Vol. 36 (2016)
No. 5

数 学 杂 志
J. of Math. (PRC)

SOME RESULTS FOR TWO KINDS OF FRACTIONAL EQUATIONS WITH BOUNDARY VALUE PROBLEMS

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Abstract: In this paper, we study the boundary value problems for two kinds of fractional differential equations, in which the nonlinear term including the derivative of the unknown function. Using the properties of the fractional calculus and the Banach contraction principle, we give the existence results of solutions for these fractional differential equations, which generalize the results of previous literatures.

Keywords: fractional differential equations; Banach contraction principle; BVPs; monotone positive solution

2010 MR Subject Classification: 34A08; 34A12; 34B15; 34B18

Document code: A **Article ID:** 0255-7797(2016)05-0889-09

1 Introduction

During the last few years the fractional calculus was applied successfully to a variety of applied problems. It drew a great applications in nonlinear oscillations of earthquakes, many physical phenomena such as seepage flow in porous media and in fluid dynamic traffic model, see [1, 2]. For more details on fractional calculus theory, one can see the monographs of Kai Diethelm [3], Kilbas et al. [4], Lakshmikantham et al. [1], Podlubny [5]. Fractional differential equations involving the Riemann-Liouville fractional derivative or the Caputo fractional derivative were paid more and more attentions [6–9].

Recently, the boundary value problems for fractional differential equations provoked a great deal attention and many results were obtained, for example [2, 10–12, 14, 16, 17].

In [2], Athinson investigated the following boundary value problem (BVP) of integral type

$$\begin{cases} x^{''}+a(t)x^{\lambda}=0, \\ \lim\limits_{t\to+\infty} x(t)=1, \\ \lim\limits_{t\to+\infty} x^{'}(t)=0, \end{cases} \tag{1.1}$$

* **Received date:** 2014-06-24 **Accepted date:** 2015-01-04

Foundation item: Supported by the National Nature Science Foundation of China (11371027); Starting Research Fund for Doctors of Anhui University (023033190249); National Natural Science Foundation of China, Tian Yuan Special Foundation (11326115); the Special Research Fund for the Doctoral Program of the Ministry of Education of China (20123401120001).

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where $a(t):[t_0,+\infty)\rightarrow(0,+\infty),\lambda>0$.

As a fractional counterpart of (1.1), some scholars introduced another kind of two-point BVP (see [14])

$$\begin{cases} {}_0D_t^{\alpha}(x^{'})+a(t)x^{\lambda}=0, \\ \lim\limits_{t\rightarrow 0}[t^{1-\alpha}x^{'}(t)]=0, \\ \lim\limits_{t\rightarrow+\infty}x(t)=1, \end{cases} \tag{1.2}$$

where ${}_0D_t^{\alpha}f(t)$ stands for the Riemann-Liouville derivative (see Section 2) of order α of some function f, here $\alpha\in(0,1)$, $x^{\lambda}=|x|^{\lambda}\mathrm{sign}x$ and $\Gamma(.)$ stands for Euler's function Gamma.

Inspired by the work of above papers, the aim of this paper is to solve the BVPs of the following equations

$$\begin{cases} {}_0D_t^{\beta}u(t)+f(t,u(t),u^{'}(t))=0, t>0, \\ \lim\limits_{t\rightarrow+\infty}u(t)=M, \\ \lim\limits_{t\rightarrow 0^+}t^{1-\alpha}u^{'}(t)=0, \end{cases} \tag{1.3}$$

where ${}_0D_t^{\beta}$ stands for the Riemann-Liouville derivative of order β, and M is a constant,$\beta\in(1,2)$. And

$$\begin{cases} u^{''}+f(t,u(t),{}_0^cD_t^{\alpha}u(t))=0, \\ \lim\limits_{t\rightarrow+\infty}u(t)=M, \\ \beta(t)\leq u^{'}(t)\leq\gamma(t), t\geq t_0>0, \end{cases} \tag{1.4}$$

where M(constant) $\in(0,+\infty)$, $\alpha\in(0,1)$, f is continuous functions, ${}_0^cD_t^{\alpha}u(t)$ is Caputo derivative of order α (see Section 2) and $\lim\limits_{t\rightarrow+\infty}\beta(t)=\lim\limits_{t\rightarrow+\infty}\gamma(t)=0,\beta(t)\leq\gamma(t),t\geq t_0>0$.

