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FINITE-TIME SYNCHRONIZATION OF COMPLEX NETWORKS WITH DISTURBANCES BY TERMINAL SLIDING MODE CONTROL

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Abstract: In this paper, we investigate the finite-time synchronization of complex networks with disturbances. Based on the terminal sliding mode control method, appropriate sliding mode surfaces and controllers are designed, and some sufficient criteria are derived to guarantee the synchronization between two different complex networks, which generalize some existing results about the synchronization of complex networks.

Keywords: complex network; finite-time synchronization; sliding mode; disturbance

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1 Introduction

Since the landmark discovery of the "small-world" and "scale-free" properties of complex networks in the end of 20th century [1, 2], it was found that complex networks widely exist in our real life. There are a great number of real world networks— such as cooperate networks, social networks, neural networks, WWW, food webs, electrical power grids and so on, all which can be described by complex networks.

Over the past decades, the modeling, statistic analysis, control and synchronization, and topology identification [3] of complex networks were hot focus for many scientists from various fields, for instance, sociology, biology, mathematics and physics. In particular, synchronization of networks is considered to be a very significant topic, much work was done for the synchronization of complex networks in the literature. In this period, many kinds of synchronization definitions are presented, for example, complete synchronization, projective synchronization, lag synchronization, phase synchronization and generalized function synchronization. Recently, finite-time synchronization attracted more and more attention from the researchers. Finite-time synchronization means the errors converge to zero within finite

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Biography: Zhang Qunjiao (1980–), female, born at Zhongxiang, Hubei, associate professor, major in complex networks and nonlinear dynamical systems.

time, which has the optimality to minimize the convergence time [4]. Thus, it was used to realize the stability or synchronization for chaotic systems and complex networks.

Meanwhile, the sliding mode control theory introduced by Utkin provides an efficient way to the robust control problem [5], which has great advantages on fast response, good transient performance and robustness to variations of system parameters or disturbances, and has been widely used to control the uncertain or disturbed systems. Wang et al. studied the finite-time chaos control via nonsingular terminal sliding mode control [6]. The authors in [7] presented some finite-time synchronization of two different chaotic systems with unknown parameters via sliding mode technique. In the same year, they also investigated the synchronization for two different chaotic systems with unknown parameters by using a robust adaptive sliding mode controller [8]. However, to the best of our knowledge, there were few results concerning finite-time synchronization for complex networks with internal and external disturbances.

Based on the above reasons, in this paper, we investigate the issue on the finite-time synchronization between two different complex networks with disturbances. By using the sliding mode control method, some criteria and corollary are obtained for the finite-time synchronization of complex networks with internal and external disturbances. Finally, the theoretical results are illustrated by complex networks composed of the chaotic unified systems and Chua's circuit systems.

Notations For a vector $\mathbf{z}=(z_1,z_2,\cdots,z_n)\in\mathbf{R}^n$, then $\|\mathbf{z}\|_r=(\sum\limits_{i=1}^{n}|z_i|^r)^{(1/r)}$. In this paper, the $\|\cdot\|_2$ is simplified as $\|\cdot\|$. For a matrix $C\in\mathbf{R}^{n\times n}$, then $\|C\|=\sqrt{\lambda_{\max}(C^TC)}$. In particular, if C is a symmetric matrix, then $\|C\|=\max\limits_{1\le i\le n}|\lambda_i|$, where λ_i $(1\le i\le n)$ are all the eigenvalues of matrix C, $\mathrm{sgn}(\cdot)$ represents the sign function.

2 Problem Formulation and Preliminaries

Let's consider a general complex network, which is disturbed by two components, one is the dynamical disturbance, and the another one is the external disturbance. The state equations of the entire networks are described by

$$\dot{x}_i=f_i(t,x_i)+\triangle f_i(t,x_i(t))+\sum_{j=1}^{N}a_{ij}\varphi_i(x_j)+d_i^{(m)},\quad i=1,2,\cdots,N,$$

where $x_i=(x_{i1},x_{i2},\cdots,x_{in})^{\mathrm{T}}\in\mathbf{R}^n$ is a state vector representing the state variables of node i, $f_i:\mathbf{R}^+\times\mathbf{R}^n\to\mathbf{R}^n$ is a dynamical function and $\triangle f_i:\mathbf{R}^+\times\mathbf{R}^n\to\mathbf{R}^n$ is a disturbed dynamical function, $d_i^{(m)}$ is the external disturbance of master network. The matrix $A=(a_{ij})_{N\times N}$ is the coupling configuration matrix of this network, φ_i is an inner coupling function in each node.

