

ROBUST STABILIZATION OF UNCERTAIN STOCHASTIC SYSTEMS WITH TIME-VARYING DELAY AND NONLINEARITY

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Abstract: In this paper, we study with robust stabilization problem of uncertain stochastic time-varying delay systems with nonlinear perturbation. Constructing a suitable Lyapunov-Krasovskii functional and employ the free weighting matrix method, in terms of the linear matrix inequality (LMI) technique, we design a memoryless state feedback controller, and obtain delay dependent robust stabilization criterion for the uncertain stochastic time-varying delay systems. A numerical example and its simulation curve are given to show that the proposed theoretical result is effective.

Keywords: free-weighting matrices; nonlinear perturbation; time-varying delay; feedback control

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1 Introduction

The problem of the stabilization of time-delayed systems was often explored in recent years. Time delays are common in engineering processes. They frequently arose in chemical processes, in long transmission lines and in pneumatic, hydraulic and rolling mill systems. The problem of stability analysis in time-delayed systems was one of the main concerns of research into the attributes of such systems. Many works on this subject were published [1–7]. Depending on the information about the size of time-delays of the systems, criteria for time-delay systems can be classified into two categories, namely, delay-independent criteria [1, 2] and delay-dependent criteria [3–7]. Generally speaking, for the cases of small delays, the latter ones are less conservative than the former ones. To obtain delay-dependent conditions, many efforts were made in the literature, among which the model transformation and bounding technique for cross terms [8] were often used. However, it is well known that these two kinds of methods are the main sources of conservatism. Recently, in order to reduce the conservatism, a free-weighting matrix method was proposed in [9, 10] to investigate

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delay-dependent stability, in which neither model transformation nor bounding technique is involved.

In recent years, the non-fragile control problem was an attractive topic in theory analysis and practical implement, because of perturbations often appearing in the controller gain, which may result from either the actuator degradations or the requirements for readjustment of controller gains. The non-fragile control concept is how to design a feedback control that will be insensitive to some error in gains of feedback control [11]. Xu et al. [12] concerned the problems of robust non-fragile stochastic stabilization and H_∞ control for uncertain time-delay stochastic systems with time-varying norm-bounded parameter uncertainties in both the state and input matrices, when the delay was assumed to be constant. Zhang et al. [13] dealt with the same problem for uncertain nonlinear stochastic systems at the time-varying delay case. However, there was the restriction that time-derivative of time-varying delay must be less than one, which limits the application scope of the existing results. Wang et al. [14] dealt with the problems of non-fragile robust stochastic stabilization and robust H_∞ control for uncertain stochastic nonlinear single time-varying delay systems. By introducing the homogeneous domination approach to stochastic systems, Liu et al. [15] investigated a class of stochastic feedforward nonlinear systems with time-varying delay. By constructing delay-partitioning dependent Lyapunov - Krasovskii functional with reciprocally convex approach, Xia et al. [16] dealt with the problem of state robust H_∞ tracking control for uncertain stochastic systems with interval time-varying delay.

In this paper, our objective is to solve the problem of robust stabilization of uncertain stochastic systems with time-varying delay and nonlinearity. Parameter uncertainty in the state and input matrices, It is assumed to be norm bounded. Time delay is unknown, but in the known range changes with time. The goal of this paper is to design a memoryless state feedback controller, for all admissible parametric uncertainties, and make the closed-loop system is robustly stochastically stable. The present results are derived by choosing an appropriate Lyapunov functional and by making use of free-weighting matrices method. Numerical example and its simulation curve are given to show the proposed theoretical result is effective.

Notation Through this paper, the superscript T stands for matrix transposition; R^n denotes the n -dimensional Euclidean space, $R^{n \times m}$ is the set of $n \times m$ real matrices, I is the identity matrix of appropriate dimensions; the notation $X > 0$ (respectively, $X \geq 0$), for $X \in R^{n \times n}$ means that the matrix X is real positive definite (respectively, positive semi-definite); the symbol $*$ is used to denote the transposed elements in the symmetric positions of a matrix. Matrices, if the dimensions are not explicitly stated, are assumed to have compatible dimensions for algebraic operation.

