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EKELAND'S PRINCIPLE FOR EXISTENCE OF SOLUTION OF VECTOR EQUILIBRIUM PROBLEM IN CONE METRIC SPACE

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Abstract: In this paper, we study the vector equilibrium problem. By using the new Ekeland's principle that we introduce in cone metric space, we derive an existence theorem of solution for vector equilibrium problem. The results obtained in this paper are new and improve the recent ones announced by many others.

Keywords: cone metric space; Ekeland's principle; vector equilibrium problem

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1 Introduction

Throughout this paper, let X be a nonempty set and R be the set of real numbers. The bi-function $\rho : X \times X \to R$ is the metric function on X and denote by (X, ρ) the metric space. Let Y be a n-dimensional Euclidean space with norm $\|\cdot\|$ and C be a cone in Y.

In 2007, Bianchi, Kassay and Pini (see [1]) introduced the vector equilibrium problem (VEP) which is to find $\bar{x} \in (X, \rho)$ such that

$$f(\bar{x}, y) \notin \operatorname{int}(C),\ \forall y \in (X, \rho), \tag{1.1}$$

where $f : (X, \rho) \times (X, \rho) \to Y$ and $\operatorname{int}(C)$ denotes the interior of C. They proved some existence results for VEP (1.1) by applying a vector version of Ekeland's principle in metric space (X, ρ). Their work improved some results of equilibrium problem (EP), which was initially introduced by Blum and Oettli (see [2]). For more details about EP, please see, for instance, [1–10].

Recently, Huang and Zhang (see [3]) introduced the concept of cone metric space, which extends the notion of metric space.

In this paper, inspired by Huang and Zhang (see [3]), Schaible, Bianchi and Kassay (see [1]), we introduce another vector version of Ekeland's principle in cone metric space. Furthermore, applying this new Ekeland's principle, we consider a new VEP in cone metric

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space and prove an existence theorem of solution for this VEP. Our results are new and improve some results announced by many others. This paper is organized as follows. In Section 2, some topological properties of cone metric space are presented. Section 3 is devoted to nonempty intersection theorem and vector Ekeland's principle in cone metric space. In Section 4, an existence result for VEP is proved in the cone metric space by applying the new Ekeland's principle.

2 Preliminaries

2.1 Some Topological Concepts and Properties in Cone Metric Space

Definition 2.1 (see [3]) Let Y be a n-dimensional Euclidean space, and θ is the zero element in Y. C is a subset of Y. C is said to be a cone if and only if:

(i) C is nonempty, closed, and $C \neq \theta$;

(ii) $\forall a, b \in R, a, b \geq 0, \forall x, y \in C \Rightarrow ax + by \in C$;

(iii) $x \in C$ and $-x \in C \Rightarrow x = \theta$.

Given a cone C, we can define a partial ordering $\preceq$ with respect to C by: $x \preceq y$ if and only if $y - x \in C$. We shall write $x \prec y$ to indicate $x \preceq y$ but $x \neq y$, and $x \prec\prec y$ stands for $y - x \in \operatorname{int}(C)$, where $\operatorname{int}(C)$ denotes the interior of C. The C is called normal if there is a real number $l > 0$ such that $\forall x, y \in Y, \theta \preceq x \preceq y$ implies $\|x\| \leq l\|y\|$. The least positive number satisfying above is called the normal constant of C.

Definition 2.2 (see [3]) Let X be a nonempty set and Y be a n-dimensional Euclidean space. If the mapping $d : X \times X \to Y$ satisfies:

(i) $\theta \preceq d(x, y)$ for all $x, y \in X$ and $d(x, y) = \theta \Leftrightarrow x = y$;

(ii) $d(x, y) = d(y, x)$ for all $x, y \in X$;

(iii) $d(x, y) \preceq d(x, z) + d(y, z)$ for all $x, y, z \in X$.

Then d is called a cone metric on X, and denote by (X, d) the cone metric space.

Remark 2.1 In the rest of this paper, we always suppose:

(i) C is the cone in n-dimensional Euclidean space Y with $\operatorname{int}(C) \neq \emptyset$, and $\preceq$ is the partial ordering with respect to C.

