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DIVIDEND PROBLEMS IN THE CLASSICAL COMPOUND POISSON RISK MODEL WITH MIXED EXPONENTIALLY DISTRIBUTED CLAIM SIZE

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Abstract: This paper studies the dividend problems in the classical compound Poisson risk model with some mixed exponentially distributed claim size. By using stochastic control theory, under the unbounded dividend intensity assumption, the explicit expression for the value function is obtained and the corresponding optimal dividend strategy is given, which generalize the results of [4].

Keywords: dividend; mixed exponentially distributed; HJB equation

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1 Introduction

Consider the classical compound Poisson risk model

$$X(t) = x + ct - S(t) = x + ct - \sum_{i=1}^{N(t)} Z_i,\ t \geq 0, \tag{1.1}$$

where $X(0) = x \geq 0$ is the initial surplus, $c > 0$ is the premium rate, and $\{S(t);\ t \geq 0\}$ represents the aggregate claims process. More specifically, $\{N(t);\ t \geq 0\}$ is a Poisson process with intensity $\lambda > 0$, which denotes the number of claims up to time t, i.e., the interclaim times $\{T_i;\ i \geq 1\}$ form a sequence of independent and identically distributed (i.i.d) positive random variables (r.v.s) and have a common exponential distribution with expectation $\frac{1}{\lambda}$. The claim sizes $\{Z_i;\ i \geq 1\}$ form a sequence of i.i.d mixed exponentially distributed r.v.s with a common density function $f_Z(z) = (1-\theta)\beta e^{-\beta z} + k\theta\beta e^{-k\beta z}(0 < \theta < 1)$, we consider for simplicity $k = 2$.

The dividend problem for an insurance risk model was first proposed by Finetti [1] who proposed to look for the expected discounted sum of dividend payments until the time of ruin. Since then the risk model in the presence of dividend payments became a more and more popular topic in risk theory. Two recent survey papers are [2] and [3].

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For classical compound Poisson risk model, the problem of looking for a strategy which maxizmizes the cumulative expected discounted dividend payments was first studied by Gerber [4]. Under the exponentially distributed assumption, the explicit expression for the value function is given and the optimal dividend strategy is proved to be a barrier strategy. Azcue and Muler [5] studied the optimal dividend problem in the compound Poisson model again by using viscosity solution method. Thonhauser and Albrecher [6] considered the model with time value of ruin and proved that the optimal dividend strategy is also a barrier strategy. Zhang and Liu [7] studied the optimal dividend payment and capital injection problem for the classical compound Poisson risk model with both proportional and fixed costs. Yao et al. [8] considered the combined optimal dividend, capital injection and reinsurance strategies for the classical compound Poisson risk model. Other interesting works can be found in Yang and Hua [9] and Wu and Wang [10]. In this paper, assuming that the surplus process is described by the classical compound Poisson risk model and the claim sizes are mixed exponentially distributed, we prove that the optimal dividend strategy is a barrier strategy. In addition, the explicit expression for the expected discounted dividend payments until ruin is obtained.

This paper is organized as follows. In Section 2, the model we discussed is introduced. In Section 3, the HJB equation for the value function is given and solved explicitly.

2 The Model

Let $L(t)$ be the accumulated dividends paid up to time t. So the controlled process $\{U^L(t);\ t\geq 0\}$ is defined by

$$U^L(t)=x+ct-\sum_{i=1}^{N(t)}Z_i-L(t),\ t\geq 0. \tag{2.1}$$

Let $\tau=\inf\{t\geq 0,\ U^L(t)<0\}$ be the ruin time. A dividend process $L=\{L(t);\ t\geq 0\}$ is called admissible if it is an adapted càglàd (previsible, $L(t-)=L(t)$) and non-decreasing process, the paying dividends cannot cause ruin, i.e., $L(t)-L(t-)\leq U^L(t-)$ and $L(0-)=0$. In addition, no dividend is paid after ruin, i.e., $\mathrm{d}L(s)=0$ for $s\geq\tau$.

Assume that the dividend intensity is unbounded and dividends are discounted at a constant force of interest δ. In this paper, we aim to identify the dividend payment strategy $L=\{L(t);\ t\geq 0\}$ which maximizes the expected discounted dividend payments until ruin

$$V(x,L)=E\Big[\int_0^\tau e^{-\delta t}\mathrm{d}L(t)|U^L(0)=x\Big], \tag{2.2}$$

i.e., we are looking for the value function

$$V(x)=\sup\nolimits_L V(x,L), \tag{2.3}$$

where the supremum is taken over all admissible strategies.