2 System Descriptions and Preliminaries

Consider the following uncertain linear stochastic differential delay system with nonlin-

ear perturbation and parameter uncertainties

$$\begin{cases} dx(t) = [A(t)x(t) + A_1(t)x(t-h(t)) + B_1(t)u(t) + \sigma(t, x(t), x(t-h(t)))]dt \\ \quad + [C(t)x(t) + C_1(t)x(t-h(t)) + B_2(t)u(t)]d\omega(t), \\ x(t) = \phi(t), \quad t \in [-h, 0], \end{cases} \quad (2.1)$$

where $x(t) \in R^n$ is the state vector, $u(t) \in R^n$ is the control input, $\phi(t)$ is a continuous-time real valued function representing the initial condition of the system, and $\omega(t)$ is one-dimensional Brownian motion defined on a complete probability space (Ω, F, P) satisfying $E\{d\omega(t)\} = 0$, $E\{d\omega(t)^2\} = dt$. In the system descriptive equation (2.1), the time-varying matrices are given by $A(t) = A + \Delta A(t)$, $A_1(t) = A_1 + \Delta A_1(t)$, $B_1(t) = B_1 + \Delta B_1(t)$, $C(t) = C + \Delta C(t)$, $C_1(t) = C_1 + \Delta C_1(t)$, and $B_2(t) = B_2 + \Delta B_2(t)$, where A, A_1, B_1, C, C_1 and B_2 are known constant matrices and $\Delta A(t), \Delta A_1(t), \Delta B_1(t), \Delta C(t), \Delta C_1(t)$ and $\Delta B_2(t)$ are unknown matrices representing time-varying parametric uncertainties in the system. They are assumed to be norm-bounded of the form

$$\begin{bmatrix} \Delta A(t) & \Delta A_1(t) & \Delta B_1(t) \\ \Delta C(t) & \Delta C_1(t) & \Delta B_2(t) \end{bmatrix} = \begin{bmatrix} D_1 \\ D_2 \end{bmatrix} F(t) \begin{bmatrix} E_1 & E_2 & E_3 \end{bmatrix}, \quad (2.2)$$

where D_1, D_2, E_1, E_2 and E_3 are known real constant matrices with appropriate dimensions and $F(t)$ is unknown time-varying matrix which is Lebesgue measurable satisfying $F^T(t)F(t) \leq I, \forall t$. The time-varying delay $h(t)$ is a differentiable function satisfying the following condition

$$0 \leq h(t) \leq h, \quad \dot{h}(t) \leq \mu < \infty, \quad (2.3)$$

where h and μ are constant scalars. The term $\sigma(t, x(t), x(t-h(t))) \in R^n$ represents the unknown nonlinear perturbation with respect to the state $x(t)$ and the delayed state $x(t-h(t))$, which is assumed to be bounded with the following form

$$\|\sigma(t, x(t), x(t-h(t)))\| \leq \alpha \|x(t)\| + \beta \|x(t-h(t))\|, \quad \forall t > 0, \quad (2.4)$$

where α, β are the known non-negative constants.

Before formulating the problems to be coped with, we first introduce the following concept of robust stability for system (2.1).

Definition 1 The uncertain stochastic system in (2.1) with $u(t) = 0$ is said to be robustly stochastically stable if there exists a positive scalar $\epsilon > 0$ such that

$$\lim_{T \rightarrow 0} E \left\{ \int_0^T \|x(t)\|^2 dt \right\} \leq \epsilon \sup_{-h \leq s \leq 0} E \|\phi(s)\|^2$$

for all admissible uncertainties $\Delta A(t), \Delta A_1(t), \Delta B_1(t), \Delta C(t), \Delta C_1(t)$ and $\Delta B_2(t)$.