(ii) (X, d) is a cone metric space defined as in Definition 2.2 and (X, ρ) is the metric space.

Definition 2.3 Let (X, d) be a cone metric space. E is a nonempty subset of (X, d). E is said to be a bounded subset of (X, d) if and only if there exists a real number $M > 0$ such that $\|d(x, y)\| \leq M$ for all $x, y \in E$. (X, d) is called bounded cone metric space if and only if it is bounded itself.

Definition 2.4 Let (X, d) be a cone metric space. E is a nonempty subset of (X, d). The diameter of E (denoted by $\operatorname{diam}(E)$) is defined as:

$$\operatorname{diam}(E) = \begin{cases} \sup\{\|d(x, y)\| : x, y \in E\}, & \text{if } E \text{ is bounded}, \\ \infty, & \text{if } E \text{ is unbounded}. \end{cases}$$

Definition 2.5 Let (X,d) be a cone metric space. $x \in X, c \in C$ and $\theta \prec\prec c$. We shall denote the set $\{y \in X : d(x,y) \prec\prec c\}$ by $B(x,c)$, and call $B(x,c)$ the neighborhood of x.

Definition 2.6 Let (X,d) be a cone metric space, $A \subset X$, $x_0 \in X$ and $c \prec\prec C$. if, for every $\theta \prec\prec c$, $A \bigcap (B(x_0,c)/x_0) \neq \emptyset$, then x_0 is called the accumulation point of A. $\eth A$ denotes the set that's consisted of all the accumulation points of A. The set $A \cup \eth A$ is called the closure of A and denoted by A^-. A is closed with respect to d if and only if $A^- = A$.

2.2 Convergences of Sequences in Cone Metric Space

Definition 2.7 (see [3]) Let (X,d) be a cone metric space. Let $\{x_n\}$ be a sequence in (X,d) and $x \in (X,d)$. Then $\{x_n\}$ is said to be:

(i) convergent to x, if, for every $c \in Y$ with $\theta \prec\prec c$, there exists a real number $N > 0$ such that $d(x_n,x) \prec\prec c$ for all $n > N$. In this case, we say x is the limit of x_n and denote this by: $x_n \to x, (n \to \infty)$.

(ii) a Cauchy sequence, if, for every $\theta \prec\prec c$, there exists a real number $N > 0$ such that $d(x_n,x_m) \prec\prec c$, for all $n,m > N$.

Definition 2.8 (see [3]) A cone metric space (X,d) is said to be:

(i) complete, if every Cauchy sequence in (X,d) is convergent in (X,d);

(ii) sequentially compact, if every convergent sequence $\{x_n\}$ has convergent subsequence in (X,d).

2.3 Semicontinuity of Vector-Valued Functions in Cone Metric Space

First, let's recall some concepts and lemmas about semicontinuity in metric space (X,ρ).

Definition 2.9 (see [1]) Let (X,ρ) be a complete metric space and Y be the n-dimensional Euclidean space ordered by the cone $C \subset Y$. The vector-valued function $h : (X,\rho) \to Y$ is said to be:

(i) quasi lower semicontinuous at $x_0 \in (X,\rho)$. If, for each $b \in Y$ such that $b \notin h(x_0)+C$, there exists a neighborhood $U \subset (X,\rho)$ of x_0 such that $b \notin h(x)+C$ for all $x \in U$. h is quasi lower semicontinuous on (X,ρ) if it is quasi lower semicontinuous at each point of (X,ρ).

(ii) upper semicontinuous at $x_0 \in (X,\rho)$. If, for each $x_0 \in (X,\rho)$ and each neighborhood V of $h(x_0)$, there exists a neighborhood U of x_0 such that $h(x) \in V - C$ for all $x \in U$.

Lemma 2.1 (see [1]) Let (X,ρ) be a complete metric space, and Y is the n-dimensional Euclidean space ordered by cone $C \subset Y$. $h : (X,\rho) \to Y$ is quasi lower semicontinuous on (X,ρ) if and only if $L(h,b) = \{x \in (X,\rho) : h(x) \in b - C\}$ is closed for all $b \in Y$.

