

SCHEDULING WITH OUTSOURCING AND VARIABLE TIME SLOT COSTS

CHEN Rong-jun¹, QIN Li-zhen¹, TANG Guo-chun²

(*1. Department of Mathematics, Changzhou Institute of Technology, Changzhou 213002, China*)

(*2. Institute of Management Engineering, Shanghai Second Polytechnic University,
Shanghai 201209, China*)

Abstract: This paper is concerned with scheduling model where each job can be either processed at a manufacturer or outsourced to a subcontractor which has a single machine for processing. The outsourcing cost that the subcontractor charges the manufacturer is determined by the cost of time slots allocated to the outsourced jobs. This paper considers that the manufacturer has a single machine or two open-shop machines environment. The manufacturer needs to determine simultaneously the set of outsourced jobs and the schedule of the all jobs such that the sum of the makespan of all jobs and the outsourcing cost is minimized. For each machine environment at the manufacturer, we show the problem is NP-hard by reduction method and derive a dynamic programming algorithm.

Keywords: scheduling; outsourcing; open shop; time slot

2010 MR Subject Classification: 90B35; 90C27

Document code: A

Article ID: 0255-7797(2015)05-1068-07

1 Introduction

In recent years, outsourcing becomes more and more popular in area of industries. By outsourcing some jobs to subcontractors, manufacturers can not only complete its production task as early as possible, but also reduce its investment of production and facilities. However, once making outsourcing decision, the manufacturers will pay an amount of outsourcing cost to its subcontractors. Thus, in order to minimize the total costs including production and outsourcing, the manufacturer will determine the set of outsourced jobs and the schedule of all jobs, simultaneously.

In the last decade, scheduling models with outsourcing decision were widely investigated. Some of them study the model where the manufacturer has a single machine, for example see [1–7]. Whereas the others focus on the model where the manufacturer has complex floor shop environment, which can be found in [8–14]. But in all of the above-mentioned papers,

* **Received date:** 2014-05-01

Accepted date: 2014-09-03

Foundation item: Supported by National Natural Science Foundation of China(71371120; 61475027).

Biography: Chen Rongjun(1971–), male, born at Changzhou, Jiangsu, professor, major in scheduling and its application.

time slot costs at subcontractors have not been taken into consideration when calculating the outsourcing cost.

In this paper, we study a scheduling model with outsourcing where the outsourcing cost is determined by the total costs of time slots allocated to the outsourced jobs at the subcontractor. Two kinds of machine environments at manufacturer, i.e. single machine and two open-shop machines, are considered. In order to deliver the jobs to the costumers as soon as possible, the manufacturer is allowed to outsource some jobs to its subcontractor which has a single machine. When undertaking the production of outsourced jobs, the subcontractor will charge the manufacturer an amount of outsourcing cost. The objective is to minimize the sum of the outsourcing cost and the makespan of the schedule of all jobs. The rest of this paper is organized as follows. In Section 2, we provide a formal description of our model and present two primary results. In Section 3, we study the model where the manufacturer has a single machine. We show the computational complexity and derive a pseudo-polynomial time algorithm to the model. In Section 4, we study the model where the manufacturer has two open-shop machines environment. Finally, we conclude our papers in Section 5.

2 Problem Description

Suppose that a manufacturer will process a set of jobs $N = \{1, 2, \dots, n\}$. Let $p_j, j \in N$ be the processing time of job j . This paper considers two kinds of machine environment at the manufacturer, i.e., (1) single machine and (2) two open-shop machines. In the second machine environment, the manufacturer has two machines $\{M_1, M_2\}$ for processing the jobs in N . Each job $j \in N$ consists of exactly two operations $\{O_{1,j}, O_{2,j}\}$, where operation $O_{i,j}$ must be processed on machine M_i for the amount of time $p_{i,j}$ and $p_j = p_{1,j} + p_{2,j}$. At any time, no machine can process more than one job and no job can be processed on more than one machine. When started, no job or operation can be interrupted before it is completed. All jobs are available at time zero. Set

$$P_i = \sum_{j=1}^n p_{ij}, i = 1, 2, \quad P = P_1 + P_2.$$

To make the jobs completed earlier, the manufacturer can outsource some jobs to its subcontractor which has a single machine. The subcontractor's machine can process both operations of any outsourced job. If job j is outsourced, the manufacturer will pay for the subcontractor an outsourcing cost which is related with the time slots allocated to j . Let $I_k = [k-1, k], k = 1, 2, \dots$, be the k -th time slot at the subcontractor and f_k be the cost of using I_k . Denote the set of time slots allocated to job j as B_j . $B_j = \emptyset$ implies that j is processed at the manufacturer's machine. Thus, the total outsourcing cost can be written as $\sum_{j \in N} \sum_{k \in B_j} f_k$. Let S be the schedule of all jobs and $C_{\max}(S)$ be the makespan of S . In this paper, the manufacturer needs to determine S such that $F(S) = C_{\max}(S) + \sum_{j \in N} \sum_{k \in B_j} f_k$ is minimized.