3 The Value Function and the Optimal Dividend Strategy

In this section, some basic properties of the value function are given and the corresponding HJB equation is derived and solved.

The next proposition was stated in Lemma 2.37 of Schmidli [11].

Proposition 3.1 The function $V(x)$ is increasing and locally Lipschitz continuous over $[0,\infty]$, and therefore absolutely continuous. For any $x \geq 0$, we have $x + \frac{c}{\lambda+\delta} \leq V(x) \leq x + \frac{c}{\delta}$ and for any $y > x$, we have $V(y) - V(x) \geq y - x$.

The next proposition gives the HJB equation which was proved in Theorem 2.39 of Schmidli [11].

Proposition 3.2 The function $V(x)$ satisfies the HJB equation

$$\max\Big\{1 - V'(x), \mathcal{A}V(x)\Big\} = 0, \tag{3.1}$$

where $\mathcal{A}V(x) = cV'(x) - (\lambda+\delta)V(x) + \lambda \int_0^x V(x-z) f_Z(z)\mathrm{d}z$.

Assume that (3.1) has a concave differentiable solution. The crucial point where the first derivative of the value function becomes smaller than one is denoted by x_0. For $x > x_0$, we have $1 - V'(x) = 0$, which immediately gives $V_2(x) = x + B_1$ for some constant B_1. For $x \leq x_0$, we have to solve

$$cV'(x) - (\lambda+\delta)V(x) + \lambda \int_0^x V(x-z) f_Z(z)\mathrm{d}z = 0. \tag{3.2}$$

Plugging $f_Z(z) = (1-\theta)\beta e^{-\beta z} + 2\theta\beta e^{-2\beta z}$ into (3.2), changing the integration variable, we get

$$cV'(x) - (\lambda+\delta)V(x) + (1-\theta)\lambda\beta e^{-\beta x} \int_0^x V(z)e^{\beta z}\mathrm{d}z + 2\theta\lambda\beta e^{-2\beta x} \int_0^x V(z) e^{2\beta z}\mathrm{d}z = 0. \tag{3.3}$$

Applying the operator $\left(\frac{\mathrm{d}}{\mathrm{d}x} + \beta\right)$ to (3.3), we have

$$cV''(x) + (\beta c - \lambda - \delta)V'(x) + \beta(\theta\lambda - \delta)V(x) - 2\theta\lambda\beta^2 e^{-2\beta x} \int_0^x V(z)e^{2\beta z}\mathrm{d}z = 0. \tag{3.4}$$

Applying the operator $\left(\frac{\mathrm{d}}{\mathrm{d}x} + 2\beta\right)$ to (3.4), we have

$$cV'''(x) + (3\beta c - \lambda - \delta)V''(x) + \beta(2\beta c + \theta\lambda - 2\lambda - 3\delta)V'(x) - 2\beta^2\delta V(x) = 0. \tag{3.5}$$

It is well known that the solution of (3.5) is of the form

$$V(x) = A_1 e^{r_1 x} + A_2 e^{r_2 x} + A_3 e^{r_3 x} \tag{3.6}$$

for some constants $A_1,\ A_2,\ A_3$, where $r_1,\ r_2,\ r_3$ ($r_1 > 0 > r_2 > r_3$) are three real roots of the characteristic equation in ξ:

$$c\xi^3 + (3\beta c - \lambda - \delta)\xi^2 + \beta(2\beta c + \theta\lambda - 2\lambda - 3\delta)\xi - 2\beta^2\delta = 0. \tag{3.7}$$

Remark 3.3 It is easy to see that

$$\begin{aligned} r_1+r_2+r_3 &= \frac{\lambda+\delta}{c}-3\beta, \\ r_1r_2r_3 &= \frac{2\beta^2\delta}{c}, \\ r_1r_2+r_2r_3+r_3r_1 &= \frac{\beta(2\beta c+\theta\lambda-2\lambda-3\delta)}{c}. \end{aligned}$$

Let

$$h_1(\xi)=c\xi^3+(3\beta c-\lambda-\delta)\xi^2+\beta(2\beta c+\theta\lambda-2\lambda-3\delta)\xi-2\beta^2\delta,$$

then $r_1,\ r_2,\ r_3$ are three real zeros of $h_1(\xi)$. Because $h_1(-2\beta)<0$, $h_1(-\beta)>0$ and $h_1(0)<0$, we have $-\beta<r_2<0$ and $-2\beta<r_3<-\beta$.