The objective of this paper is to develop delay-dependent stochastic stabilization criterion for the existence of a memoryless state feedback controller for system (2.1) satisfying the time-varying delay (2.3). The state feedback controller is given by

$$u(t) = Kx(t), \quad (2.5)$$

where K being the controller gain to be designed. Following lemma is indispensable for deriving the criterion.

Lemma 1 For any symmetric positive-definite matrices G and Z , of appropriate dimensions, the following inequality holds

$$-GZ^{-1}G \leq Z - 2G.$$

Proof Since $Z > 0$, we have $(Z - G)Z^{-1}(Z - G) \geq 0$. The proof follows immediately.

Lemma 2 [17] Given appropriately dimensioned matrices ψ, D, E with $\psi = \psi^T$. Then

$$\psi + DF(t)E + E^T F^T(t)D^T < 0$$

holds for all $F(t)$ satisfying $F^T(t)F(t) \leq I$ if and only if for some $\eta > 0$,

$$\psi + \eta DD^T + \eta^{-1} E^T E < 0.$$

3 Main Results

Now we provide a novel delay-dependent stabilization criterion for system (2.1) as follows

Theorem 1 For given positive scalars h, μ and λ , if there exist symmetric positive-definite matrices X, S_1, S_2, Z , appropriately dimensioned matrices $Y, U_j, V_j (j = 1, 2, 3)$, and positive scalars $\varepsilon_1, \varepsilon_2$, such that the following LMI hold

$$\begin{bmatrix} \Theta_{11} & \Theta_{12} & \Theta_{13} & \rho I & \Theta_{15} & \Theta_{16} & XE_1^T & Y^T E_3^T & \alpha X & 0 & hU_1 & hV_1 \\ * & \Theta_{22} & \Theta_{23} & 0 & \Theta_{25} & \Theta_{26} & XE_2^T & 0 & 0 & \beta X & hU_2 & hV_2 \\ * & * & \Theta_{33} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & hU_3 & hV_3 \\ * & * & * & -\rho I & 0 & h\rho I & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & \Theta_{55} & \Theta_{56} & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & * & \Theta_{66} & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & * & * & -\varepsilon_1 I & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & * & * & * & -\varepsilon_2 I & 0 & 0 & 0 & 0 \\ * & * & * & * & * & * & * & * & -\frac{\rho I}{2} & 0 & 0 & 0 \\ * & * & * & * & * & * & * & * & * & -\frac{\rho I}{2} & 0 & 0 \\ * & * & * & * & * & * & * & * & * & * & \Theta_{11,11} & 0 \\ * & * & * & * & * & * & * & * & * & * & * & \Theta_{12,12} \end{bmatrix} < 0, \tag{3.1}$$

where

$$\begin{aligned} \Theta_{11} &= AX + XA^T + B_1 Y + Y^T B_1^T + S_1 + S_2 + U_1 + U_1^T + (\varepsilon_1 + \varepsilon_2) D_1 D_1^T, \\ \Theta_{12} &= A_1 X - U_1 + V_1 + U_2^T, \\ \Theta_{22} &= -(1 - \mu) S_1 - U_2 - U_2^T + V_2 + V_2^T, \\ \Theta_{13} &= -V_1 + U_3^T, \\ \Theta_{23} &= -V_2 - U_3^T + V_3^T, \end{aligned}$$

$$\begin{aligned}
\Theta_{33} &= -S_2 - V_3 - V_3^T, \\
\Theta_{15} &= XC^T + Y^T B_2^T + (\varepsilon_1 + \varepsilon_2)D_1 D_2^T, \\
\Theta_{25} &= XC_1^T, \\
\Theta_{55} &= -X + (\varepsilon_1 + \varepsilon_2)D_2 D_2^T, \\
\Theta_{16} &= hXA^T + hY^T B_1^T + h(\varepsilon_1 + \varepsilon_2)D_1 D_1^T, \\
\Theta_{26} &= hXA_1^T, \\
\Theta_{56} &= h(\varepsilon_1 + \varepsilon_2)D_2 D_1^T, \\
\Theta_{66} &= -hZ + h^2(\varepsilon_1 + \varepsilon_2)D_1 D_1^T, \\
\Theta_{11,11} &= h\lambda^2 Z - 2h\lambda X, \\
\Theta_{12,12} &= h\lambda^2 Z - 2h\lambda X.
\end{aligned}$$

