the predator population is considered as the time delay τ. Then we considerd $g(x(t), y(t)) = x(t)$ and

$$k(t-\tau_0) = \frac{1}{\tau}\exp(-\frac{1}{\tau}(t-\tau_0)),$$

then the prey biomass in predator's equation is replaced as follows

$$z(t) = \int_{-\infty}^{t} x(t)\frac{1}{\tau}\exp(-\frac{1}{\tau}(t-\tau_0))d\tau_0.$$

From the above assumptions we consider the following system

$$\begin{cases} \frac{dx}{dt} = rx(1-\frac{x}{k}) - \frac{a_1xy}{n+x} - \frac{qEx}{m_1E+m_2x}, \\ \frac{dy}{dt} = sy(-1+\frac{a_2z}{n+z}), \\ z(t) = \displaystyle\int_{-\infty}^{t} x(t)\frac{1}{\tau}exp(-\frac{1}{\tau}(t-\tau_0))d\tau_0, \\ 0 = \frac{qE}{m_1E+m_2x}(px-c) - m. \end{cases} \tag{2.4}$$

Then the nonlinear integro-differential algebraic system can be transformed into the following set of nonlinear ordinary differential algebraic system, we obtain a delayed differential algebraic predator-prey system with nonlinear harvesting in prey

$$\begin{cases} \frac{dx}{dt} = rx(1-\frac{x}{k}) - \frac{a_1xy}{n+x} - \frac{qEx}{m_1E+m_2x}, \\ \frac{dy}{dt} = sy(-1+\frac{a_2z}{n+z}), \\ \frac{dz}{dt} = \frac{1}{\tau}(x-z), \\ 0 = \frac{qE}{m_1E+m_2x}(px-c) - m. \end{cases} \tag{2.5}$$

In this paper, we mainly discuss the effects of the economic profit on the dynamic of system (2.5) in the region $R_+^4 = \{(x,y,z,E)|x \geq 0, y \geq 0, z \geq 0, E \geq 0\}$. For convenience, let

$$f(m, X_1, E) = \begin{pmatrix} f_1(m, X_1, E) \\ f_2(m, X_1, E) \\ f_3(m, X_1, E) \end{pmatrix} = \begin{pmatrix} x(r - \frac{r}{k}x - \frac{a_1y}{n+x} - \frac{qE}{m_1E+m_2x}) \\ sy(-1+\frac{a_2z}{n+z}) \\ \frac{1}{\tau}(x-z) \end{pmatrix},$$

$$g(m, X_1, E) = \frac{qE}{m_1E+m_2x}(px-c) - m,$$

where $X_1 = (x, y, z)^T$.

3 Hopf Bifurcation and Stability for Positive Equilibrium Point

From system (2.5), we know that $P := (\hat{x}, \hat{y}, \hat{z}, \hat{E})$ is a positive equilibrium of system (2.5) if and if only this point P is a solution of the following equations

$$\begin{cases} rx(1-\frac{x}{k}) - \frac{a_1xy}{n+x} - \frac{qEx}{m_1E+m_2x} = 0, \\ sy(-1+\frac{a_2z}{n+z}) = 0, \\ \frac{1}{\tau}(x-z) = 0, \\ \frac{qE}{m_1E+m_2x}(px-c) - m = 0. \end{cases} \tag{3.1}$$

Through a simple calculation, we obtain

$$P := (\hat{x}, \hat{y}, \hat{z}, \hat{E}) = (\frac{n}{a_2 - 1}, \frac{n + x_0}{a_1}(r - \frac{r}{k}x_0 - \frac{qE_0}{m_1E_0 + m_2x_0}), \frac{n}{a_2 - 1}, \frac{mm_2x_0}{qpx_0 - qc - mm_1}).$$