2 Preliminaries

In this section, we introduce some definitions about fractional differential equation and theorems that are useful to the proof of our main results. For more details, one can see [3, 5].

Definition 2.1 The fractional Riemann-Liouville integral of order $\alpha\in(0,1)$ of a function $f:[0,+\infty)\rightarrow R$ given by

$${}_aD_t^{-\alpha}f(t)=\frac{1}{\Gamma(\alpha)}\int_a^t(t-\tau)^{\alpha-1}f(\tau)d\tau, \tag{2.1}$$

where $\Gamma(.)$ denotes the Gamma function.

Definition 2.2 The Riemann-Liouville and Caputo fractional derivatives are defined respectively as

$${}_aD_t^{p}f(t)=\begin{cases} f(t), & p=0, \\ \frac{d^n}{dt^n}[{}_aD_t^{p-n}f(t)], & p>0 \end{cases} \tag{2.2}$$

and

$$ {}_a^cD_t^p f(t) = {}_a D_t^{p-n}[\frac{d^n}{dt^n} f(t)], n-1 < p < n, $$

where n is the first integer which is not less than p, $D^{(\cdot)}$ and ${}^cD^{(\cdot)}$ are Riemann-Liouville and Caputo fractional derivatives, respectively.

Definition 2.3 For measurable functions $m : R \to R$, define the norm

$$ \|m\|_{L^p(R)} := \begin{cases} (\int_R |m(t)|^p dt)^{\frac{1}{p}}, & 1 \le p < \infty, \\ \inf_{\mu(\widetilde{R})=0}\{\sup_{t \in R - \widetilde{R}} |m(t)|\}, & p = \infty, \end{cases} \tag{2.3} $$

where $\mu(\widetilde{R})$ is the Lebesgue measure on $\widetilde{R}$. Let $L^p(R, R)$ be the Banach space of all Lebesgue measurable functions $m : R \to R$ with $\|m\|_{L^p(R)} < \infty$.

We give some useful theorems to illustrate the relation between Riemann-Liouvill and Caputo fractional derivative and the operational formula about Riemann-Liouvill derivative.

Theorem 2.1 (see [3, p.54]) Assume that $\eta \ge 0, m = \lceil \eta \rceil$, and $f \in A^m[a, b]$. Then

$$ {}_aD_t^{-\eta}({}_a^cD_t^{\eta} f(t)) = f(t) - \sum_{k=0}^{m-1} \frac{D^k f(a)}{k!}(t-a)^k. \tag{2.4} $$

Theorem 2.2 (see [5, p.74]) The composition of two fractional Riemann-Liouville derivative operators: ${}_aD_t^p$ $(m-1 \le p < m)$, and ${}_aD_t^q$ $(n-1 \le q < n)$, m, n are both positive integer,

$$ {}_aD_t^p({}_aD_t^q f(t)) = {}_aD_t^{p+q} f(t) - \sum_{j=1}^{n} [{}_aD_t^{q-j} f(t)]_{t=a} \frac{(t-a)^{-p-j}}{\Gamma(1-p-j)}. \tag{2.5} $$

Theorem 2.3 (Banach's fixed point theorem) Assume (U, d) to be a nonempty complete metric space, let $0 \le \alpha < 1$ and let the mapping A: $U \to U$ satisfy the inequality

$$ d(Au, Av) \le \alpha d(u, v) $$

for every $u, v \in U$. Then A has a uniquely determined fixed point u^*. Furthermore, for any $u_0 \in U$, the sequence $(A^j u_0)_{j=1}^{\infty}$ converges to this fixed point u^*.

3 Main Results

The following lemma 3.1 we proved will be used to solve (1.3).

Lemma 3.1 In eq. (1.3), let $\beta = 1 + \alpha, \alpha \in (0, 1)$, and we can have

$$ v(t) = -\frac{1}{\Gamma(\alpha)} \int_0^t (t-\tau)^{\alpha-1} f(\tau, M - \int_{\tau}^{+\infty} v(s)ds, v(\tau))d\tau, \tag{3.1} $$

where $v(t) = u^{'}(t)$.