Next, we extend the concepts of quasi lower semicontinuous and upper semicontinuous from metric space to cone metric space.

Definition 2.10 Let (X,d) is complete cone metric space where the cone is normal, and Y is the n-dimensional Euclidean space ordered by cone $C \subset Y$. The vector-valued function $h : (X,d) \to Y$ is said to be:

(i) quasi lower semicontinuous at x_0, if, for each $\forall b \in Y$ such that $b \notin h(x_0) + C$, there exists a neighborhood U of x_0 such that $b \notin h(x) + C$ for all $x \in U$. h is quasi lower semicontinuous on (X,d) if it is quasi lower semicontinuous at each point of (X,d).

(ii) upper semicontinuous at $x_0 \in (X,d)$. If, for each $x_0 \in (X,d)$ and each neighborhood V of $h(x_0)$, there exists a neighborhood U of x_0 such that $h(x) \in V - C$ for all $x \in U$.

Lemma 2.2 (see [4]) Let (X,d) is complete cone metric space where the cone is normal, if we denote

$$\tau_d = \{U : \forall x \in U, \text{there exist } K_c(x) \text{ such that } x \in K_c(x) \subset U\}$$

and

$$\tau_D = \{K_\varepsilon(x) : x \in X, \varepsilon > 0\},$$

where $D(x,y) = \|d(x,y)\|$ and $K_\varepsilon(x) = \{y \in X : D(y,x) < \varepsilon\}, K_c(x) = \{y \in X : d(x,y) \prec\prec c, c \in C \text{ and } c \succ\succ \theta\}$. Then

(a) Both τ_d and τ_D are the topology on (X,d), and $\tau_d = \tau_D$;

(b) $K_c(x) \in \tau_d$ for all $x \in X$.

Applying Lemma 2.2, one can extend the Lemma 2.1 from metric space to cone metric space.

Lemma 2.3 (see [1]) Let (X,d) be a complete cone metric space, and Y is the n-dimensional Euclidean space ordered by cone $C \subset Y$. $h : (X,d) \to Y$ is quasi lower semicontinuous on (X,d) if and only if $L(h,b) = \{x \in (X,d) : h(x) \in b - C\}$ is closed for all $b \in Y$.

3 Main Results

In this section, we prove a nonempty intersection theorem in cone metric space (X,d), which can be used to extend the Ekeland's principle from metric space to cone metric space.

Lemma 3.1 Let (X,d) be complete cone metric space, where the cone is normal. $E_1, E_2, \cdots, E_i, \cdots$ are the subsets of (X,d) such that $E_1 \supset E_2 \supset \cdots \supset E_i \supset \cdots$ and $\mathrm{diam}(E_i) \to 0, (n \to \infty)$. Then, there exists a unique element in $\cap_{i \in N^+} E_i^-$.

Proof Suppose $E_1, E_2, \cdots, E_i, \cdots$ are the subsets of (X,d) such that $E_1 \supset E_2 \supset \cdots \supset E_i \supset \cdots$ and $\mathrm{diam}(E_i) \to 0, (i \to \infty)$. Let $x_i \in E_i^-$ for $i \in N^+$, and we shall prove that $\{x_i\}_{i \in N^+}$ is a Cauchy sequence. In fact, since $\mathrm{diam}(E_i) \to 0, (i \to \infty)$. For any $\varepsilon > 0$, there exists a real number $N > 0$ such that $\mathrm{diam}(E_k) < \varepsilon$ when $k > N$. For the above given $N > 0$, we have

$$\|d(x_i,x_j)\| \le \sup_{x_i,x_j \in E_{\min\{i,j\}}} \|d(x_i,x_j)\| = \mathrm{diam}(E_{\min\{i,j\}}) < \varepsilon, \text{ for } i,j > N,$$

which implies

$$\|d(x_i,x_j)\| \to 0, (i,j \to \infty).$$

From the continuity of norm, we get $d(x_i,x_j) \to \theta, (i,j \to \infty)$. That is, $\{x_i\}_{i \in N^+}$ is a Cauchy sequence.