Plugging (3.6) into (3.3) and (3.4), then let x tend to 0 from the left, we have

$$[cr_1-\lambda-\delta]A_1+[cr_2-\lambda-\delta]A_2+[cr_3-\lambda-\delta]A_3=0 \tag{3.8}$$

and

$$\begin{aligned} &\left[cr_1^2-(\lambda+\delta)r_1+(1+\theta)\lambda\beta\right]A_1+\left[cr_2^2-(\lambda+\delta)r_2+(1+\theta)\lambda\beta\right]A_2 \\ &+\left[cr_3^2-(\lambda+\delta)r_3+(1+\theta)\lambda\beta\right]A_3=0. \end{aligned} \tag{3.9}$$

Equations (3.8) and (3.9) imply that

$$A_2=A_1R_2,\ A_3=A_1R_3, \tag{3.10}$$

where

$$\begin{aligned} &R_2=\frac{R_{31}}{R_{23}},\quad R_3=\frac{R_{12}}{R_{23}}, \\ &R_{31}=[cr_3-\lambda-\delta][cr_1^2-(\lambda+\delta)r_1+(1+\theta)\lambda\beta]-[cr_1-\lambda-\delta][cr_3^2-(\lambda+\delta)r_3+(1+\theta)\lambda\beta], \\ &R_{12}=[cr_1-\lambda-\delta][cr_2^2-(\lambda+\delta)r_2+(1+\theta)\lambda\beta]-[cr_2-\lambda-\delta][cr_1^2-(\lambda+\delta)r_1+(1+\theta)\lambda\beta], \\ &R_{23}=[cr_2-\lambda-\delta][cr_3^2-(\lambda+\delta)r_3+(1+\theta)\lambda\beta]-[cr_3-\lambda-\delta][cr_2^2-(\lambda+\delta)r_2+(1+\theta)\lambda\beta]. \end{aligned}$$

We need to find a differentiable solution, so the differentiability of $V(x)$ over $x=x_0$ gives that $B_1=-x_0+V_1(x_0)$ and $V_1'(x_0)=1$, hence we have

$$A_1=\frac{1}{r_1e^{r_1x_0}+R_2r_2e^{r_2x_0}+R_3r_3e^{r_3}x_0}.$$

Therefore we get the form of $V(x)$ that

$$V(x)=\begin{cases}\dfrac{g(x)}{g'(x_0)}, & x\le x_0,\\ x-x_0+V(x_0), & x>x_0,\end{cases} \tag{3.11}$$

where $g(t)=e^{r_1t}+R_2e^{r_2t}+R_3e^{r_3t}$.

In order to determine $V(x)$, we are still short of an additional condition to determine x_0. Noting that $\{A_1, A_2, A_3\}$ are the functions of the barrier x_0, it is easy to see that the optimal barrier x_0 can be determined by minimizing $g'(t) = r_1 e^{r_1 t} + R_2 r_2 e^{r_2 t} + R_3 r_3 e^{r_3 t}$, i.e., if $x_0 > 0$, then x_0 supplies the equation

$$g''(t) = r_1^2 e^{r_1 t} + R_2 r_2^2 e^{r_2 t} + R_3 r_3^2 e^{r_3 t} = 0. \tag{3.12}$$

In the following we show that the equation $g''(t) = 0$ has a unique root if $x_0 > 0$.

Lemma 3.4 $R_{31} > 0$, $R_{12} > 0$ and $R_{23} < 0$, and therefore $R_2 < 0$ and $R_3 < 0$.