Proof Using formula (2.5) and the equation ${}_0D_t^{\beta}u(t)+f(t,u(t),u^{'}(t))=0$, we can get

$$\begin{aligned}{}_0D_t^{-\alpha}({}_0D_t^{\beta}u(t)) &= u^{'}(t)-\sum_{j=1}^{2}[{}_0D_t^{\beta-j}u(t)]_{t=0}\tfrac{t^{\alpha-j}}{\Gamma(1+\alpha-j)}\\ &= -\tfrac{1}{\Gamma(\alpha)}\int_0^t(t-\tau)^{\alpha-1}f(\tau,u,u^{'})d\tau,\end{aligned}$$

that is

$$\begin{aligned}u^{'}(t)= & -\tfrac{1}{\Gamma(\alpha)}\int_0^t(t-\tau)^{\alpha-1}f(\tau,u,u^{'})d\tau\\ & +\tfrac{t^{\alpha-1}}{\Gamma(\alpha)}\lim_{t\to0^+}\int_0^t\frac{(t-\tau)^{-\beta}}{\Gamma(1-\beta)}f(\tau,u,u^{'})d\tau\\ & +\tfrac{t^{\alpha-2}}{\Gamma(\alpha-1)}\lim_{t\to0^+}\int_0^t\frac{(t-\tau)^{1-\beta}}{\Gamma(2-\beta)}f(\tau,u,u^{'})d\tau.\end{aligned}\tag{3.2}$$

We now prove the following result can hold

$$\lim_{t\to0^+}\int_0^t(t-\tau)^{-\beta}f(\tau,u,u^{'})d\tau=\lim_{t\to0^+}\int_0^t(t-\tau)^{1-\beta}f(\tau,u,u^{'})d\tau=0.\tag{3.3}$$

Due to $\lim\limits_{t\to0^+}t^{1-\alpha}u^{'}(t)=0$, then $\lim\limits_{t\to0^+}t^{2-\alpha}u^{'}(t)=0$. With (3.2), we can easily get (3.3). So (3.2) can be changed into

$$u^{'}(t)=-\frac{1}{\Gamma(\alpha)}\int_0^t(t-\tau)^{\alpha-1}f(\tau,u,u^{'})d\tau,$$

and remember $\displaystyle\int_t^{+\infty}v(s)ds=M-u(t)$, we can obtain (3.1). The proof is completed.

Now, we introduce the set X and the metric $d(v_1,v_2)$ of X. We define

$$X=\{v(t)|v(t)\in(C\cap L^{\infty}\cap L^1)([0,+\infty),R)\}$$

and

$$d(v_1,v_2)=\|v_1-v_2\|_{L^1((0,+\infty);R)}+\sup_{t\geq0}|v_1-v_2|\tag{3.4}$$

for any $v_1,v_2\in X$. One can easily prove that (X,d) is a complete metric space by the Lebesgue dominated convergence theorem [13].

Some hypothesis will be introduced here.

[H1]: f meets weak Lipschitz condition with the second and the third variables on X:

$$|f(t,u_1,w_1)-f(t,u_2,w_2)|\leq a(t)(|u_1-u_2|+|w_1-w_2|),\tag{3.5}$$

especially, $|f|\leq a(t)d(v,0)=a(t)(\|v\|_{L^1}+\sup_{t\geq0}|v|)$, $v\in X$. And $a(t)$ can be some nonnegative continuous functions that can make [H2] and [H3] hold.

[H2]: Let $\omega_0(t) = \int_0^t (t-\tau)^{\alpha-1}B(\tau)d\tau + \int_t^{+\infty}\int_0^s (s-\tau)^{\alpha-1}B(\tau)d\tau ds$, we assume $0 < \omega_0(t) < 1$, where $B(t) = a(t)[\Gamma(\alpha)]^{-1}$.

Theorem 3.1 Assume that [H1], [H2] are satisfied.Then the problem (1.3) has a solution $u(t) = M - \int_0^{+\infty} v(s)ds$, $t > 0$, where $v \in (C \cap L^\infty \cap L^1)([0,+\infty), R)$.

Proof From Lemma 3.1, the operator T can be defined by $T : X \to (C \cap L^\infty \cap L^1)([0,+\infty), R)$ with the formula

$$(Tv)(t) = -\frac{1}{\Gamma(\alpha)}\int_0^t (t-\tau)^{\alpha-1} f(\tau, M - \int_\tau^{+\infty} v(s)ds, v(\tau))d\tau, t \geq 0. \tag{3.6}$$

From [H1] and (3.5), $(Tv)(t)$ satisfies

$$|(Tv)(t)| \leq (\|v\|_{L^1} + \sup_{t\geq 0}|v|)\frac{1}{\Gamma(\alpha)}\int_0^t (t-\tau)^{\alpha-1}a(\tau)d\tau,$$

so $TX \subseteq X$.