Denote $F_i(\mathbf{x})=a_{i1}\varphi_i(x_1)+a_{i2}\varphi_i(x_2)+\cdots+a_{iN}\varphi_i(x_N)$, where $\mathbf{x}=(x_1^T,\cdots,x_N^T)^T$, then the above master network can be clearly written as

$$\dot{x}_i=f_i(t,x_i)+\triangle f_i(t,x_i(t))+F_i(\mathbf{x})+d_i^{(m)},\quad i=1,2,\cdots,N. \tag{2.1}$$

The slave network with different dynamics and different configuration is given by

$$\dot{y}_i = g_i(t, y_i) + \triangle g_i(t, y_i(t)) + \sum_{j=1}^{N} b_{ij}\psi_i(y_j) + d_i^{(s)} + u_i, \quad i = 1, 2, \cdots, N,$$

where $y_i = (y_{i1}, y_{i2}, \cdots, y_{in})^{\mathrm{T}} \in \mathbf{R}^n$ is a state vector, $g_i, \triangle g_i : \mathbf{R}^+ \times \mathbf{R}^n \to \mathbf{R}^n$ are dynamical function and disturbed dynamical function respectively, $d_i^{(s)}$ are external disturbances for this slave network. The matrix $B = (b_{ij})_{N\times N}$ is the coupling configuration matrix of the network, ψ_i is inner connecting matrix in each node.

Similarly, it can be rewritten as

$$\dot{y}_i = g_i(t, y_i) + \triangle g_i(t, y_i(t)) + G_i(\mathbf{y}) + d_i^{(s)} + u_i, \quad i = 1, 2, \cdots, N, \tag{2.2}$$

where $G_i(\mathbf{y}) = b_{i1}\psi_i(y_1) + b_{i2}\psi_i(y_2) + \cdots + b_{iN}\psi_i(y_N)$ and $\mathbf{y} = (y_1^T, \cdots, y_N^T)^T$.

To discuss the finite-time synchronization between networks (2.1) and (2.2), define the errors $e_i = y_i - x_i \in \mathbf{R}^n$ and subtract (2.1) from (2.2), one gets the error dynamics as

$$\begin{aligned} \dot{e}_i \;\; = \;\; & g_i(t, y_i) - f_i(t, x_i) + \triangle g_i(t, y_i(t)) - \triangle f_i(t, x_i(t)) + G_i(\mathbf{y}) \\ & -F_i(\mathbf{x}) + d_i^{(s)} - d_i^{(m)} + u_i, \quad i = 1, 2, \cdots, N. \end{aligned} \tag{2.3}$$

Remark 2.1 In this model, the networks are perturbed by both internal and external disturbances. Particularly, it includes the case that the disturbances are induced by the uncertain or disturbed parameters [6].

Definition 2.1 For the error systems (2.3), if there exists a constant $T > 0$ such that

$$\lim_{t\to T} \|e_i(t)\| = 0 \;\; (1 \le i \le N)$$

and $\|e_i(t)\| \equiv 0$, if $t \ge T$, then the origin of (2.3) is finite-time stable, i.e., the networks (2.1) and (2.2) is finite-time synchronous.

Before the main results, the following lemmas and assumptions will be introduced.

Lemma 2.1 (see [7]) Suppose $a_1, a_2, \cdots, a_n$ and $0 < q < 2$ are all real numbers, then the following inequality holds

$$(a_1^2 + a_2^2 + \cdots + a_n^2)^{q/2} \le |a_1|^q + |a_2|^q + \cdots + |a_n|^q.$$

In particular, when $q = 1$, there is

$$\sqrt{a_1^2 + a_2^2 + \cdots + a_n^2} \le |a_1| + |a_2| + \cdots + |a_n|.$$

Let a vector $\mathbf{a} = (a_1, a_2, \cdots, a_n)$, that means $\|\mathbf{a}\| \le \|\mathbf{a}\|_1$.