Then the uncertain linear stochastic differential delay system (2.1) with time-varying parametric uncertainties (2.2) and nonlinear perturbation (2.4) is robust stabilization, in this case, an appropriate memoryless state feedback controller can be chosen by

$$u(t) = YX^{-1}x(t).$$

Proof Substituting the state feedback controller (2.5) into system (2.1), we obtain the resulting closed-loop system as

$$dx(t) = f(t)dt + g(t)d\omega(t),$$

where

$$\begin{aligned}
f(t) &= (A(t) + B_1(t)K)x(t) + A_1(t)x(t-h(t)) + \sigma(t, x(t), x(t-h(t))), \\
g(t) &= (C(t) + B_2(t)K)x(t) + C_1(t)x(t-h(t)).
\end{aligned}$$

Now, choose a Lyapunov functional candidate as

$$\begin{aligned}
V(x(t), t) &= x^T(t)Px(t) + \int_{t-h(t)}^t x^T(s)Q_1x(s)ds + \int_{t-h}^t x^T(s)Q_2x(s)ds \\
&\quad + \int_{-h}^0 \int_{t+s}^t f^T(v)Rf(v)dvds,
\end{aligned}$$

where P, Q_1, Q_2 and R are symmetric positive-definite matrices to be chosen.

By Itô's differential formula, we obtain stochastic differential as follows

$$dV(x(t), t) = LV(x(t), t)dt + 2x^T(t)Pg(t)d\omega(t), \quad (3.2)$$

where

$$\begin{aligned}
LV(x(t), t) &\leq 2x^T(t)Pf(t) + g^T(t)Pg(t) + x^T(t)(Q_1 + Q_2)x(t) \\
&\quad - (1 - \mu)x^T(t-h(t))Q_1x(t-h(t)) - x^T(t-h)Q_2x(t-h) \\
&\quad + hf^T(t)Rf(t) - \int_{t-h}^t f^T(s)Rf(s)ds.
\end{aligned} \quad (3.3)$$

From the Leibniz-Newton formula, the following equations are true for any matrices M and N with appropriate dimensions

$$2\xi^T(t)M \left[x(t) - x(t - h(t)) - \int_{t-h(t)}^t f(s)ds - \int_{t-h(t)}^t g(s)d\omega(s) \right] = 0, \tag{3.4}$$

$$2\xi^T(t)N \left[x(t - h(t)) - x(t - h) - \int_{t-h}^{t-h(t)} f(s)ds - \int_{t-h}^{t-h(t)} g(s)d\omega(s) \right] = 0, \tag{3.5}$$

where

$$\begin{aligned} \xi^T(t) &= [x^T(t) \quad x^T(t - h(t)) \quad x^T(t - h) \quad \sigma^T(t, x(t), x(t - h(t)))] , \\ M^T &= [M_1^T \quad M_2^T \quad M_3^T \quad 0] , \quad N^T = [N_1^T \quad N_2^T \quad N_3^T \quad 0] . \end{aligned}$$

On the other hand, the following equation is also true

$$- \int_{t-h}^t f^T(s)Rf(s)ds = - \int_{t-h(t)}^t f^T(s)Rf(s)ds - \int_{t-h}^{t-h(t)} f^T(s)Rf(s)ds. \tag{3.6}$$

For any positive scalar δ , it follows from (2.4) that

$$\delta [2\alpha^2 x^T(t)x(t) + 2\beta^2 x^T(t - \tau_i(t))x(t - \tau_i(t)) - \zeta^T(t)\zeta(t)] \geq 0, \tag{3.7}$$

where $\zeta(t) = \sigma(t, x(t), x(t - h(t)))$.