In this paper, we concentrate on the interior equilibrium $P(\hat{x}, \hat{y}, \hat{z}, \hat{E})$ of system (2.5), since the biological meaning of the interior equilibrium implies that the prey, the predator and the harvest effort on prey all exist, which are relevant to our study. Thus throughout the paper, we assume that

$$a_2 - 1 > 0, r - \frac{r}{k}x_0 - \frac{qE_0}{m_1E_0 + m_2x_0} > 0,\ qpx_0 - qc - mm_1 > 0,\ px_0 - 1 > 0.$$

Then we study the Hopf bifurcation stability of positive point $P(\hat{x}, \hat{y}, \hat{z}, \hat{E})$, which is the interior equilibrium of system (2.5). Combined with the above analysis of the model of system (2.5), we have the following matrix from system (2.5)

$$\begin{aligned} \overline{M} &= D_{X_1}f - D_Ef(D_Eg)^{-1}D_{X_1}g \\ &= \begin{pmatrix} r - \frac{2rx}{k} - \frac{a_1ny}{(n+x)^2} + \frac{qcE}{(px-c)(m_1E+m_2x)} & -\frac{a_1x}{n+x} & 0 \\ 0 & -s + \frac{a_1sz}{n+z} & \frac{a_2sny}{(n+z)^2} \\ \frac{1}{\tau} & 0 & -\frac{1}{\tau} \end{pmatrix}. \end{aligned}$$

As $P(\hat{x}, \hat{y}, \hat{z}, \hat{E})$ is the interior equilibrium points of system (2.5), and it is satisfied with equation (3.1), then we get the following matrix

$$M = \begin{pmatrix} \frac{a_1\hat{x}\hat{y}}{(n+\hat{x})^2} - \frac{r}{k}\hat{x} + \frac{pm\hat{x}}{(p\hat{x}-c)^2} & -\frac{a_1\hat{x}}{n+\hat{x}} & 0 \\ 0 & 0 & \frac{a_2sn\hat{y}}{(n+\hat{z})^2} \\ \frac{1}{\tau} & 0 & -\frac{1}{\tau} \end{pmatrix}.$$

Thus the characteristic polynomial of the matrix M is given by

$$v^3 + l_1v^2 + l_2v + l_3 = 0,$$

where

$$\begin{aligned} l_1 &= \frac{r}{k}\hat{x} - \frac{a_1\hat{x}\hat{y}}{(n+\hat{x})^2} - \frac{pm\hat{x}}{(p\hat{x}-c)^2} + \frac{1}{\tau}, \\ l_2 &= \frac{1}{\tau}(\frac{r}{k}\hat{x} - \frac{a_1\hat{x}\hat{y}}{(n+\hat{x})^2} - \frac{pm\hat{x}}{(p\hat{x}-c)^2}), \\ l_3 &= \frac{sa_1a_2n\hat{x}\hat{y}}{\tau(n+\hat{x})(n+\hat{z})^2}. \end{aligned}$$

We obtain $l_1 > 0, l_2 > 0$ if $\frac{r}{k}\hat{x} - \frac{a_1\hat{x}\hat{y}}{(n+\hat{x})^2} - \frac{pm\hat{x}}{(p\hat{x}-c)^2 > 0}$. And assuming $B(\tau) = l_1l_2 - l_3$, then

$$B(\tau) = \frac{1}{\tau^2}(t_1 + t_1\tau),$$

where

$$t_1 = \frac{r}{k}\hat{x} - \frac{a_1\hat{x}\hat{y}}{(n+\hat{x})^2} - \frac{pm\hat{x}}{(p\hat{x}-c)^2},$$
$$t_1 = (\frac{r}{k}\hat{x} - \frac{a_1\hat{x}\hat{y}}{(n+\hat{x})^2} - \frac{pm\hat{x}}{(p\hat{x}-c)^2})^2 - \frac{sa_1a_2n\hat{x}\hat{y}}{(n+\hat{x})(n+\hat{z})^2}.$$

Now we have the following theorem which ensures the local stability of the intreior equilibrium point, $P(\hat{x}, \hat{y}, \hat{z}, \hat{E})$ of the model system (2.5).

