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CENTRAL INVARIANTS OF GENERALZED HOM-LIE ALGEBRAS

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Abstract: Let L be a generalized Hom-Lie algebra, V a H-Hom-Lie ideal of $[L, L]$. In this paper, we mainly discuss the central invariant of L. Using the method of Hopf algebras, we obtain that H-invariant of V is contained in H-invariant of the center of L. It generalizes the main results by Cohen and Westreich (1994).

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1 Introduction

Hom-algebras were firstly studied by Hartwig, Larsson and Silvestrov in [4], where they introduced the structure of Hom-Lie algebras in the context of the deformations of Witt and Virasoro algebras. Determination of derivation algebras is an important task in Lie algbra, see [11]. Later, Larsson and Silvestrov extended the notion of Hom-Lie algebras to quasi-Hom Lie algebras and quasi-Lie algebras, see [5] and [6]. Wang et al. (see [10]) studied the structure of the generalized Hom-Lie algebras (i.e., the Hom-Lie algebras in Yetter-Drinfeld category ${}^H_H\mathcal{YD}$).

Let H be a Hopf algebra, and A be an H-module algebra. Cohen and Westreich [2] showed that if H is quasitriangular and A is quantum commutative with respect to H, then $A_0 \subseteq Z(A)$. It is now a naive but natural question to ask whether we can obtain same results for the generalized Hom-Lie algebras that are analogous to [2]. This becomes our main motivation of the paper.

To give a positive answer to the question above, we organize this paper as follows.

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Biography: Dong Lihong (1980–), female, born at Xinxiang, Henan, doctor, major in Hopf algebra and quantum group.

In Section 2, we recall some basic definitions about Yetter-Drinfeld modules, (monoidal) Hom-Lie algebras and generalized Hom-Lie algebras. In Section 3, we discuss the central invariant of generalized Hom-Lie algebras (see Theorem 3.8).

2 Preliminaries

In this section we recall some basic definitions and results related to our paper. Throughout the paper, all algebraic systems are supposed to be over a field k. The reader is referred to [1] as general references about monoidal Hom-algebras and monoidal Hom-Lie algebras, to [7] and [9] about Hopf algebras, and [3] about Yetter-Drinfeld categories. If C is a coalgebra, we use the Sweedler-type notation for the comultiplication: $\Delta(c) = c_1 \otimes c_2$ for all $c \in C$.

From now on we always assume that H is a Hopf algebra with a bijective antipode S. The Yetter-Drinfeld category ${}^H_H\mathcal{YD}$ is a braided monoidal category whose objects M are both left H-modules and left H-comodules, morphisms are both left H-linear and H-colinear maps and satisfy the compatibility condition

$$h_1 m_{(-1)} \otimes h_2 \cdot m_0 = (h_1 \cdot m)_{(-1)} h_2 \otimes (h_1 \cdot m)_0$$

or equivalently $\rho(h \cdot m) = h_1 m_{(-1)} S(h_3) \otimes (h_2 \cdot m_0)$, where the H-module action is denoted $h \cdot m$ and the H-comodule structure map is denoted by $\rho_M : M \to H \otimes M, \rho(m) = m_{(-1)} \otimes m_0$ for all $h \in H, m \in M$. The braiding τ is given by $\tau(m \otimes n) = m_{(-1)} \cdot n \otimes m_0$ for all $m \in M, n \in N$, M, N are objects in ${}^H_H\mathcal{YD}$.

Let A be an object in ${}^H_H\mathcal{YD}$, the braiding τ is called symmetric on A if the following condition holds, for any $a, b \in A$, $((a_{(-1)} \cdot b)_{(-1)} \cdot a_0) \otimes (a_{(-1)} \cdot b)_0 = a \otimes b$, which is equivalent to the following condition

$$a_{(-1)} \cdot b \otimes a_0 = b_0 \otimes S^{-1}(b_{(-1)}) \cdot a. \tag{2.1}$$