Combining (3.3)–(3.7), we can obtain the following inequality

$$\begin{aligned} LV(x(t), t) &\leq \xi^T(t)(\Xi(t) + hMR^{-1}M^T + hNR^{-1}N^T)\xi(t) + F(d\omega(t)) \\ &\quad - \int_{t-h(t)}^t (\xi^T(t)M + f^T(s)R) R^{-1} (M^T\xi(t) + Rf(s)) ds \\ &\quad - \int_{t-h}^{t-h(t)} (\xi^T(t)N + f^T(s)R) R^{-1} (N^T\xi(t) + Rf(s)) ds, \end{aligned} \tag{3.8}$$

where

$$\Xi(t) = \begin{bmatrix} \Xi_{11} & \Xi_{12} & \Xi_{13} & P + h(A(t) + B_1(t)K)^T R \\ * & \Xi_{22} & \Xi_{23} & hA_1^T(t)R \\ * & * & \Xi_{33} & 0 \\ * & * & * & hR - \delta I \end{bmatrix},$$

$$\begin{aligned} \Xi_{11} &= P(A(t) + B_1(t)K) + (A(t) + B_1(t)K)^T P + Q_1 + Q_2 + M_1 + M_1^T + 2\alpha^2\delta I \\ &\quad + (C(t) + B_2(t)K)^T P(C(t) + B_2(t)K) + h(A(t) + B_1(t)K)^T R(A(t) + B_1(t)K), \end{aligned}$$

$$\Xi_{12} = PA_1(t) - M_1 + N_1 + M_2^T + (C(t) + B_2(t)K)^T PC_1(t) + h(A(t) + B_1(t)K)^T RA_1(t),$$

$$\Xi_{22} = -(1 - \mu)Q_1 - M_2 - M_2^T + N_2 + N_2^T + C_1^T(t)PC_1(t) + hA_1^T(t)RA_1(t) + 2\beta^2\delta I,$$

$$\Xi_{13} = -N_1 + M_3^T,$$

$$\Xi_{23} = -N_2 - M_3^T + N_3^T,$$

$$\Xi_{33} = -Q_2 - M_3 - M_3^T,$$

$$F(d\omega(t)) = -2\xi^T(t)M \int_{t-h(t)}^t g(s)d\omega(s) - 2\xi^T(t)N \int_{t-h}^{t-h(t)} g(s)d\omega(s).$$

Since $R > 0$, then the last two parts in inequality (3.8) are all less than 0. So, taking the mathematical expectation on both sides of equation (3.2) and using inequality (3.8), since $E\{F(d\omega(t))\} = 0$, we can obtain that

$$E \left\{ \frac{d}{dt} V(x(t), t) \right\} = ELV(x(t), t) \leq E \left\{ \xi^T(t)(\Xi(t) + hMR^{-1}M^T + hNR^{-1}N^T)\xi(t) \right\}. \tag{3.9}$$

It remains to show that $\Xi(t) + hMR^{-1}M^T + hNR^{-1}N^T < 0$. Using Schur complement formula, we see that $\Xi(t) + hMR^{-1}M^T + hNR^{-1}N^T < 0$ if and only if the following matrix inequality holds

$$\begin{bmatrix} \Sigma_{11} & \Sigma_{12} & \Xi_{13} & P & \Sigma_{15} & \Sigma_{16} & hM_1 & hN_1 \\ * & \Sigma_{22} & \Xi_{23} & 0 & C_1^T(t)P & hA_1^T(t)R & hM_2 & hN_2 \\ * & * & \Xi_{33} & 0 & 0 & 0 & hM_3 & hN_3 \\ * & * & * & -\delta I & 0 & hR & 0 & 0 \\ * & * & * & * & -P & 0 & 0 & 0 \\ * & * & * & * & * & -hR & 0 & 0 \\ * & * & * & * & * & * & -hR & 0 \\ * & * & * & * & * & * & * & -hR \end{bmatrix} < 0, \tag{3.10}$$

where

$$\begin{aligned} \Sigma_{11} &= P(A(t) + B_1(t)K) + (A(t) + B_1(t)K)^T P + Q_1 + Q_2 + M_1 + M_1^T + 2\alpha^2\delta I, \\ \Sigma_{12} &= PA_1(t) - M_1 + N_1 + M_2^T, \\ \Sigma_{22} &= -(1 - \mu)Q_1 - M_2 - M_2^T + N_2 + N_2^T + 2\beta^2\delta I, \\ \Sigma_{15} &= (C(t) + B_2(t)K)^T P, \\ \Sigma_{16} &= h(A(t) + B_1(t)K)^T R. \end{aligned}$$