Proof Because

$$\begin{aligned}
R_{31} =& c^2 r_1 r_3 (r_1 - r_3) + c(1+\theta)\lambda\beta(r_3 - r_1) \\
& + (\lambda+\delta)\big[c(r_3 - r_1)(r_3 + r_1) + (\lambda+\delta)(r_1 - r_3)\big] \\
=& (r_1 - r_3)\big[c^2 r_1 r_3 - c(\lambda+\delta)(r_1 + r_3) - c(1+\theta)\lambda\beta + (\lambda+\delta)^2\big] \\
=& \frac{c(r_1 - r_3)}{r_2}\Big[2\beta^2\delta + (\lambda+\delta)r_2^2 + \beta[3(\lambda+\delta) - (1+\theta)\lambda]r_2\Big] \qquad (3.13) \\
=& \frac{c(r_1 - r_3)}{r_2}\Big[c r_2^3 + 3\beta c r_2^2 + \beta[2\beta c + \theta\lambda - 2\lambda - 3\delta]r_2 + \beta[3(\lambda+\delta) - (1+\theta)\lambda]r_2\Big] \\
=& c^2 (r_1 - r_3)(r_2 + \beta)(r_2 + 2\beta), \\
R_{12} =& \frac{c(r_2 - r_1)}{r_3}\Big[2\beta^2\delta + (\lambda+\delta)r_3^2 + \beta[3(\lambda+\delta) - (1+\theta)\lambda]r_3\Big] \qquad (3.14) \\
=& c^2 (r_2 - r_1)(r_3 + \beta)(r_3 + 2\beta), \\
R_{23} =& \frac{c(r_3 - r_2)}{r_1}\Big[2\beta^2\delta + (\lambda+\delta)r_1^2 + \beta[3(\lambda+\delta) - (1+\theta)\lambda]r_1\Big] \qquad (3.15) \\
=& c^2 (r_3 - r_2)(r_1 + \beta)(r_1 + 2\beta),
\end{aligned}$$

we have $R_{31} > 0$, $R_{12} > 0$ and $R_{23} < 0$, and therefore $R_2 < 0$ and $R_3 < 0$.

Lemma 3.5 $g'''(t) > 0$ for any $t \ge 0$.

Proof As

$$g'''(t) = r_1^3 e^{r_1 t} + R_2 r_2^3 e^{r_2 t} + R_3 r_3^3 e^{r_3 t},$$

we know that $g'''(t) > 0$ for any $t \ge 0$ by Lemma 3.4.

Lemma 3.6 If

$$(\lambda+\delta)^2 \ge (1+\theta)\lambda\beta c,$$

then the equation $g''(t) = 0$ has no positive root.

If

$$(\lambda+\delta)^2 < (1+\theta)\lambda\beta c,$$

then the equation $g''(t) = 0$ has a unique positive root.

Proof By (3.13), (3.14) and (3.15), we have

$$\begin{aligned}
g''(0) =& r_1^2 + R_2 r_2^2 + R_3 r_3^2 \\
=& \frac{1}{R_{23}}(R_{23}r_1^2 + R_{31}r_2^2 + R_{12}r_3^2) \\
=& \frac{c}{R_{23}}\Big[r_1(r_3 - r_2)\Big[2\beta^2\delta + (\lambda+\delta)r_1^2 + \beta[3(\lambda+\delta) - (1+\theta)\lambda]r_1\Big]\Big] \\
&+ \Big[r_2(r_1 - r_3)\Big[2\beta^2\delta + (\lambda+\delta)r_2^2 + \beta[3(\lambda+\delta) - (1+\theta)\lambda]r_2\Big]\Big] \\
&+ \Big[r_3(r_2 - r_1)\Big[2\beta^2\delta + (\lambda+\delta)r_3^2 + \beta[3(\lambda+\delta) - (1+\theta)\lambda]r_3\Big]\Big] \\
=& \frac{c}{R_{23}}\Big[(\lambda+\delta)\big[r_1r_2(r_2 - r_1)(r_2 + r_1) + r_1r_3(r_1 - r_3)(r_1 + r_3) + r_2r_3(r_3 - r_2)(r_3 + r_2)\big] \\
&+ \beta[3(\lambda+\delta) - (1+\theta)\lambda](r_1 - r_2)(r_2 - r_3)(r_3 - r_1)\Big] \\
=& \frac{c}{R_{23}}\Big[(\lambda+\delta)\big[r_1r_2(r_2 - r_1)(-\frac{3\beta c - \lambda - \delta}{c} - r_3) \\
&+ r_1r_3(r_1 - r_3)(-\frac{3\beta c - \lambda - \delta}{c} - r_2) + r_2r_3(r_3 - r_2)(-\frac{3\beta c - \lambda - \delta}{c} - r_1)\big] \\
&+ \beta[3(\lambda+\delta) - (1+\theta)\lambda](r_1 - r_2)(r_2 - r_3)(r_3 - r_1)\Big] \\
=& \frac{1}{R_{23}}[(\lambda+\delta)^2 - (1+\theta)\lambda\beta c](r_1 - r_2)(r_2 - r_3)(r_3 - r_1).
\end{aligned}$$

If $(\lambda+\delta)^2 \geq (1+\theta)\lambda\beta c$, we have $g''(0) \geq 0$, we know that the equation $g''(t) = 0$ has no positive root by Lemma 3.5. If $(\lambda+\delta)^2 < (1+\theta)\lambda\beta c$, we have $g''(0) < 0$, which together with the fact $\lim_{t\to\infty} g''(t) = \infty$, implies that the equation $g''(t) = 0$ has a unique root.