Since (X,d) is compete, there exists $x \in (X,d)$ such that $\{x_i\}_{i \in N^+}$ is convergent to x. Observing that $x_i, x_{i+1}, \cdots \in E_i^-$ for each $i \in N^+$, we obtain $x \in E_i^-$. Hence $x \in \cap_{i \in N^+} E_i^-$.

Suppose $y \in \cap_{i \in N^+} E_i^-$, then $\|d(x,y)\| \leq \operatorname{diam}(E_i) \to 0, (i \to \infty)$, which indicates $d(x,y) = \theta$. That is, $x = y$. Applying the Lemma 3.1, we introduce a new vector version of Ekeland's principle in cone metric as follows.

Theorem 3.1 Let (X,d) be complete cone metric space, and Y is the n-dimensional Euclidean space ordered by a normal cone C, whose normal constant is l. Let $e^* : Y \to R$ be a linear functional and $f : X \times X \to Y$. Suppose the following conditions are satisfied:

(i) $f(t,t) = \theta$ for all $t \in (X,d)$;

(ii) $e^*(f(x,\cdot))$ is lower bounded for all $x \in (X,d)$;

(iii) $f(z,y) + f(y,x) \in f(z,x) + C$ for all $x,y,z \in (X,d)$;

(iv) $f(x,\cdot)$ is quasi lower semicontinuous for all $x \in (X,d)$;

(v) $e^*(d(x,z)) \geq \|d(x,z)\|$ for all $x,z \in (X,d)$, and $e^*(y) \geq 0$ for any $y \in C$.

Then, for every $\varepsilon > 0$ and every $x_0 \in X$, there exists $\bar{x} \in (X,d)$ such that

(a) $f(x_0,\bar{x}) + \varepsilon d(x_0,\bar{x}) \in -C$;

(b) $f(\bar{x},x) + \varepsilon d(\bar{x},x) \notin -C, \forall x \neq \bar{x}, x \in (X,d)$.

Proof Without loss of generality , we consider the case $\varepsilon = 1$. Let

$$F(x) = \{y \in (X,d) : f(x,y) + d(x,y) \in -C\}$$

for each $x \in (X,d)$. From the conditions (i), (iv) and Lemma 2.3, we have $F(x)$ is nonempty and closed for every $x \in (X,d)$.

Next, we divide the rest of proof into three steps.

Step 1 Show that if $y \in F(x)$, then $F(y) \subset F(x)$.

Assume $y \in F(x)$, then

$$f(x,y) + d(x,y) \in -C, \tag{3.1}$$

and taking $z \in F(y)$, we can get

$$f(y,z) + d(y,z) \in -C. \tag{3.2}$$

By (3.1), (3.2), (iii) and the triangle inequality of the cone metric space, we get $z \in F(x)$. Hence $F(y) \subset F(x)$.

Step 2 Estimate the $\operatorname{diam}(F(x))$.

Based on condition (ii), we can define real-valued function $v(x) = \inf_{z \in F(x)} e^*(f(x,z))$. If $z \in F(x)$, then there exists a $k \in C$ such that $d(x,z) = -f(x,z) - k$. Since e^* is a linear functional satisfying condition (v), we have

$$e^*(d(x,z)) = e^*(-f(x,z) - k) = -e^*(f(x,z)) - e^*(k) \leq -e^*(f(x,z)) \tag{3.3}$$

and

$$\|d(x,z)\| \leq e^*(d(x,z)) \leq -e^*(f(x,z)) \leq -\inf_{z \in F(x)} e^*(f(x,z)) = -v(x). \tag{3.4}$$

Generally, for every $x_1, x_2 \in F(x)$, since $d(x_1, x_2) \preceq d(x_1, x) + d(x, x_2)$ and together with (3.4), we get

$$\|d(x_1, x_2)\| \leq l\|d(x_1, x) + d(x, x_2)\| \leq l\|d(x_1, x)\| + l\|d(x, x_2)\| \leq -2lv(x),$$

which implies

$$\operatorname{diam}(F(x)) \leq -2lv(x) \tag{3.5}$$

Step 3 Prove $\operatorname{diam}(F(x_n)) \to 0, (n \to \infty)$.