2.1 Monoidal Hom-Algebra

Recall from [1] that a monoidal Hom-algebra is a triple (A, μ, α) consisting of a linear space A, a k-linear map $\mu : A \otimes A \to A$ and a homomorphism $\alpha : A \to A$ for all $a, b, c \in A$, such that

$$\alpha(ab) = \alpha(a)\alpha(b),\ \alpha(1_A) = 1_A,\ \alpha(a)(bc) = (ab)\alpha(c),\ 1_A a = a 1_A = \alpha(a).$$

2.2 Monoidal Hom-Lie Algebra

Recall from [1] that a monoidal Hom-Lie algebra is a triple $(L, [,], \alpha)$ consisting of a linear space L, a bilinear map $[,] : L \otimes L \to L$ and a homomorphism $\alpha : L \to L$ satisfying

$$\begin{aligned}
&\alpha[l, l'] = [\alpha(l), \alpha(l')],\\
&[l, l'] = -[l', l] \ \ (\mathrm{Skew-symmetry}),\\
&\circlearrowleft_{l,l',l''} [\alpha(l), [l', l'']] = 0 \ \ (\mathrm{Hom-Jacobi\ identity})
\end{aligned}$$

for all $l, l', l'' \in L$, where $\circlearrowleft$ denotes the summation over the cyclic permutation on l, l', l''.

2.2 Generalized Hom-Lie Algebra

Let H be a Hopf algebra. Recall from [10] that a generalized Hom-Lie algebra is a triple $(L, [,], \alpha)$, which is a monoidal Hom-Lie algebra in a Yetter-Drinfeld category ${}_H^H\mathcal{YD}$, where L is an object in ${}_H^H\mathcal{YD}$, $\alpha : L \to L$ is a homomorphism in ${}_H^H\mathcal{YD}$ and $[,] : L \otimes L \to L$ is a morphism in ${}_H^H\mathcal{YD}$ satisfying

(1) H-Hom-skew-symmetry

$$[l, l'] = -[l_{(-1)} \cdot l', l_0],\ l, l' \in L. \tag{2.2}$$

(2) H-Hom-Jacobi identity

$$\{l \otimes l' \otimes l''\} + \{(\tau \otimes 1)(1 \otimes \tau)(l \otimes l' \otimes l'')\} + \{(1 \otimes \tau)(\tau \otimes 1)(l \otimes l' \otimes l'')\} = 0 \tag{2.3}$$

for all $l, l', l'' \in L$, where $\{l \otimes l' \otimes l''\}$ denotes $[\alpha(l), [l', l'']]$ and τ the braiding for L.

3 Main Results

In this section we always assume that the braiding τ is symmetric. We consider some H-analogous of classical concepts of ring theory and of Lie theory as follows.

Let A be a monoidal Hom-algebra in ${}_H^H\mathcal{YD}$. An H-Hom-ideal U of A is not only H-stable but also H-costable such that $\alpha(U) \subseteq U$ and $(AU)A = A(UA) \subseteq U$.

Let L be a generalized Hom-Lie algebra. An H-Hom-Lie ideal U of L is not only H-stable but also H-costable such that $\alpha(U) \subseteq U$ and $[U, L] \subseteq U$.

Define the center of L to be $Z_H(L) = \{l \in L | [l, L]_H = 0\}$. It is easy to see that $Z_H(L)$ is not only H-stable but also H-costable.

L is called H-prime if the product of any two non-zero H-Hom-ideals of L is non-zero. It is called H-semiprime if it has no non-zero nilpotent H-Hom-ideals, and is called H-simple if it has no nontrivial H-Hom-ideals.

Definition 3.1 If A is a monoidal Hom-algebra in ${}_H^H\mathcal{YD}$, the monoidal Hom-subalgebra of H-invariant is the set

$$A_0 = \{a \in A | h \cdot a = \varepsilon(h)a, \alpha(a) = a\}.$$

Example 3.2 Let $\{x_1, x_2, x_3\}$ be a basis of a 3-dimensional linear space A. The following multiplication m and linear map α on A define a monoidal Hom-algebra (see [8]):