Then premultiplying and postmultiplying inequality (3.10) by

$$\text{diag} [P^{-1}, P^{-1}, P^{-1}, \delta^{-1}I, P^{-1}, R^{-1}, P^{-1}, P^{-1}],$$

and defining $X = P^{-1}$, $Z = R^{-1}$, $XQ_1X = S_1$, $XQ_2X = S_2$, $XM_iX = U_i$, $XN_iX = V_i$, $i = 1, 2, 3$, $\rho = \delta^{-1}$, we have

$$\begin{bmatrix} \Gamma_{11} & \Gamma_{12} & \Theta_{13} & \rho I & \Gamma_{15} & \Gamma_{16} & hU_1 & hV_1 \\ * & \Gamma_{22} & \Theta_{23} & 0 & XC_1^T(t) & hXA_1^T(t) & hU_2 & hV_2 \\ * & * & \Theta_{33} & 0 & 0 & 0 & hU_3 & hV_3 \\ * & * & * & -\rho I & 0 & h\rho I & 0 & 0 \\ * & * & * & * & -X & 0 & 0 & 0 \\ * & * & * & * & * & -hZ & 0 & 0 \\ * & * & * & * & * & * & -hXZ^{-1}X & 0 \\ * & * & * & * & * & * & * & -hXZ^{-1}X \end{bmatrix} < 0, \tag{3.11}$$

where

$$\begin{aligned} \Gamma_{11} &= (A(t) + B_1(t)K)X + X(A(t) + B_1(t)K)^T + S_1 + S_2 + U_1 + U_1^T + 2\alpha^2\rho^{-1}XX, \\ \Gamma_{12} &= A_1(t)X - U_1 + V_1 + U_2^T, \\ \Gamma_{22} &= -(1 - \mu)S_1 - U_2 - U_2^T + V_2 + V_2^T + 2\beta^2\rho^{-1}XX, \\ \Gamma_{15} &= X(C(t) + B_2(t)K)^T, \\ \Gamma_{16} &= hX(A(t) + B_1(t)K)^T, \end{aligned}$$

and Θ_{13} , Θ_{23} and Θ_{33} are defined in inequality (3.1).

Noting equation (2.2), and let $Y = KX$, inequality (3.11) can be written as

$$\begin{bmatrix} \Pi_{11} & \Pi_{12} & \Theta_{13} & \rho I & \Pi_{15} & \Pi_{16} & hU_1 & hV_1 \\ * & \Pi_{22} & \Theta_{23} & 0 & XC_1^T & hXA_1^T & hU_2 & hV_2 \\ * & * & \Theta_{33} & 0 & 0 & 0 & hU_3 & hV_3 \\ * & * & * & -\rho I & 0 & h\rho I & 0 & 0 \\ * & * & * & * & -X & 0 & 0 & 0 \\ * & * & * & * & * & -hZ & 0 & 0 \\ * & * & * & * & * & * & -hXZ^{-1}X & 0 \\ * & * & * & * & * & * & * & -hXZ^{-1}X \end{bmatrix} + L_1F(t)L_2^T + L_2F^T(t)L_1^T + L_1F(t)L_3^T + L_3F^T(t)L_1^T < 0, \tag{3.12}$$