Lemma 2.2 (see [9]) Assume that a continuous, positive-definite function $V(t)$ satisfies the following differential inequality

$$\dot{V}(t) \le -pV^{\eta}(t), \quad \forall t \ge t_0, \; V(t_0) \ge 0,$$

where $p>0, 0<\eta<1$ are two constants. Then, for any given t_0, $V(t)$ satisfies the following inequality

$$V^{1-\eta}(t)\le V^{1-\eta}(t_0)-p(1-\eta)(t-t_0),\quad t_0\le t\le t_1,$$

and $V(t)\equiv 0, \forall\ t\ge t_1$ with t_1 given by $t_1=t_0+\frac{V^{1-\eta}(t_0)}{p(1-\eta)}$.

Assumption 1 (A1) Assume the dynamical disturbances $\triangle f_i(t,x_i(t)), \triangle g_i(t,y_i(t))$ are norm bounded, then $\|\triangle g_i(t,y_i(t))-\triangle f_i(t,x_i(t))\|$ is bounded, that is, there exists a constant $\alpha_i>0$ such that

$$\|\triangle g_i(t,y_i(t))-\triangle f_i(t,x_i(t))\|\le\alpha_i\ (i=1,2,\cdots,N). \tag{2.4}$$

Assumption 2 (A2) Assume the external disturbances $d_i^{(m)}$, $d_i^{(s)}$ are norm bounded, then $\|d_i^{(s)}-d_i^{(m)}\|$ is bounded, that is, there exists a constant $\beta_i>0$ such that

$$\|d_i^{(s)}-d_i^{(m)}\|\le\beta_i\ (i=1,2,\cdots,N). \tag{2.5}$$

3 Finite-Time Synchronization of the Networks with Disturbances

The following section is about how to design appropriate finite-time sliding mode controller to realize the reachability of the sliding mode surface.

Theorem 3.1 Suppose (A1) and (A2) hold, the constant $0<\gamma<1$, choose the controllers as

$$u_i=f_i(t,x_i)-g_i(t,y_i)-G_i(\mathbf{y})+F_i(\mathbf{x})-C_i^{-1}[\mathrm{sgn}(e_i)|e_i|^\gamma]-k_i\mathrm{sgn}(s_i), \tag{3.1}$$

if $k_i\ge\dfrac{(\alpha_i+\beta_i)\|C_i\|+1}{\min\limits_{1\le l\le n}\{c_{il}\}}$, then the sliding mode surface

$$s_i=C_ie_i+\int_0^t[\mathrm{sgn}(e_i)|e_i|^\gamma]d\tau\quad(i=1,2,\cdots,N) \tag{3.2}$$

will reach $s_i\equiv 0$ after finite time $T_1^{(i)}=\frac{1}{\varepsilon}\sqrt{2V_1^{(i)}(0)}$, where $C_i=\mathrm{diag}\{c_{i1},c_{i2},\cdots,c_{in}\}\in\mathbf{R}^{n\times n}$ is a diagonal matrix with $c_{il}>0$ $(1\le l\le n)$, $s_i=(s_{i1},s_{i2},\cdots,s_{in})^{\mathrm{T}}$ and $\mathrm{sgn}(s_i)=(\mathrm{sgn}(s_{i1}),\mathrm{sgn}(s_{i2}),\cdots,\mathrm{sgn}(s_{in}))^{\mathrm{T}}$. The denotation $[\mathrm{sgn}(e_i)|e_i|^\gamma]\triangleq[\mathrm{sgn}(e_{i1})|e_{i1}|^\gamma,\ \mathrm{sgn}(e_{i2})|e_{i2}|^\gamma,\ \cdots,\mathrm{sgn}(e_{in})|e_{in}|^\gamma]^{\mathrm{T}}$.