According to the machine environment at manufacturer and the classical three-field notation [15], our model can be denoted by $M|\text{slotcost, outsourcing}|C_{\max} + \sum_j \sum_{k \in B_j} f_k$, where $M \in \{1, O2\}$, 1, O2 describe the single machine and two open-shop machines environment at manufacturer, respectively.

If the time slot costs $\{f_k\}$ are non-decreasing with k , the outsourced jobs should be processed as early as possible at subcontractor's machine. Thus, our model reduces to the corresponding classical scheduling with outsourcing which was studied [12]. So, in this paper, we assume that $\{f_k\}$ are non-increasing with k . For simplicity, we also assume that all the jobs will be completed by P . We have the following two lemmas.

Lemma 1 For problem $M|\text{slotcost, outsourcing}|C_{\max} + \sum_j \sum_{k \in B_j} f_k$, where $M \in \{1, O2\}$, there exists an optimal schedule without idle between any two outsourced jobs.

Proof Assume that there exists an idle between two outsourced jobs i and j where i is before j . Let t be the length of the idle. Moving i backward t units will not increase the makespan of all jobs and possibly reduce the slot time costs of i since $\{f_k\}$ are in non-increasing with k .

Lemma 2 In any optimal solution of problem $M|\text{slotcost, outsourcing}|C_{\max} + \sum_j \sum_{k \in B_j} f_k$, where $M \in \{1, O2\}$, any arbitrary sequence of the outsourced jobs leads to the same outsourcing cost.

$$3 \quad 1|\text{slotcost, outsourcing}|C_{\max} + \sum_j \sum_{k \in B_j} f_k$$

In this section, we first analyze computational complexity of

$$1|\text{slotcost, outsourcing}|C_{\max} + \sum_j \sum_{k \in B_j} f_k.$$

For the sake of convenience, we refer the problem as R1.

Theorem 1 Problem R1 is NP-hard in ordinary sense.

Proof If set $f_k = 0$ for all k s, then R1 can be reduced to the classical scheduling $P2||C_{\max}$. Thus R1 is NP-hard in binary sense.

In the following, we design a dynamic programming algorithm DP1 to solve R1. For $j = 1, 2, \dots, n$, define a state variable (j, a, t, k) to denote a sub-schedule for jobs in $A_j = \{1, 2, \dots, j\}$, where a is the total processing time of jobs at manufacturer's machine, k is the number of outsourced jobs with the last completion time t . Let $G(j, a, t, k)$ be the minimum makespan of the sub-schedules described by (j, a, t, k) . According to job j processed in-house or outsourced, we have the following two cases

$$G(j, a, t, k) = \begin{cases} \max\{G(j-1, a-p_j, t, k), a\}, & \text{if } j \text{ is processed in-house,} \\ \max\{G(j-1, a, t-p_j, k-1), t\}, & \text{otherwise.} \end{cases}$$

The initial conditions for $j = 1$ are the following:

$$G(1, a, t, k) = \begin{cases} p_1, & \text{if } a = p_1, t = 0, k = 0, \\ t, & \text{if } a = 0, t \geq p_1, k = 1, \\ \infty, & \text{else.} \end{cases}$$

The boundary conditions are $G(j, a, t, k) = \infty$ for $a < 0, t < 0, k < 0$.

The objective value is gotten by

$$\min\{G(n, a, t, k) + \sum_{l=t-P+a+1}^t f_l | a = 0, 1, \dots, P; k = 0, 1, \dots, n; t = P - a, \dots, P\}.$$

It's easy to verify that the overall running time of DP1 is in $O(n^2 P^3)$.

Instance 1 Consider three jobs $\{J_1, J_2, J_3\}$ with the processing times 1, 2 and 3, respectively. Let the time slot costs $\{f_k\}$ be in the following:

$$f_k = \begin{cases} \frac{1}{2}, & \text{if } k = 1, 2, 3, \\ \frac{1}{4}, & \text{if } k = 4, 5, 6, \\ 0, & \text{else.} \end{cases}$$

Using DP1, we get the optimal schedule with the value $G(3, 3, 3, 1) = 4.5$, where J_3 is outsourced, or $G(3, 3, 3, 2) = 4.5$ where J_1, J_2 are outsourced.

$$4 \quad O(2|\text{slotcost, outsourcing}|C_{\max} + \sum_j \sum_{k \in B_j} f_k)$$

In this section, we refer the problem as R2 and analyze its computational complexity by Partition problem in the following.