where

$$\begin{aligned} \Pi_{11} &= AX + XA^T + B_1Y + Y^TB_1^T + S_1 + S_2 + U_1 + U_1^T + 2\alpha^2\rho^{-1}XX, \\ \Pi_{12} &= A_1X - U_1 + V_1 + U_2^T, \\ \Pi_{22} &= -(1 - \mu)S_1 - U_2 - U_2^T + V_2 + V_2^T + 2\beta^2\rho^{-1}XX, \\ \Pi_{15} &= XC^T + Y^TB_2^T, \\ \Pi_{16} &= hXA^T + hY^TB_1^T, \\ L_1^T &= \begin{bmatrix} D_1^T & 0 & 0 & 0 & D_2^T & hD_1^T & 0 & 0 \end{bmatrix}, \\ L_2^T &= \begin{bmatrix} E_1X & E_2X & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \\ L_3^T &= \begin{bmatrix} E_3Y & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}. \end{aligned}$$

For given scalar $\lambda > 0$, the nonlinear term $-hXZ^{-1}X$ in the matrix inequality (3.12) can be rewritten as $-h(\lambda X)(\lambda^2 Z)^{-1}(\lambda X)$. Therefore, by Lemma 1, we have the inequality $-hXZ^{-1}X \leq h\lambda^2 Z - 2h\lambda X$. Applying Lemma 2 and Schur complement to inequality (3.12), we can obtain the LMI (3.1) stated in Theorem 1, which means that system (2.1) under control law $u(t) = YX^{-1}x(t)$ is robust stabilization. This completes the proof.

Remark 1 When the differential of $h(t)$ is unknown, and the delay $h(t)$ satisfies $0 \leq h(t) \leq h$, by setting $S_1 = 0$, a delay-dependent and rate-independent criterion for robust stabilization of systems (2.1) from Theorem 1 can be obtained.

Table 1: (MAUB) h of the time-varying delay $h(t)$ for different μ .

μ	0	0.1	0.5	0.9
C.Wang [14]	0.1370	0.1246	0.0766	0.0650
$\lambda = 2.0$	0.0685	0.0623	0.0456	0.0456
$\lambda = 1.0$	0.1370	0.1246	0.0766	0.0650
$\lambda = 0.5$	0.2740	0.2492	0.1431	0.0757
$\lambda = 0.2$	0.6850	0.6230	0.3108	0.0649
$\lambda = 0.1$	1.3700	1.2461	0.6096	0.0646

Remark 2 When $\alpha = 0, \beta = 0$, a uncertain linear stochastic differential delay system criterion without nonlinear perturbation for robust stabilization of systems (2.1) from Theorem 1 can be obtained.

4 Numerical Example

In this section, in order to demonstrate the effectiveness of the proposed method, we provide the following numerical example.

Example 1 Consider the uncertain nonlinear single time-delay system (2.1) with the following parameters

$$\begin{aligned}
 A &= \begin{bmatrix} -3 & 0 \\ 1 & 4 \end{bmatrix}, \quad A_1 = \begin{bmatrix} 1 & 0 \\ -1 & 3 \end{bmatrix}, \quad B_1 = \begin{bmatrix} 1 \\ 3 \end{bmatrix}, \quad C = \begin{bmatrix} 0.5 & 0.2 \\ 0.1 & 0.3 \end{bmatrix}, \\
 C_1 &= \begin{bmatrix} -1 & 1.5 \\ -0.5 & 2 \end{bmatrix}, \quad B_2 = \begin{bmatrix} 0.2 \\ -0.1 \end{bmatrix}, \quad D_1 = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.2 \end{bmatrix}, \quad D_2 = \begin{bmatrix} 0.2 & 0 \\ 0 & 0.3 \end{bmatrix}, \\
 E_1 &= \begin{bmatrix} 0.3 & 0 \\ 0 & 0.4 \end{bmatrix}, \quad E_2 = \begin{bmatrix} 0.4 & 0 \\ 0 & 0.5 \end{bmatrix}, \quad E_3 = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \quad \alpha = 0.1, \beta = 0.5.
 \end{aligned}$$