Theorem 3.1 If $P(\hat{x}, \hat{y}, \hat{z}, \hat{E})$ exists with $\frac{r}{k} > \frac{a_1\hat{y}}{(n+\hat{x})^2} + \frac{pm}{(p\hat{x}-c)^2}$ and $t_2\tau + t_1 > 0$, then $P(\hat{x}, \hat{y}, \hat{z}, \hat{E})$ is locally asymptotically stable.

Proof With
$$\frac{r}{k} > \frac{a_1\hat{y}}{(n+\hat{x})^2} + \frac{pm}{(p\hat{x}-c)^2}$$
and
$$t_2\tau + t_2 > 0,$$
we can get that $l_1 > 0$, $l_3 > 0$. Then $t_2\tau + t_1 > 0$ implies that $B(\tau) = l_1l_2 - l_3 > 0$. Hence by Hurwitz criterion, the theorem follows.

Delayed differential algebraic predator-prey system with nonlinear harvesting in prey with constant parameters are often found to approach a steady state in which the species coexist in equilibrium. But if parameters used in the model are changed, other types of dynamical behavior may occur and the critical parameter values at which such transitions happen are called bifurcations.

According to this, we have the following theorem which uses to analyze the Hopf bifurcation of system (2.5) assuming τ as the bifurcation parameter.

Theorem 3.2 If $P(\hat{x}, \hat{y}, \hat{z}, \hat{E})$ exists with $\frac{r}{k} > \frac{a_1\hat{y}}{(n+\hat{x})^2} + \frac{pm}{(p\hat{x}-c)^2}$ and $\tau < \frac{2f}{\frac{2a_1^2n\hat{x}\hat{y}}{(n+\hat{x})(n+\hat{z})^2} - f^2}$, then a simple Hopf bifurcation occurs at the positive unique value $\tau = \tau^*$, where $\tau^* = \frac{2f}{\frac{2a_1^2n\hat{x}\hat{y}}{(n+\hat{x})(n+\hat{z})^2} - f^2}$.

Proof We know that the characteristic equation of system (2.5) at $P(\hat{x}, \hat{y}, \hat{z}, \hat{E})$ is given by
$$v^3 + l_1(\tau)v^2 + l_2(\tau)v + l_3(\tau) = 0. \tag{3.2}$$

Equation (3.2) has two purely roots if and if $l_1l_2 = l_3$ for a unique value of τ_* at which we have a Hopf bifurcation. And characteristic (2.5) can't have real roots in the neighborhood of τ_*.

Then we can get
$$(v^2 + l_2)(v + l_1) = 0.$$

This equation has two purely imaginary roots and a real root
$$v_1 = i\sqrt{l_2}, v_2 = -i\sqrt{l_2}, v_1 = -l_1.$$

Then the roots are of the following form

$$v_1(\tau) = p(\tau) + iq(\tau), v_2(\tau) = p(\tau) - iq(\tau), v_3(\tau) = -l_1(\tau).$$

Applying Hopf bifurcation theorem, we need substitute $v_1(\tau) = p(\tau) + iq(\tau)$ in eq. (3.2) and setting $p(\tau) = 0$ and $q(\tau) = \sqrt{l_2}$, we obtain the transversality condition at $\tau = \tau^*$ as

$$(\frac{dp(\tau)}{d\tau})_{\tau=\tau^*} = (-\frac{l_2(l_1 l_2' - l_3' + l_1' l_2)}{2(l_2^2 + l_1^2 l_2)})_{\tau=\tau^*}.$$

According to the expressions of l_1, l_2 and l_3 we find

$$(\frac{dp(\tau)}{d\tau})_{\tau=\tau^*} = \frac{2f + f^2\tau - \frac{2a_1^2 n\hat{x}\hat{y}}{(n+\hat{x})(n+\hat{z})^2}\tau}{2\tau^3(\frac{1}{\tau^2} + \frac{3f}{\tau} + f^2)} > 0,$$

if $\tau < \frac{2f}{\frac{2a_1^2 n\hat{x}\hat{y}}{(n+\hat{x})(n+\hat{z})^2} - f^2}$, where $f = \frac{r}{k} - \frac{a_1\hat{y}}{(n+\hat{x})^2} - \frac{pm}{(p\hat{x}-c)^2}$.