Proof Let the Lyapunov function be in the form of

$$V_1^{(i)}=\frac{1}{2}\|s_i\|^2=\frac{1}{2}s_i^{\mathrm{T}}s_i,$$

then its derivation along (3.2) is

$$\dot V_1^{(i)}=s_i^{\mathrm{T}}\dot s_i=s_i^{\mathrm{T}}(C_i\dot e_i+[\mathrm{sgn}(e_i)|e_i|^\gamma])$$

with the error systems (2.3) and controllers (3.1), we further have

$$\dot V_1^{(i)}=s_i^{\mathrm{T}}C_i((\triangle g_i(t,y_i(t))-\triangle f_i(t,x_i(t))+(d_i^{(s)}-d_i^{(m)})-k_i\mathrm{sgn}(s_i)),$$

according to (A1) and (A2), then

$$\dot{V}_1^{(i)} \leq (\alpha_i + \beta_i)\|C_i\|\|s_i\| - k_i s_i^{\mathrm{T}} C_i \mathrm{sgn}(s_i). \tag{3.3}$$

In fact, along with Lemma 2.1, we have

$$\begin{aligned} s_i^{\mathrm{T}} C_i \mathrm{sgn}(s_i) &= c_{i1}|s_{i1}| + c_{i2}|s_{i2}| + \cdots + c_{in}|s_{in}| \\ &\geq \min_{1\leq l\leq n}\{c_{il}\}(|s_{i1}| + |s_{i2}| + \cdots + |s_{in}|) \\ &= \min_{1\leq l\leq n}\{c_{il}\}\|s_i\|_1 \geq \min_{1\leq l\leq n}\{c_{il}\}\|s_i\|. \end{aligned}$$

Notice that $k_i > 0$, therefore $-k_i s_i^{\mathrm{T}} C_i \mathrm{sgn}(s_i) \leq -k_i \min\limits_{1\leq l\leq n}\{c_{il}\}\|s_i\|$.

Thus from inequality (3.3), we further have

$$\dot{V}_1^{(i)} \leq ((\alpha_i + \beta_i)\|C_i\| - k_i \min_{1\leq l\leq n}\{c_{il}\})\|s_i\|, \tag{3.4}$$

if $k_i \geq \dfrac{(\alpha_i + \beta_i)\|C_i\| + \varepsilon}{\min\limits_{1\leq l\leq n}\{c_{il}\}}$, then

$$\dot{V}_1^{(i)} \leq -\varepsilon\|s_i\| = -\varepsilon\sqrt{2}\sqrt{V_1^{(i)}}. \tag{3.5}$$

From Lemma 2.2, we know that $s_i \equiv 0$ when $t \geq T_1^{(i)} = \frac{1}{\varepsilon}\sqrt{2V_1^{(i)}(0)}$. It means that the sliding surface (3.2) will achieve $s_i = 0$ after finite time $T_1^{(i)}$.

Remark 3.1 Because the matrix C_i is a diagonal matrix, then

$$\|C_i\| = \lambda_{\max}(C_i), \ \min_{1\leq l\leq n}\{c_{il}\} = \lambda_{\min}(C_i),$$

thus the satisfying k_i in Theorem 3.1 can be expressed as $k_i \geq \dfrac{(\alpha_i + \beta_i)\lambda_{\max}(C_i) + 1}{\lambda_{\min}(C_i)}$.

After the sliding mode surface arrives at $s_i = 0$, according to the sliding mode control theory in [10], with suitable equivalent controllers, there also will be $\dot{s}_i = 0$, that is,

$$\dot{s}_i = C_i\dot{e}_i + [\mathrm{sgn}(e_i)|e_i|^\gamma] = 0 \ \ (i = 1, 2, \cdots, N), \tag{3.6}$$

it implies that

$$\dot{e}_i = -C_i^{-1}[\mathrm{sgn}(e_i)|e_i|^\gamma] \ \ (i = 1, 2, \cdots, N). \tag{3.7}$$

Theorem 3.2 After the sliding mode surface $s_i = 0$ is achieved, the errors e_i ($1 \leq i \leq N$) on the sliding mode surface will converge to zero in a finite time $T_2^{(i)} = \dfrac{2(V_2^{(i)}(0))^{\frac{1-\gamma}{2}}}{\rho(1-\gamma)}$.