According to (3.4), we get

$$d(Tv_1, Tv_2) = \sup_{t\geq 0}|(Tv_1)(t) - (Tv_2)(t)| + \|(Tv_1) - (Tv_2)\|_{L^1((0,+\infty);R)}$$

for any $v_1, v_2 \in X$.

First, from (3.5) and (3.6), we have

$$\begin{aligned}
|(Tv_1)(t) - (Tv_2)(t)| &= \frac{1}{\Gamma(\alpha)}|\int_0^t (t-\tau)^{\alpha-1}[f(\tau, M - \int_\tau^\infty v_1(s)ds, v_1) \\
&\quad - f(\tau, M - \int_\tau^\infty v_2(s)ds, v_2)]d\tau| \\
&\leq \int_0^t \frac{a(\tau)}{\Gamma(\alpha)}(t-\tau)^{\alpha-1}(\int_\tau^{+\infty}|v_1(s) - v_2(s)|ds + |v_1 - v_2|)d\tau \\
&\leq d(v_1, v_2)\int_0^t (t-\tau)^{\alpha-1}B(\tau)d\tau. \qquad (3.7)
\end{aligned}$$

Second,

$$\begin{aligned}
\int_t^{+\infty}|(Tv_1)(\tau) - (Tv_2)(\tau)|d\tau &\leq \int_t^{+\infty}\int_0^s (s-\tau)^{\alpha-1}B(\tau)(\int_\tau^{+\infty}|v_1(h) - v_2(h)|dh \\
&\quad + |v_1(\tau) - v_2(\tau)|)d\tau ds \\
&\leq d(v_1, v_2)\int_t^{+\infty}\int_0^s (s-\tau)^{\alpha-1}B(\tau)d\tau ds. \qquad (3.8)
\end{aligned}$$

Then (3.7) and (3.8) yield

$$d(\tau v_1, \tau v_2) \leq d(v_1, v_2)\omega_0(t).$$

By using [H2] and Theorem 2.3, Tv has a fixed point v_0. This v_0 function is the solution of problem (1.3). The proof is completed.

Remark 3.1 In fact, problem (1.2) is the special case of Theorem 3.1. One can easily get the existence result from the procedure of proving Theorem 3.1.

Remark 3.2 In the following part, we will prove that some conditions supplied can make $0<\omega_0(t)<1$ true. We can choose a simple candidate for $a(t)$ which is provided by the restriction $a(t)\leq c\cdot t^{-2}, t\geq t_0>0$, c is undetermined coefficient. Using the restriction, we acquire that

$$0<\omega_0(t)\leq\frac{1}{\Gamma(\alpha)}[\int_0^t(t-\tau)^{\alpha-1}c\tau^{-2}d\tau+\int_t^{+\infty}\int_0^s(s-\tau)^{\alpha-1}c\tau^{-2}d\tau ds].$$

It is easily found that the integral mean value theorem can be used in here to obtain that

$$\begin{aligned}0<\omega_0(t)\quad&\leq\tfrac{c}{\Gamma(\alpha)}[(t-\theta t)^{\alpha-1}(\theta t)^{-2}t+\int_t^{+\infty}(s-\theta s)^{\alpha-1}(\theta s)^{-2}sds]\\&=\tfrac{c}{\Gamma(\alpha)}[(1-\theta)^{\alpha-1}\theta^{-2}t^{\alpha-2}+(1-\theta)^{\alpha-1}\theta^{-2}\tfrac{s^{\alpha-1}}{\alpha-1}|_t^{+\infty}]\\&=\tfrac{c(1-\theta)^{\alpha-1}\theta^{-2}}{\Gamma(\alpha)}(t_0^{\alpha-2}+\tfrac{t_0^{\alpha-1}}{1-\alpha})\end{aligned}$$

in which $\theta\in(0,1)$. So we can select $0<c<\frac{(1-\alpha)\Gamma(\alpha)(1-\theta)^{1-\alpha}\theta^2}{(1-\alpha)t_0^{\alpha-2}+t_0^{\alpha-1}}$ make that true. In fact, if $a(t)\leq ct^{-(\alpha+1)}$, we can find a suitable c which achieve that. So if we can find appropriate $a(t)$ which satisfy [H1] and [H2], we can solve problem (3) with some supplied conditions.