By using matlab solver feasp, for given $\mu = 0.5, \lambda = 0.2$, the feasibility upper bound of $h(t)$ is 0.3108. Choosing $h = 0.3$, according to Theorem 1, solve LMI in inequality (3.1), and get a set of solutions as follows

$$\begin{aligned}
 X &= \begin{bmatrix} 2.0317 & 0.8467 \\ 0.8467 & 1.4721 \end{bmatrix}, \quad S_1 = \begin{bmatrix} 10.4893 & 10.4257 \\ 10.4257 & 21.0523 \end{bmatrix}, \quad S_2 = \begin{bmatrix} 0.6142 & 0.2361 \\ 0.2361 & 0.2781 \end{bmatrix}, \\
 Z &= \begin{bmatrix} 11.8632 & 6.6515 \\ 6.6515 & 14.1387 \end{bmatrix}, \quad U_1 = \begin{bmatrix} 0.3290 & 0.0884 \\ 1.4220 & 0.3305 \end{bmatrix}, \quad U_2 = \begin{bmatrix} 0.0423 & -0.0058 \\ -0.7172 & -0.1623 \end{bmatrix}, \\
 U_3 &= \begin{bmatrix} 0.0754 & 0.0148 \\ 0.0114 & 0.0016 \end{bmatrix}, \quad V_1 = \begin{bmatrix} -0.4550 & -0.0974 \\ -0.6036 & -0.1512 \end{bmatrix}, \quad V_2 = \begin{bmatrix} -0.4343 & -0.0913 \\ 0.2040 & 0.0317 \end{bmatrix}, \\
 V_3 &= \begin{bmatrix} 0.5853 & 0.1225 \\ 0.0947 & 0.0434 \end{bmatrix}, \quad Y = \begin{bmatrix} -4.4817 & -12.7889 \end{bmatrix}, \\
 \varepsilon_1 &= 0.0349, \quad \varepsilon_2 = 1.5686, \quad \rho = 1.1266.
 \end{aligned}$$

Therefore the robust problem is solvable, and the memoryless feedback gains in control are computed as

$$K = \begin{bmatrix} 1.8603 & -9.7574 \end{bmatrix}.$$

Using the controller $K = \begin{bmatrix} 1.8603 & -9.7574 \end{bmatrix}$ on system (2.1) simulation, the state response curve as shown Figure 1. This indicates that the design of the memoryless state feedback controller can ensure the robust stabilization of stochastic system.

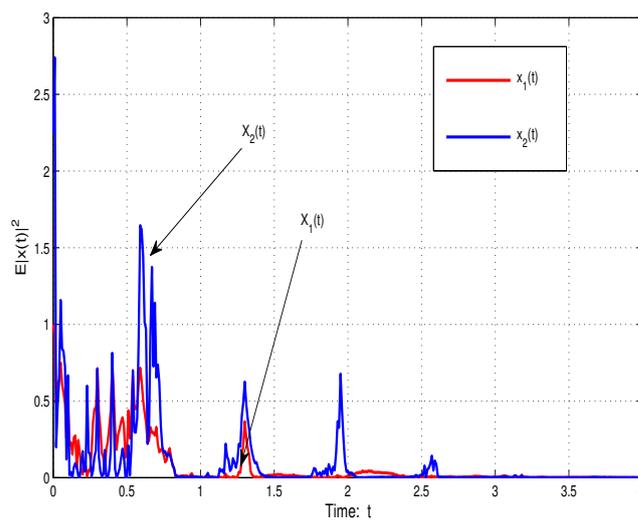


Figure 1: Trajectory of the solution to such system in Example 1

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具有非线性扰动的不确定随机时变时滞系统的鲁棒镇定

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摘要: 本文研究了具有非线性扰动的不确定随机时变时滞系统的鲁棒镇定的问题. 构造了适当的Lyapunov-Krasovskii泛函并利用自由权矩阵方法, 借助于线性矩阵不等式(LMI)技术, 设计了一个无记忆状态反馈控制器, 并获得了不确定随机时变时滞系统的时滞依赖鲁棒镇定判据. 数值例子及其仿真曲线表明所提出的理论结果是有效的.

关键词: 自由权矩阵; 非线性扰动; 时变时滞; 反馈控制

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