Proof Construct a Lyapunov function as

$$V_2^{(i)} = \frac{1}{2}\|e_i\|^2 = \frac{1}{2}e_i^{\mathrm{T}} e_i,$$

then its derivation along (3.7) is

$$\dot{V}_2^{(i)} = e_i^{\mathrm{T}}\dot{e}_i = -e_i^{\mathrm{T}}C_i^{-1}[\mathrm{sgn}(e_i)|e_i|^{\gamma}] \le -\min_{1\le l\le n}\{\frac{1}{c_{il}}\}\cdot(\sum_{j=1}^{n}|e_{ij}|^{\gamma+1}).$$

From Lemma 2.1, we further have

$$\dot{V}_2^{(i)} \le -\min_{1\le l\le n}\{\frac{1}{c_{il}}\}\cdot(\sum_{j=1}^{n}|e_{ij}|^2)^{\frac{\gamma+1}{2}} = -\min_{1\le l\le n}\{\frac{1}{c_{il}}\}\cdot 2^{\frac{\gamma+1}{2}}\cdot(V_2^{(i)})^{\frac{\gamma+1}{2}}, \tag{3.8}$$

then from Lemma 2.2, the errors e_{ij} will converge to zero in finite time $T_2^{(i)} = \dfrac{2(V_2^{(i)}(0))^{\frac{1-\gamma}{2}}}{\rho(1-\gamma)}$, where $\rho = \min\limits_{1\le l\le n}\{\frac{1}{c_{il}}\}\cdot 2^{\frac{\gamma+1}{2}}$.

Remark 3.2 From Theorem 3.1 and Theorem 3.2, we know that networks (2.1) and (2.2) with internal and external disturbances will be synchronized after finite time $T = \max\limits_{1\le i\le n}\{T^{(i)}\}$, where $T^{(i)} = T_1^{(i)} + T_2^{(i)}$.

4 A Simulation Example

In the section, the synchronization between two different networks with 6 nodes are given as an example.

The first network is composed with the unified chaotic systems, which are described by only one parameter $\theta \in [0,1]$. It has some special features and advantages because it unifies both the Lorenz system (when $\theta = 0$) and the Chen system (when $\theta = 1$). Here, assume the internal disturbances are induced by the disturbed parameter $\triangle\theta = 0.1$, that is,

$$\dot{x}_i = \begin{pmatrix} (25(\theta+\triangle\theta)+10)(x_{i2}-x_{i1}) \\ (28-35(\theta+\triangle\theta))x_{i1} - x_{i1}x_{i3} - (29(\theta+\triangle\theta)-1)x_{i2} \\ x_{i1}x_{i2} - \frac{\theta+\triangle\theta+8}{3}x_{i3} \end{pmatrix} + \sum_{j=1}^{N} a_{ij}\varphi(x_j) + d_i^{(m)}, \tag{4.1}$$

or

$$\dot{x}_i = \begin{pmatrix} (25\theta+10)(x_{i2}-x_{i1}) \\ (28-35\theta)x_{i1} - x_{i1}x_{i3} - (29\theta-1)x_{i2} \\ x_{i1}x_{i2} - \frac{\theta+8}{3}x_{i3} \end{pmatrix} + \triangle\theta \begin{pmatrix} 25(x_{i2}-x_{i1}) \\ -35x_{i1} - 29x_{i2} \\ -\frac{1}{3}x_{i3} \end{pmatrix} + \sum_{j=1}^{N} a_{ij}\varphi(x_j) + d_i^{(m)}, \tag{4.2}$$

where

$$\varphi(x_j) = \begin{pmatrix} x_{j1} \\ x_{j2}^2 \\ x_{j3} \end{pmatrix},\ d_i^{(m)} = \begin{pmatrix} \sin(t) \\ \sin(2t) \\ \sin(3t) \end{pmatrix},\ A = (a_{ij}) = \begin{pmatrix} -5 & 1 & 2 & 1 & 0 & 1 \\ 1 & -2 & 1 & 0 & 0 & 0 \\ 2 & 1 & -5 & 1 & 1 & 0 \\ 1 & 0 & 1 & -4 & 1 & 1 \\ 0 & 0 & 1 & 1 & -3 & 1 \\ 1 & 0 & 0 & 1 & 1 & -3 \end{pmatrix}.$$