Thus form the investigation ,we can get that the equilibrium point $P(\hat{x}, \hat{y}, \hat{z}, \hat{E})$ is locally asympotically stable for $\tau < \tau^*$. Furthermore, according to the Liu's [30] criterion simple Hopf bifurcation occurs at $\tau = \tau^*$ and for $\tau > \tau^*$.

4 Numerical Simulations

In this section, we assign numerical values to illustrate the effectiveness of our analytical results. The category consists of the results where system (2.5) undergoes a Hopf bifurcation with respect to bifurcation parameter τ^* around the equilibrium point $P(\hat{x}, \hat{y}, \hat{z}, \hat{E})$.

According to the part of Section 3, let us consider the parameters of system (2.5) as

$$r = 3, k = 1, a_1 = 1, a_1 = 2, s = 1, n = 0.5, p = 4, c = 1, m_1 = 1, m_1 = 2, q = 5, m = 0.4,$$

then system (12) becomes

$$\begin{cases} \frac{dx}{dt} = 3x(1-x) - \frac{xy}{0.5+x} - \frac{5Ex}{E+2x}, \\ \frac{dy}{dt} = y(-1 + \frac{2x}{0.5+x}), \\ \frac{dz}{dt} = \frac{1}{\tau}(x - z), \\ 0 = \frac{5E}{E+2x}(4x-1) - 0.4. \end{cases} \tag{4.1}$$

According system (2.5) and Theorem 2, we can obtain that system (4.1) exists equilibrium point $P(0.5, 1, 0.5, \frac{1}{9})$, and the bifurcation value $\tau^* = 0.263$.

If we consider the value of $\tau = 0.239 < \tau^*$, then it is observed from Figure 1 that $P(\hat{x}, \hat{y}, \hat{z}, \hat{E})$ is locally asymptotically stable and the population of prey and predator converge to their steady states in finite time. Now if we gradually increase value of τ, from Theorem 2 we have got that the $P(\hat{x}, \hat{y}, \hat{z}, \hat{E})$ loses its stability as $\tau = 0.263 = \tau^*$ by Figure 2. Also we can note that the positive equilibrium point $P(\hat{x}, \hat{y}, \hat{z}, \hat{E})$ is unstable while $\tau = 0.26316 > \tau^*$ from Figure 3.