$$\begin{aligned}
&m(x_1, x_1) = x_1, m(x_1, x_2) = x_2, m(x_1, x_3) = bx_3,\\
&m(x_2, x_1) = x_2, m(x_2, x_2) = x_2, m(x_2, x_3) = bx_3,\\
&m(x_3, x_1) = bx_3, m(x_3, x_2) = 0, m(x_3, x_3) = 0,\\
&\alpha(x_1) = x_1, \alpha(x_2) = x_2, \alpha(x_3) = bx_3,
\end{aligned}$$

where b is a parameter in k. Let G be the cyclic group of order 2 generated by g. The group algebra $H = kG$ is a Hopf algebra in the usual way,

$$\rho(x_1) = e \otimes x_1, \rho(x_2) = e \otimes x_2, \rho(x_3) = g \otimes x_3,$$
$$e \cdot x_i = x_i, g \cdot x_1 = x_1, g \cdot x_2 = x_2, g \cdot x_3 = -x_3, i = 1, 2, 3,$$

where e is the unit of the group G. It is not hard to check that A is a monoidal Hom-algebra in ${}^H_H\mathcal{YD}$.

We assume that A_0 is a linear space spanned by $\{x_1, x_2\}$, then A_0 is the monoidal Hom-subalgebra of H-invariant.

From a monoidal Hom-algebra (L, α) in ${}^H_H\mathcal{YD}$, Wang et al. [10] gave a derived monoidal Hom-Lie algebra $(L, [,], \alpha)$ in ${}^H_H\mathcal{YD}$ (that is a generalized Hom-Lie algebra) as follows for all $a, b \in L$,

$$[,] : L \otimes L \to L, \quad [a, b] = ab - (a_{(-1)} \cdot b)a_0. \tag{3.1}$$

In what follows, we always assume that the generalized Hom-Lie algebra means the above generalized Hom-Lie algebra.

The following lemma is referred to [10].

Lemma 3.3 Let $(L, [,], \alpha)$ be a generalized Hom-Lie algebra. Then

(1) $[\alpha(a), bc] = [a, b]\alpha(c) + (a_{-1} \cdot \alpha(b))[a_0, c]$;

(2) $[ab, \alpha(c)] = \alpha(a)[b, c] + [a, b_{(-1)} \cdot c]\alpha(b_0)$;

(3) $[ab, \alpha(c)] = [\alpha(a), bc] + [a_{(-1)} \cdot \alpha(b), (a_{0(-1)} \cdot c)a_{00}]$ for all $a, b, c \in L$.

Define $ad_x(l) = [x, l]$ for all $x, l \in L$. By Lemma 3.3 (1), we have

$$ad_{\alpha(x)}(lm) = ad_x(l)\alpha(m) + (x_{(-1)} \cdot \alpha(l))ad_{x_0}(m).$$

Lemma 3.4 Let $(L, [,], \alpha)$ be a generalized Hom-Lie algebra, and let $x \in L_0$. Then

(1) $\tau_{L,L}(x \otimes y) = y \otimes x$, $\tau_{L,L}(y \otimes x) = x \otimes y$;

(2) $ad_x(y) = xy - yx$;

(3) $ad_x(yz) = ad_x(y)\alpha(z) + \alpha(y)ad_x(z)$;

(4) $ad_x^2(yz) = ad_x^2(y)\alpha^2(z) + 2\alpha(ad_x(y)ad_x(z)) + \alpha^2(y)ad_x^2(z)$, for all $y, z \in L$.

Lemma 3.5 Let $(L, [,], \alpha)$ be a generalized Hom-Lie algebra. Assume that L is H-simple. Then $Z_H(L)_0$ is a field.

Proof Note that $Z_H(L)_0 = Z_H(L) \cap L_0 = Z(L) \cap L_0 = Z(L)_0$, where $Z(L)$ is the usual center of L. Taking $0 \neq x \in Z_H(L)_0$, we have that $Lx = I \neq 0$ is an H-Hom-ideal, thus $I = L$. That is to say that for some $y \in L$, we obtain $xy = yx = 1$. Since

$$\begin{aligned}
\alpha^2(h \cdot y) &= \alpha(h \cdot y)1 = \alpha(h \cdot y)(xy) = \alpha(h_1 \cdot y)(\varepsilon(h_2)xy) \\
&= \alpha(h_1 \cdot y)((h_2 \cdot x)y) = (h \cdot (yx))\alpha(y) \\
&= (h \cdot 1)\alpha(y) = \varepsilon(h)1\alpha(y) = \varepsilon(h)\alpha^2(y).
\end{aligned}$$

We can get $h \cdot y = \varepsilon(h)y$, that is, $y \in L_0$.