In order to obtain existence theorems of equation (1.4), we introduce the following definitions and assumption.

Let

$$X_1=\{v\in C([t_0,+\infty];R)\mid\beta(t)\leq v(t)\leq\gamma(t),t\geq t_0>0\}$$

and consider $\lim\limits_{t\to+\infty}u(t)=M$, we have

$$X_2=\{u\in C([t_0,+\infty];R)\mid M-\int_t^{+\infty}\gamma(s)ds\leq u(t)\leq M-\int_t^{+\infty}\beta(\tau)d\tau,t\geq t_0>0\},$$

where $v(t)=u^{'}(t)$.

We still need to give [H3] to solve (1.4).

[H3]: $a(t)$ satisfies the following inequalities

$$0<\omega_1(t)=\int_t^{+\infty}a(\tau)d\tau+\int_t^{+\infty}(\tau-t)a(\tau)d\tau<1$$

and

$$0<\omega_2(t)=\frac{1}{\Gamma(1-\alpha)}\int_t^{+\infty}a(\tau)\tau^{1-\alpha}d\tau+\frac{1}{\Gamma(1-\alpha)}\int_t^{+\infty}a(\tau)(\tau-t)\tau^{1-\alpha}d\tau<1,$$

$a(t)$ is nonnegative continuous function.

Theorem 3.2 Assume that [H1], [H3] hold. Problem (1.4) has a solution on X_1.

Proof From (2.5) and the conditions with $\beta(t), \gamma(t)$ supplied, and $\lim\limits_{t\to+\infty} u(t) = M$, we can obtain that

$$v(t) = \int_t^{+\infty} f(\tau, M - \int_\tau^{+\infty} v(s)ds, \quad \frac{1}{\Gamma(1-\alpha)} \int_0^\tau v(s)(\tau - s)^{-\alpha} ds) d\tau.$$

Similar to the definition about d in Theorem 3.1, we have

$$d(v_1, v_2) = \|v_1 - v_2\|_{L^1((t_0,+\infty);R)} + \sup_{t\geq t_0} |v_1 - v_2|. \tag{3.9}$$

One can easily prove the matric space $E = (X_1, d)$ is complete by using Lebesgue's dominated convergence theorem.

In fact, we can define $T : X_1 \to C([t_0, +\infty); R)$ given by the formula

$$(Tv)(t) = \int_t^{+\infty} f(\tau, M - \int_\tau^{+\infty} v(s)ds, \frac{1}{\Gamma(1-\alpha)} \int_0^\tau v(s)(\tau - s)^{-\alpha} ds) d\tau, t \geq t_0, \tag{3.10}$$

where $v(t) \in X_1$. It is easy to see that $TX_1 \subseteq X_1$ from the definition about X_1. The operator T is contraction in X_1, so we have

$$d(Tv_1, Tv_2) = \sup_{t\geq t_0} |(Tv_1)(t) - (Tv_2)(t)| + \|(Tv_1) - (Tv_2)\|_{L^1((t_0,+\infty);R)}. \tag{3.11}$$

Considering [H1],

$$\begin{aligned} |(Tv_1(t) - Tv_2(t))| &\leq \int_t^{+\infty} a(\tau)(\int_\tau^{+\infty} |v_2(s) - v_1(s)|ds \\ &\quad + \frac{1}{\Gamma(1-\alpha)} \int_0^\tau |v_2(s) - v_1(s)|(\tau - s)^{-\alpha} ds) d\tau \\ &\leq \|v_1 - v_2\|_{L^1((0,+\infty);R)} \int_t^{+\infty} a(\tau)d\tau \\ &\quad + \sup_{t\geq t_0} |v_1 - v_2| \frac{1}{\Gamma(1-\alpha)} \int_t^{+\infty} a(\tau)\tau^{1-\alpha} d\tau \end{aligned} \tag{3.12}$$