Starting from x_0, construct a sequence x_n which satisfies $x_{n+1} \in F(x_n)$ and

$$e^*(f(x_n, x_{n+1})) \leq v(x_n) + \frac{1}{2^{n+1}}. \tag{3.6}$$

It follows from condition (iii) that

$$e^*(f(z, y)) + e^*(f(y, x)) \geq e^*(f(z, x)). \tag{3.7}$$

Applying the inequality (3.7) and the definition of $v(x)$, we obtain

$$\begin{aligned} v(x_{n+1}) &\geq \inf_{y \in F(x_n)} e^*(f(x_{n+1}, y)) \\ &\geq (\inf_{y \in F(x_n)} e^*(f(x_n, y))) - e^*(f(x_n, x_{n+1})) \\ &= v(x_n) - e^*(f(x_n, x_{n+1})). \end{aligned}$$

From the above inequality chain and (3.6), we get

$$-v(x_n) \leq -e^*(f(x_n, x_{n+1})) + \frac{1}{2^{n+1}} \leq v(x_{n+1}) - v(x_n) + \frac{1}{2^{n+1}},$$

which indicates $\operatorname{diam}(F(x_n)) \leq -2lv(x_n) \leq 2l \cdot 2^{-n}$, that is, $\operatorname{diam}(F(x_n)) \to 0, (n \to \infty)$.

Since the set $F(x_n)$ is closed and $F(x_{n+1}) \subset F(x_n)$, from the Lemma 3.1, there exists unique element $\bar{x} \in X$ such that $\cap_{n=0}^{\infty} F(x_n) = \{\bar{x}\}$. Since $\bar{x} \in F(x_0)$,we get (a). Moreover, if $x \neq \bar{x}$, then $x \notin F(\bar{x})$,and we have (b).

Remark 3.1 Actually, the linear functional e^* that satisfied the condition (v) in Theorem 3.1 may exists. Let's give a example as follows:

Let $X = Y = R^n$ and $C = \{(y_1, y_2, \cdots, y_n) \in Y : y_i \geq 0, i = 1, 2, \cdots, n\}$. Define $d : X \times X \to Y$ such that $d(x, z) = (|x_1 - z_1|, |x_2 - z_2|, \cdots, |x_n - z_n|)$ for any $x, z \in X$, where $x = (x_1, x_2, \cdots, x_n)$ and $z = (z_1, z_2, \cdots, z_n)$. Obviously, (X, d) is a cone metric space. Define a linear functional $e^* : Y \to R$ such that $e^*(y) = y_1 + y_2 +, \cdots, +y_n$ for any $y \in Y$, where $y = (y_1, y_2, \cdots, y_n)$. From this definition of d, we get $e^*(y) \geq 0$ for any $y \in C$. Noting that

$$|x_1 - z_1| + |x_2 - z_2| +, \cdots, +|x_n - z_n| \geq \sqrt{|x_1 - z_1|^2 + |x_2 - z_2|^2 +, \cdots, +|x_n - z_n|^2},$$

we have $e^*(d(x, z)) \geq \|d(x, z)\|$ for any $x, z \in (X, d)$.

Next, we consider the vector equilibrium problem (VEP) as follows: let (X,d) be a cone metric space and $f:(X,d)\times(X,d)\to Y$. The VEP that we consider in this paper is to find $\bar{x}\in(X,d)$ such that

$$f(\bar{x},y)\notin \mathrm{int}(C),\ \forall y\in(X,d). \tag{3.8}$$

Remark 3.2 The difference between VEP (3.8) and VEP (1.1) is: In VEP (1.1), the authors study the problems in metric space; However, the space that we consider in VEP (3.8) is cone metric space which is more general than VEP (1.1).

Using Theorem 3.1, we are in a position to state and prove the existence of solution for VIP (3.8) in cone metric space.