We need to show $y \in Z_H(L)$. For any $z \in L$, by Lemma 3.4 (1), $[z, x] = zx - xz = 0$. Thus $yz - zy = 0$, i.e., $[y, z] = yz - zy = 0$ by Lemma 3.4 (2). This shows that $y \in Z_H(L)$.

Lemma 3.6 Let $(L, [,], \alpha)$ be a generalized Hom-Lie algebra, and let $x \in L_0$, $l, m \in L$. Then

(1) $ad_x^2(xl) = \alpha^2(x)ad_x^2(l)$;

(2) if $ad_x^2(L) = 0$ and $\mathrm{char}(k) \neq 2$, then $ad_x(l)(Lad_x(m)) = 0$.

Proof (1) It is easy to show that (1) holds by Lemma 3.4 (4).

(2) For all $l, m \in L$, we have

$$\begin{aligned} 0 &= ad_x^2(lm) = ad_x^2(l)\alpha^2(m) + 2\alpha(ad_x(l)ad_x(m)) + \alpha^2(l)ad_x^2(m) \\ &= 2ad_x(\alpha(l))ad_x(\alpha(m)), \end{aligned}$$

and so $ad_x(l)ad_x(m) = 0$ since $\mathrm{char}(k) \neq 0$. Thus by Lemma 3.4 (3), one gets

$$ad_x(l)(Lad_x(m)) = 0$$

for all $l, m \in L$.

Lemma 3.7 Let $(L, [,], \alpha)$ be a generalized Hom-Lie algebra. If L is H-simple with $\mathrm{char}(k) \neq 2$, assume that I is an H-Hom-Lie ideal of $[L, L]$. Let $x \in I_0$ satisfying

(i) $ad_x(I) = 0$;

(ii) $ad_x^2([L, L]) = 0$.

Thus $x \in Z_H(L)$.

Proof Let $x \in I_0$. For any $m \in L$, $l \in [L, L]$ and $y \in I$. By Lemma 3.3 (1),

$$0 = ad_x^2([\alpha(l), my]) = ad_x^2([l, m]\alpha(y)) + ad_x^2((l_{(-1)} \cdot \alpha(m))[l_0, y]). \tag{3.2}$$

First, we have

$$\begin{aligned} & ad_x^2([l, m]\alpha(y)) \\ = \;& ad_x^2([l, m])\alpha^3(y) + 2\alpha(ad_x([l, m])ad_x(\alpha(y))) + \alpha^2([l, m])ad_x^2(\alpha(y)) \\ \overset{(i)}{=} \;& ad_x^2([l, m])\alpha^3(y) \overset{(ii)}{=} 0. \end{aligned}$$

Hence

$$ad_x^2([l, m]\alpha(y)) = 0. \tag{3.3}$$

Similarly,

$$\begin{aligned} & ad_x^2((l_{(-1)} \cdot \alpha(m))[l_0, y]) \\ = \;& ad_x^2(l_{(-1)} \cdot \alpha(m))\alpha^2([l_0, y]) + 2\alpha(ad_x(l_{(-1)} \cdot \alpha(m))ad_x([l_0, y])) \\ & + \alpha^2(l_{(-1)} \cdot \alpha(m))ad_x^2([l_0, y]). \end{aligned}$$

Since $l \in [L, L]$ and $[,]$ is H-colinear, $l_0 \in [L, L]$, $ad_x([l_0, y]) \overset{(i)}{=} 0$ and $ad_x^2([l_0, y]) \overset{(ii)}{=} 0$. Hence

$$ad_x^2((l_{(-1)} \cdot \alpha(m))[l_0, y]) = ad_x^2(l_{(-1)} \cdot \alpha(m))\alpha^2([l_0, y]). \tag{3.4}$$

Substituting (3.3) and (3.4) into (3.2), we obtain

$$ad_x^2(l_{(-1)} \cdot \alpha(m))\alpha^2([l_0, y]) = 0 \tag{3.5}$$

for all $y \in I$, $l \in [L, L]$, $m \in L$. We now consider two cases.