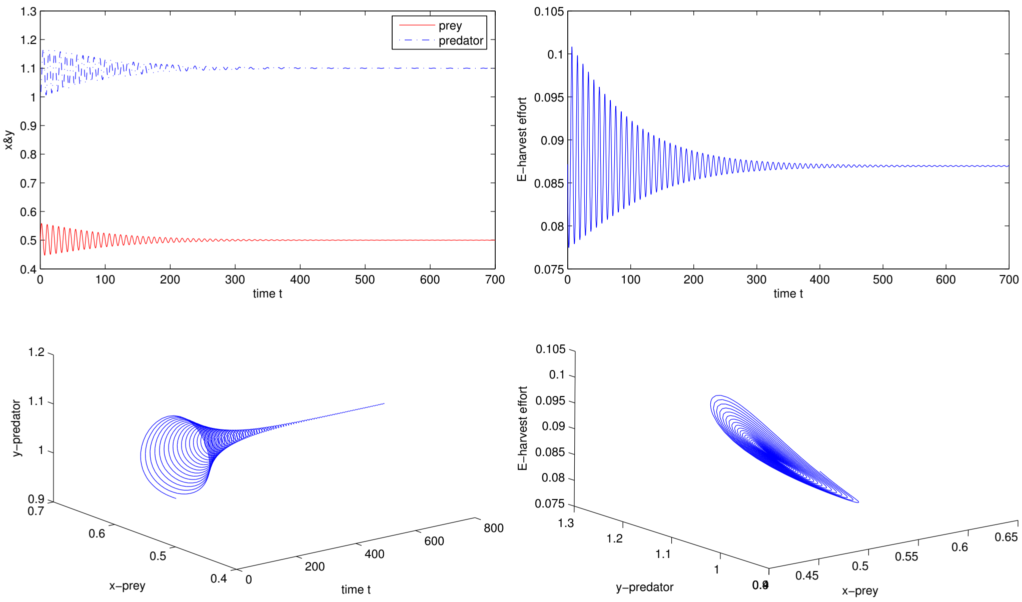

Figure 1: When $\tau = 0.239 < \tau^*$ and with the initial condition $\hat{x} = 0.499, \hat{y} = 0.999, \hat{z} = 0.499, \hat{E} = 0.111$, that show the positive equilibrium point $P(\hat{x}, \hat{y}, \hat{z}, \hat{E})$ of system (4.1) is locally asymptotically stable.

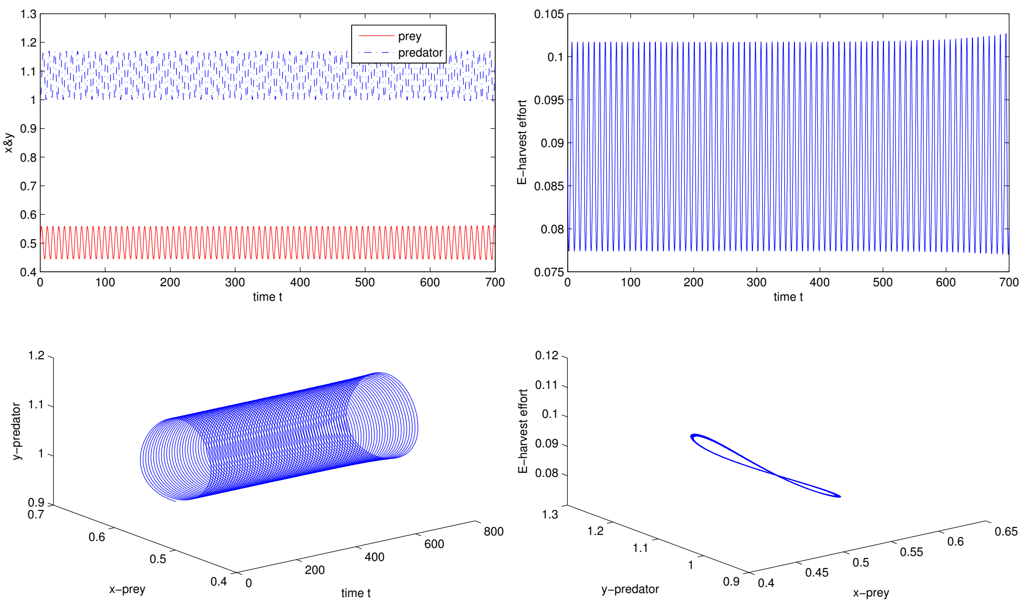

Figure 2: When $\tau = 0.26315 = \tau^*$ and with the initial condition $\hat{x} = 0.499, \hat{y} = 0.999, \hat{z} = 0.499, \hat{E} = 0.111$, that show system (4.1) taking Hopf bifurcation at the positive equilibrium point $P(\hat{x}, \hat{y}, \hat{z}, \hat{E})$.