Partition Given n integers $a_1, a_2, \dots, a_n$ such that $\sum_{j=1}^n a_j = 2A$, is there a subset $Q \subseteq \{1, 2, \dots, n\}$ such that $\sum_{j \in Q} a_j = A$?

Theorem 2 Problem R2 is NP-hard in ordinary sense.

Proof We construct an instance of R2 with $n + 1$ jobs as follows. Set

$$\begin{aligned} p_{1j} &= a_j, \quad p_{2j} = 0, \quad j = 1, 2, \dots, n. \\ p_{1, n+1} &= 0, \quad p_{2, n+1} = A. \end{aligned}$$

For the time slot costs, let $f_k = x$ for $k = 1, 2, \dots, A$, where $x < 1$ and the others $f_k = \infty$. Clearly, this reduction can be done in polynomial time. We now show that P has a solution with $C_{\max} + \sum_j \sum_{k \in B_j} f_k \leq (1 + x)A$ if and only if Partition has a solution.

Necessity. Suppose there exists a subset $Q \subseteq \{1, 2, \dots, n\}$ such that $\sum_{j \in Q} a_j = A$. Let jobs in $Q \cup \{n + 1\}$ schedule in-house and outsource jobs in $N \setminus Q$ to the subcontractor. Clearly,

$$C_{\max} + \sum_j \sum_{k \in B_j} f_k = (1 + x)A.$$

Sufficiency. Suppose there is a solution to the defined instance such that

$$C_{\max} + \sum_j \sum_{k \in B_j} f_k \leq (1+x)A.$$

First, job $n+1$ cannot be outsourced. Otherwise, suppose $n+1$ is outsourced and let δ be the total processing time of operations on M_1 . We have

$$C_{\max} + \sum_j \sum_{k \in B_j} f_k = \max\{2A - \delta, A + \delta\} + x(A + \delta).$$

If $\delta \leq \frac{A}{2}$, then

$$C_{\max} + \sum_j \sum_{k \in B_j} f_k = 2A - \delta + x(A + \delta) > A + xA.$$

Otherwise, $\delta > \frac{A}{2}$ results in that

$$C_{\max} + \sum_j \sum_{k \in B_j} f_k = A + \delta + x(A + \delta) > A + xA.$$

Thus, $n+1$ must be processed in-house.

Next, Consider the situation where job $n+1$ is processed at manufacturer's machine and let Δ be the total outsourced processing time. Thus

$$C_{\max} + \sum_j \sum_{k \in B_j} f_k = \max\{2A - \Delta, A, \Delta\} + x\Delta.$$

For $\Delta > A$ or $\Delta < A$, it's not difficult to verify that

$$C_{\max} + \sum_j \sum_{k \in B_j} f_k > A + xA.$$

This implies that $\Delta = A$ and Partition has a solution.

Based on Lemma 1 and Lemma 2, we design a dynamic programming DP2 to solve R2. For $j = 1, 2, \dots, n$, we define a state variable (j, a, b, t, k) to denote a sub-schedule for jobs in $A_j = \{1, 2, \dots, j\}$, where a is the total processing time of jobs on M_1 , k is the number of jobs outsourced with the total processing time b , t is the starting time of jobs outsourced. Let $H(j, a, b, t, k)$ be the minimum makespan of the sub-schedules described by (j, a, b, t, k) .

According to job j produced in-house or outsourced, we have the following two cases:

$$H(j, a, b, t, k) = \begin{cases} \max\{H(j-1, a - p_{1j}, b, t, k), a, W_j - a - b, p_j\}, & \text{if } j \text{ is processed in-house,} \\ \max\{H(j-1, a, b - p_j, t, k-1), t + b\}, & \text{otherwise,} \end{cases}$$

where $W_j = \sum_{j \in A_j} p_j$. The initial conditions for $j = 1$ are the following:

$$H(1, a, b, t, k) = \begin{cases} p_1, & \text{if } a = p_{11}, b = 0, t = 0, k = 0, \\ t + b, & \text{if } a = 0, b = p_1, t \geq 0, k = 1, \\ \infty, & \text{else.} \end{cases}$$

The boundary conditions are $H(j, a, b, t, k) = \infty$ for $a < 0, b < 0, t < 0, k < 0$.

The objective value of R2 can be obtained by calculating

$$\min \left\{ H(n, a, b, t, k) + \sum_{l=t+1}^{t+b} f_l \mid a = 0, 1, \dots, P_1; \ b = 0, \dots, P; \right. \\ \left. t = 0, 1, \dots, P - b; \ k = 0, \dots, n; \ a + b \leq P \right\}.$$

Evidently, the overall running time of DP2 is in $O(n^2 P^4)$.