Theorem 3.2 Let (X,d) be a sequentially compact complete cone metric space whose cone is normal. Suppose that the function $f:(X,d)\times(X,d)\to Y$ and $e^*:Y\to R$ satisfy the following conditions:

(i) $f(t,t)=\theta$ for all $t\in(X,d)$;

(ii) $e^*(f(x,\cdot))$ is lower bounded for all $x\in(X,d)$;

(iii) $f(z,y)+f(y,x)\in f(z,x)+C$ for all $x,y,z\in(X,d)$;

(iv) $f(x,\cdot)$ is quasi lower semicontinuous for all $x\in(X,d)$ and $f(\cdot,y)$ is upper semicontinuous for all $y\in(X,d)$;

(v) $e^*(d(x,z))\geq\|d(x,z)\|$ for all $x,z\in(X,d)$, and $e^*(y)\geq 0$ for any $y\in C$.

Then, the set of solutions of VEP (3.8) is nonempty.

Proof It follows from the conditions (i–v) that the Theorem 3.1 holds. Taking $\varepsilon=\frac{1}{n}$, from Theorem 3.1 (b), we can find a sequence $\{x_n\}$ such that

$$f(x_n,y)+\frac{1}{n}d(x_n,y)\notin -C,\forall y\neq x_n.$$

Since (X,d) is sequentially compact, without loss of generality, we can assume x_n converges to $\bar{x}\in(X,d)$. Assume there exists some $\bar{y}\in X$ such that$f(\bar{x},\bar{y})\in-\mathrm{int}(C)$. Take a neighborhood V of $f(\bar{x},\bar{y})$ such that $V\subset-\mathrm{int}(C)$. According to condition (iv), there exists a real number $N>0$ such that $f(x_n,\bar{y})\in V-C$ when $n\geq N$. Furthermore, $\frac{1}{n}d(x_n,\bar{y})+V\subset-\mathrm{int}(C)$ if n is big enough. So

$$f(x_n,\bar{y})+\frac{1}{n}d(x_n,\bar{y})\in V-C+\frac{1}{n}d(x_n,\bar{y})\subset-\mathrm{int}(C)$$

which is a contradiction. Hence $f(\bar{x},y)\notin-\mathrm{int}(C)$ for all $y\in X$.

References

[1] Bianchi M, Kassay G, Pini R. Ekeland's principle for vector equilibrium problems[J]. Nonlinear Analysis, 2007, 66(7): 1454–1464.

[2] Blum E, Oettli W. From optimization and variational inequalities to equilibrium problems[J]. The Mathematics Student, 1994, 63: 123–145.

[3] Huang Longguang, Zhang Xian. Cone metric spaces and fixed point theorem of contractive mappings of contractive mappings[J]. J. Math. Anal. Appl., 2007, 332(2): 1468–1476.

[4] Jankovi S, Kadelburg Z, Radenovi S. On cone metric spaces: A survey[J]. Nonlinear Analysis, 2011, 74(7): 2591–2601.

[5] Bianchi M, Hadjisavvas N, Schaible S. Vector equilibrium problems with generalized monotone bifunctions[J]. J. Optim. Theory Appl., 1997, 92(3): 527–542.

[6] Bianchi M, Kassay G, Pini R. Existence of equilibria via Ekeland's principle[J]. J. Math. Appl., 2005, 305(2): 502–512.

[7] Filipovi M, Paunovi L, Radenovi S, Rajovi M. Remarks on "Conemetric spaces and fixed point theorems of T-Kannan and T-Chatterjea contractive mappings"[J]. Math. Compu. Model., 2011, 54(5): 1467–1472.

[8] Radenovi S, Kadelburg Z. Quasi-contractions on symmetric and cone symmetric spaces and cone symmetric spaces[J]. J. Math. Anal., 2011, 5(1): 38–50.

[9] Karamardian S, Schaible S. Seven kinds of monotone maps[J]. J. Optim. Theory Appl., 1990, 66(1): 37–46.

[10] Oettli W. A remark on vector-valued equilibria and generalized monotonicity mapping[J]. Acta Math. Vietnamica, 1997, 22(1): 213–221.

锥度量空间中基于Ekeland变分原理的向量均衡问题的解的存在性

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摘 要: 本文研究了向量均衡问题. 利用在锥度量空间中给出的Ekeland变分原理, 我们推导了向量均衡问题解的存在性定理. 本文的结论是新的并推广了相关文献中的结论.

关 键 词: 锥度量空间; Ekeland变分原理; 向量均衡问题

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