(1) If $[I, [L, L]] = 0$, then we have $ad_x^2(L) = 0$. By Lemma 3.6 (2), $ad_x(l)(Lad_x(m)) = 0$. Since L is H-simple, we get $ad_x(L) = 0$, $\forall\ l \in L$, and hence $x \in Z_H(L)$.

(2) Now assume $U = [I, [L, L]] \neq 0$. It is easy to see that U is an H-Hom-Lie ideal of $[L, L]$. By (3.5) we have $ad_x^2(L)U = 0$. Let $Q = \{y \in L | yU = 0\}$, then Q is an H-stable H-costable left ideal of L, we claim $Q = 0$. If not, then $L = QL$ since L is H-simple. By (2.1) we have

$$QL \subseteq [Q, L] + LQ \subseteq [Q, L] + Q.$$

Thus $L = Q + [Q, L]$. Let $y \in Q$, $l \in [L, L]$ and $u \in U$. Then

$$[y, l]u = ylu = y[l, u].$$

Since $[l, u] \in U$, $y[l, u] = 0$, and thus $[Q, [L, L]] \subseteq Q$ and $Q[L, L] \subseteq Q$. Hence

$$L = QL = Q(Q + [Q, L]) \subseteq Q.$$

This implies $LU = 0$, which contradicts the assumption $U \neq 0$. Hence, $Q = 0$, and so $ad_x^2(L) = 0$. Similarly to case (1), one gets $x \in Z_H(L)$.

Theorem 3.8 Let $(L, [,], \alpha)$ be a generalized Hom-Lie algebra. Let L be H-simple with $\mathrm{char}(k) \neq 2$, and assume that V is an H-Hom-Lie ideal of $[L, L]$ such that $[V_0, V] \subseteq Z_H(L)_0$. Then $V_0 \subseteq Z_H(L)_0$.

Proof Let V be an H-Hom-Lie ideal of $[L, L]$ such that $[V_0, V] \subseteq Z_H(L)_0$. Let $x \in V_0$. We consider the following two cases:

(1) $ad_x(V) = 0$, which implies that $x \in Z_H(L)_0$ by Lemma 3.7.

(2) $ad_x(V) \neq 0$. For $v \in V$, we have

$$\begin{aligned}
& [[x, [x, L]], \alpha(v)] \\
\overset{(2.2)}{=} \quad & -[[x, [x, L]]_{(-1)} \cdot \alpha(v), [x, [x, L]]_0] \\
\overset{(2.1)}{=} \quad & -[\alpha(v_0), [x, [x, S^{-1}(v_{(-1)}) \cdot L]]] \\
\overset{(2.3)}{=} \quad & [[x, v_{0(-1)} S^{-1}(v_{(-1)}) \cdot \alpha(L)], [\alpha(v_{00}), x]] + [x, [[x, L], v]] \\
= \quad & [[x, \alpha(L)], [\alpha(v), x]] + [x, [[x, L], v]] \\
\subseteq \quad & [[x, L], [V, x]] + [x, [[x, L], v]] \\
\subseteq \quad & 0 + [x, [[L, L], v]] \subseteq [x, V] \subseteq Z_H(L)_0.
\end{aligned}$$

We obtain $[ad_x^2(L), V] \subseteq Z_H(L)_0$. By Lemma 3.6 (1), we have $ad_x^2(xl) = \alpha^2(x)ad_x^2(l)$. If $ad_x^2(l) \neq 0$ for some $l \in L$, then $(ad_x^2(l))^{-1} \in Z_H(L)_0$ by Lemma 3.5. In this case, it is easy to see that $x \in Z_H(L)_0$. Now we assume $ad_x^2(L) \subsetneq Z_H(L)_0$. Let $y \in L$ with $ad_x^2(y) \notin Z_H(L)_0$.