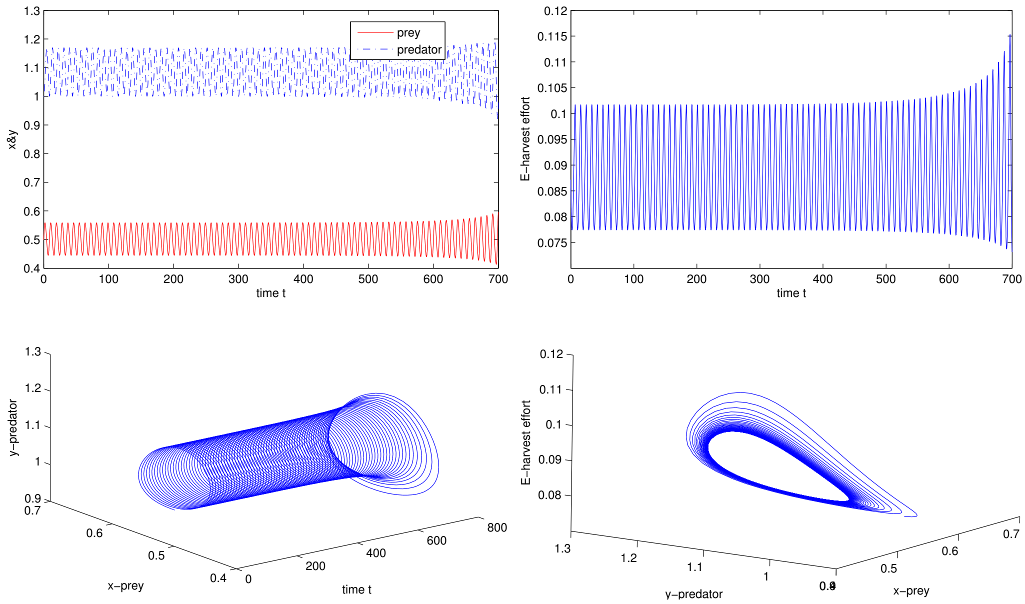

Figure 3: When $\tau = 0.26316 > \tau^*$ and with the initial condition $\hat{x} = 0.499, \hat{y} = 0.999, \hat{z} = 0.499, \hat{E} = 0.111$, that show the positive equilibrium point $P(\hat{x}, \hat{y}, \hat{z}, \hat{E})$ of system (4.1) is unstable stable.

5 Discussion

It is mainly concerned with the bifurcation analysis of a nonlinear harvested differential algebraic predator-prey system with time delay in this paper. As harvesting has a strong impact on the dynamical behavior of a predator-prey system, our predator-prey system is combined with nonlinear harvesting. It shows that nonlinear harvesting is more realistic from biological and points of view through our analysis. Also in general, delay differential algebraic equations exhibit much more complicated dynamics than ordinary differential algebraic equations, them the continuous gestation delay of predator population is incorporated in our system. We study the impact of delay, as a bifurcation parameter, and here proved that the time delay can cause a stable equilibrium to become unstable. According to Theorem 1, Theorem 2 and figures, we can know that the stability of the interior equilibrium point P changes from stable to unstable while bifurcation parameter $\tau \geq \tau *$. With the above discuss, in order to keep the population of predator, the population of prey and the economic profit at an ideal level, it needs to let τ satisfy $0 < \tau < \tau^*$.

For this paper, we only study the stability and bifurcation of system (2.5), in order to control the system, the state feedback control method should be incorporated into our model, it is good for us to control the bifurcation of the system. So we can improve our research on this aspect in the future.

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带有非线性收获和妊娠时滞的微分代数捕食者-食饵系统的稳定性及Hopf分支

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摘要: 本文研究了一类同时带有非线性食饵收获和捕食者妊娠时滞的微分代数捕食者-食饵系统的稳定性及Hopf分支问题. 利用了分支理论和稳定性理论, 以捕食者妊娠时滞作为系统的分支参数, 获得了所提出的新系统在正平衡点处系统稳定性的相关判据条件和Hopf分支的产生条件. 推广了一般带有线性收获和时滞的微分代数捕食者-食饵系统的结论.

关键词: 生物经济学; 微分代数系统; 非线性收获; 妊娠时滞; Hopf 分支; 稳定性

MR(2010)**主题分类号**: 34C23; 34C25; 34D20; 34K15; 92B05 **中图分类号**: O175.13