Instance 2 Consider three jobs $\{J_1, J_2, J_3\}$ with the processing times specified by vectors $P1, P2, P3$, respectively.

$$P1 = (0, 2m + 1), \quad P2 = (2m + 1, 0), \quad P3 = (m, m),$$

where m is an integer. Set

$$f_k = \begin{cases} m, & \text{if } k = 1, \\ \frac{1}{m}, & \text{if } 2 \leq k \leq 6m + 2, \\ 0, & \text{else.} \end{cases}$$

By DP2, the optimal value is given by $H(3, 2m + 1, 2m, 1, 1) = 2m + 3$, where J_1, J_2 are processed in-house, and J_3 is outsourced and completed at time $2m + 1$.

5 Conclusion

This paper has studied a scheduling model with outsourcing decision where the objective is to minimize the total outsourcing cost and the makespan of the schedule of all jobs. The outsourcing cost is the total costs of time slots allocated to the outsourced jobs at the subcontractor. Two kinds of machine environment at the manufacturer are considered in this paper. For each machine environment, we showed the problem is NP-hard in binary sense and developed a dynamic programming. For the future research, it will be of interest to study the model with multiple available open-shop machines at manufacturer and more complex objective functions.

References

- [1] Bertrand J W M, Sridharan V. A study of simple rules for subcontracting in make-to-order manufacturing[J]. European J. Oper. Resear., 2001, 128: 509–531.
- [2] Cai X, Lee C Y, Vairaktarakis G L. Optimization of processing and delivery decisions involving third-party machines[J]. Nonl. Anal., 2005, 63: 2269–2278.
- [3] Chen Z L, Li C L. Scheduling with subcontracting options[J]. IIE Transactions, 2008, 40: 1171–1184.
- [4] Lee I S, Sung C S. Minimizing due date related measures for a single machine scheduling problem with outsourcing allowed[J]. European J. Oper. Resear., 2008, 186: 931–952.
- [5] Lee I S, Sung C S. Single machine scheduling with outsourcing allowed[J]. Intern. J. Prod. Econ., 2008, 101: 623–634.

- [6] Qi X T. Coordinated Logistics scheduling for in-house production and outsourcing[J]. IEEE Trans. Autom. Sci. Engin., 2008, 5(1): 188–192.
- [7] Qi X T. Production scheduling with subcontracting: The subcontractor's pricing game[J]. J. Sched., 2012, 15: 773–781.
- [8] Lee Y H, Jeong C S, Moon C. Advanced planning and scheduling with outsourcing in manufacturing supply chain[J]. Comp. Indus. Engin., 2002, 43: 351–374.
- [9] Chung D, Lee K, Shin K, Park J. A new approach to job shop scheduling problems with due date constraints considering operation subcontracts[J]. Intern. J. Prod. Econ., 2005, 98(2): 238–250.
- [10] Qi X T. Two-stage production scheduling with an option of outsourcing from a remote supplier[J]. J. Sys. Sci. Sys. Engin., 2009, 18(1): 1–15.
- [11] Qi X T. Outsourcing and production scheduling for a two-stage flow shop[J]. Int. J. Prod. Econ., 2011, 129: 43–50.
- [12] Chen R J, Zhang F, Tang G C. Scheduling with contracting options under parallel and open-shop machines[J]. J. Sys. Engin., 2011, 26(5): 649–655.
- [13] Choi B C, Chung J. Two-machine flop shop scheduling problem with an outsourcing option[J]. European J. Oper. Resear., 2011, 213: 66–72.
- [14] Choi B C, Lee K. Two-stage production scheduling problem with an outsourcing option[J]. European J. Oper. Resear., 2011, 213: 489–497.
- [15] Pinedo M. Scheduling theory, algorithms and systems[M]. New Jersey: Prentice-Hall, Inc., 2002.

转包且具有不同费用时间段的排序问题

陈荣军¹, 秦立珍¹, 唐国春²

(1.常州工学院数学系, 江苏 常州 213002)

(2.上海第二工业大学管理工程研究所, 上海 201209)

摘要: 本文研究制造商可以将工件转包给承包商加工的排序模型, 承包商仅有一台机器, 转包费用由分配给转包工件的不同时间段费用确定. 本文分别研究制造商有一台单机及两台自由作业机器环境情形, 需要确定被转包工件集及全部工件的加工顺序, 使得工件最大完工时间与转包费用和最小. 本文利用归约方法对制造商每个机器环境, 证明问题NP困难性, 并提出动态规划算法.

关键词: 排序; 转包; 自由作业; 时间段

MR(2010)主题分类号: 90B35; 90C27

中图分类号: O221.7; O224