Then we choose $z \in V$ such that $0 \neq ad_z(x) = u \in Z_H(L)_0$. Thus there exist $v_1, v_2, v_3 \in Z_H(L)_0$ such that $[z, ad_x^2(y)] = v_1$, $[\alpha(z), ad_x^2(xy)] = v_2$ and $[\alpha^2(z), ad_x^2(x^2y)] = v_3$. Now we have

$$\begin{aligned} v_2 &= [\alpha(z), ad_x^2(xy)] = [\alpha(z), xad_x^2(y)] \\ &= [z, x]\alpha(ad_x^2(y)) + x[z, ad_x^2(y)] = u\alpha(ad_x^2(y)) + xv_1. \end{aligned}$$

By Lemma 3.5, u is invertible. Thus $ad_x^2(y) = \alpha^{-1}(u^{-1}v_2 - u^{-1}(xv_1))$. However, by $v_1 \in Z_H(L)$, $x \in V_0$ and Lemma 3.4 (1), we have $xv_1 = v_1x$, and so $ad_x^2(y) = \alpha^{-1}(u^{-1}v_2 - u^{-1}(v_1x))$. Similarly, we have

$$\begin{aligned} v_3 &= [\alpha^2(z), ad_x^2(x^2y)] = [\alpha(\alpha(z)), xad_x^2(xy)] \\ &= [\alpha(z), x]\alpha(ad_x^2(xy)) + x[\alpha(z), \alpha(ad_x^2(xy))] \\ &= [\alpha(z), \alpha(x)]\alpha(ad_x^2(xy)) + x[\alpha(z), \alpha(ad_x^2(xy))] \\ &= \alpha(u)\alpha(ad_x^2(xy)) + xv_2 \\ &= u\alpha(ad_x^2(xy)) + xv_2, \end{aligned}$$

and thus $ad_x^2(xy) = \alpha^{-1}(u^{-1}v_3 - u^{-1}(v_2x))$. Using Lemma 3.6 (1), we have

$$\begin{aligned} ad_x^2(xy) &= xad_x^2(y) = \alpha^{-1}(\alpha(x)(u^{-1}v_2) - \alpha(x)(u^{-1}(v_1x))) \\ &= \alpha^{-1}((xu^{-1})\alpha(v_2) - (xu^{-1})\alpha(v_1x)) = \alpha^{-1}(u^{-1}(v_2x) - u^{-1}(v_1x^2)). \end{aligned}$$

Hence $v_1x^2 - 2v_2x + v_3 = 0$, that is, $x^2 + \theta^1x + \theta^0 = 0$, where $\theta^1 = -2v_2/v_1$, $\theta^0 = v_3/v_1$, and $\theta^1, \theta^0 \in Z_H(L)$. And so by Lemma 3.3 (2) and Lemma 3.4 (1) we have

$$\begin{aligned} 0 &= [-\theta^0, \alpha(z)] = [x^2, \alpha(z)] + [\theta^1x, \alpha(z)] \\ &= \alpha([x^2, z]) + \alpha(\theta^1)[x, z] + [\theta^1, x_{(-1)} \cdot z]\alpha(x_0) \\ &= \alpha([x^2, z]) + \alpha(\theta^1)[x, z]. \end{aligned}$$

By Lemma 3.4 (1), one has $\alpha([x^2, z]) = -\alpha(\theta^1)[x, z] = \alpha(\theta^1)u$, and similarly we have

$$\alpha([x^2, z]) = \alpha(x[x, z] + [x, z]x) = 2\alpha([x, z]x) = -2\alpha(ux) = -2ux.$$

Since $u \in Z_H(L)_0$, $\alpha(\theta^1) = -2x$. Since $\mathrm{char}(k) \neq 2$, $x = -(1/2)\theta^1 \in Z_H(L)$.

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广义Hom-李代数的中心不变量

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摘要: 设L是一个广义Hom-李代数, V是$[L, L]$的一个H-Hom-李理想. 本文主要研究了L的中心不变量问题. 利用Hopf代数中的方法, 得到了V的H-不变量包含在L的中心H-不变量中, 这推广了1994年Cohen和Westreich的主要结论.

关键词: 张量Hom-代数; 广义Hom-李代数; Yetter-Drinfeld范畴

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