and

$$\begin{aligned} \int_t^{+\infty} |(Tv_1)(\tau) - (Tv_2)(\tau)|d\tau &\leq \int_t^{+\infty} \int_s^{+\infty} a(\tau)(\int_\tau^{+\infty} |v_2(h) - v_1(h)|dh \\ &\quad + \frac{1}{\Gamma(1-\alpha)} \int_0^\tau |v_2(h) - v_1(h)|(\tau - h)^{-\alpha} dh) d\tau ds \\ &\leq \|v_1 - v_2\|_{L^1((0,+\infty);R)} \int_t^{+\infty} (\tau - t)a(\tau)d\tau \\ &\quad + \sup_{t\geq t_0} |v_1 - v_2| \int_t^{+\infty} \frac{a(\tau)(\tau - t)\tau^{1-\alpha}}{\Gamma(1-\alpha)} d\tau \end{aligned} \tag{3.13}$$

and [H3], we yield

$$\begin{aligned} d(Tv_1, Tv_2) &\le \|v_1 - v_2\|_{L^1((0,+\infty);R)}\omega_1(t) + \sup_{t\ge t_0}|v_1 - v_2|\omega_2(t) \\ &< d(v_1, v_2) \end{aligned} \tag{3.14}$$

for all $v_1, v_2 \in X_1$.

At last, by Theorem 2.3, the function $u(t) = M - \int_t^{+\infty} v(s)ds, t \ge t_0 > 0$, where v is the fixed point of operator T, which is the solution we want.

Remark 3.3 If $\alpha = 1$, we will obtain the equation $u^{''}(t) + f(t, u(t), u^{'}(t)) = 0, t \ge t_0 > 0$. This kind of equation was studied by Octavian and Mustafa (see [13]) with some supplied conditions of $u(t)$ and $u^{'}(t)$

$$\begin{cases} u(t) > 0, \\ \lim\limits_{t\to+\infty} u(t) = M, M \in (0, +\infty), \\ \beta(t) \le u^{'}(t) \le \gamma(t), t \ge t_0, \end{cases}$$

where $\beta(t), \gamma(t)$ are continuous nonnegative functions satisfying

$$\begin{cases} \lim\limits_{t\to+\infty} \beta(t) = \lim\limits_{t\to+\infty} \gamma(t) = 0, \\ \beta(t) \le \gamma(t), t \ge t_0 > 0. \end{cases}$$

The terminal value problem included $u^{''} + f(t, u, u^{'}) = 0, t \ge t_0 > 0, \lim\limits_{t\to+\infty} u(t) = M \in R$ has a long history as part of the general asymptotic integration theory of ordinary differential equaitions. In fact, if $\alpha = 1$, and we know that $\lim\limits_{t\to 0}|\Gamma(t)| = +\infty$, we can get the result in [13] from Theorem 3.2.

Remark 3.4 From Theorem 2.1, if we have some initial value about $u(t)$ and use ${}_0D_t^\alpha u(t)$ instead of ${}_0^cD_t^\alpha u(t)$ in problem (1.4), we still can solve that problem by using the similar way in Theorem 3.2.

Remark 3.5 We concern about that if there exists $a(t)$ can make [H3] true. In fact, in the $u(t)$ definition domain, let $t \ge t_0 > 0$, $a(t) \le ct^{-h}, h > 3 - \alpha, \alpha \in (0, 1)$ and select $0 < c < \min\{\frac{(h-1)(h-2)}{t_0^{1-h}(t_0+h-2)}, \frac{\Gamma(1-\alpha)(\alpha+h-2)(\alpha+h-3)}{t_0^{2-\alpha-h}(\alpha+h+t_0-3)}\}$, one can easily find that [H3] can hold. The process to get range of c is similar with Remark 3.2.

4 Conclusion

In this paper, we solve two kinds of boundary value problems which include results in [13, 14]. In fact, some equations with some boundary value conditions can also include results in this paper. Our future work is just to solve equation as the following one: ${}_0D_t^\beta u(t) + f(t, u(t), u^\alpha(t)) = 0$, α, β are some fractional numbers. To solve this kind of equation also need some boundary value conditions.

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两类分数阶微分方程的边值问题

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摘要: 本文研究了两类非线性项含有未知函数导数的分数阶微分方程的边值问题. 利用分数阶微积分的性质及Banach不动点定理, 获得了解的存在唯一性等有关结果, 推广了已有文献的结论.

关键词: 分数阶微分方程; 巴拿赫压缩定理; 边值问题; 单调正解

MR(2010)**主题分类号**: 34A08; 34A12; 34B15; 34B18 **中图分类号**: O175.1