Another different network is consisted of Chua's circuits, which is given by

$$\dot{y}_i = \begin{pmatrix} \beta(y_{i2} - y_{i1} - h(y_{i1})) \\ y_{i1} - y_{i2} + y_{i3} \\ -\gamma y_{i2} \end{pmatrix} + \triangle g_i(t, y_i(t)) + \sum_{j=1}^{N} b_{ij}\psi(y_j) + d_i^{(s)} + u_i, \tag{4.3}$$

where $h(y_{i1}) = ny_{i1} + \frac{1}{2}(m-n)(|y_{i1}+1| - |y_{i1}-1|)$, the parameters (β, γ, m, n) are chosen to be $(9, 100/7, -8/7, -5/7)$, and

$$\triangle g_i(t, y_i(t)) = \begin{pmatrix} \cos(\pi y_{i1}) \\ \cos(2\pi y_{i2}) \\ \cos(3\pi y_{i3}) \end{pmatrix}, \ \psi(y_j) = \begin{pmatrix} y_{j1} \\ y_{j2} \\ -y_{j2}y_{j3} \end{pmatrix}, \ d_i^{(s)} = \begin{pmatrix} \tanh(t) \\ \tanh(2t) \\ \tanh(3t) \end{pmatrix}$$

and

$$B = (b_{ij}) = \begin{pmatrix} -5 & 1 & 1 & 1 & 1 & 1 \\ 1 & -1 & 0 & 0 & 0 & 0 \\ 1 & 0 & -1 & 0 & 0 & 0 \\ 1 & 0 & 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 & -1 & 0 \\ 1 & 0 & 0 & 0 & 0 & -1 \end{pmatrix}.$$

Let $c_{i1} = 2,\ c_{i2} = 3,\ c_{i3} = 4\ (i = 1, 2, \cdots, 6)$, Figure 1 shows that all the errors are converging to zero quickly after they arrive onto the sliding mode surface, with appropriate equivalent controllers.

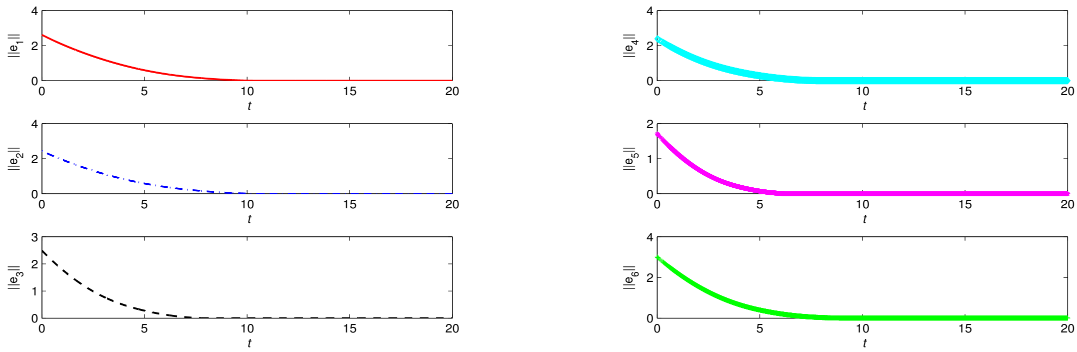

Figure 1: The error variables of $\|e_i\|$ $(i = 1, 2, \cdots, 6)$.

5 Conclusion

The finite-time synchronization between two different complex networks with disturbances is studied in this paper. Based on the sliding mode control method, some criteria and corollary are obtained to guarantee the finite-time synchronization. Finally, some numerical simulations for two complex network consisting of the unified chaotic systems and Chua's circuit systems are given to verify the correctness of the theoretical results.

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基于终端滑模控制的扰动复杂网络的有限时间同步

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摘要: 本文研究了扰动的复杂网络的有限时间同步问题. 利用终端滑模控制的方法, 设计了能保证网络同步的滑模面和控制器, 得到了两个不同的复杂网络之间达到有限时间同步的充分条件. 这些理论结果推广了复杂网络同步的一些已有结论.

关键词: 复杂网络; 有限时间同步; 滑模; 扰动

MR(2010)**主题分类号**: 93D09; 93C10 **中图分类号**: O231.2