Lemma 3.7 If $(\lambda+\delta)^2 \geq (1+\theta)\lambda\beta c$, then for any $x \geq 0$, we have

$$I(x) = \lambda(1-\theta)\Big(\frac{1}{\beta} - \frac{c}{\lambda+\delta}\Big)e^{-\beta x} + \lambda\theta\Big(\frac{1}{2\beta} - \frac{c}{\lambda+\delta}\Big)e^{-2\beta x} - \delta x + \frac{\lambda c}{\lambda+\delta} + \frac{\lambda\theta}{2\beta} - \frac{\lambda}{\beta} \leq 0. \tag{3.16}$$

Proof It is easy to see that $I(0) = 0$ and

$$I'(x) = -\lambda(1-\theta)(1 - \frac{\beta c}{\lambda+\delta})e^{-\beta x} - \lambda\theta(1 - \frac{2\beta c}{\lambda+\delta})e^{-2\beta x} - \delta.$$

If $\beta c \geq \lambda + \delta$, we have

$$\begin{aligned}
I(x) \leq& \lambda(1-\theta)\Big(\frac{1}{\beta} - \frac{c}{\lambda+\delta}\Big)(1 - \beta x) + \lambda\theta\Big(\frac{1}{2\beta} - \frac{c}{\lambda+\delta}\Big)(1 - 2\beta x) - \delta x + \frac{\lambda c}{\lambda+\delta} + \frac{\lambda\theta}{2\beta} - \frac{\lambda}{\beta} \\
=& \Big[(1+\theta)\frac{\lambda\beta c}{\lambda+\delta} - (\lambda+\delta)\Big]x \leq 0.
\end{aligned}$$

If $\beta c \leq \frac{\lambda+\delta}{2}$, we have $I'(x) \leq 0$, hence

$$I(x) \leq I(0) = 0.$$

If $\frac{\lambda+\delta}{2} < \beta c < \lambda + \delta$, setting $t = e^{-\beta x}$, we have

$$I'(x) := J(t) = \lambda\theta\Big(\frac{2\beta c}{\lambda+\delta} - 1\Big)t^2 - \lambda(1-\theta)\Big(1 - \frac{\beta c}{\lambda+\delta}\Big)t - \delta.$$

For any $0 \le t \le 1$, we have $J(t) \le \max\{J(0), J(1)\}$. Since $J(0) = -\delta \le 0$ and

$$J(1) = \lambda\theta\Big(\frac{2\beta c}{\lambda+\delta} - 1\Big) - \lambda(1-\theta)\Big(1 - \frac{\beta c}{\lambda+\delta}\Big) - \delta = \Big[(1+\theta)\frac{\lambda\beta c}{\lambda+\delta} - (\lambda+\delta)\Big] \le 0,$$

we get $I'(x) \le 0$ for any $x \ge 0$, hence we have $I(x) \le I(0) = 0$.

Theorem 3.8 If

$$(\lambda+\delta)^2 \ge (1+\theta)\lambda\beta c,$$

then $V(x) = x + \frac{c}{\lambda+\delta}$ is a solution to (3.1). If

$$(\lambda+\delta)^2 < (1+\theta)\lambda\beta c,$$

then $V(x)$ defined by (3.11) is a twice continuously differentiable concave solution to (3.1), where x_0 is the unique root of the equation (3.12).

Proof Let's first consider the case $(\lambda+\delta)^2 \ge (1+\theta)\lambda\beta c$. It is obvious that $V(x) = x + \dfrac{c}{\lambda+\delta}$ solves $1 - V'(x) = 0$. Thus we have to show that

$$cV'(x) - (\lambda+\delta)V(x) + (1-\theta)\lambda\beta e^{-\beta x}\int_0^x V(z)e^{\beta z}\mathrm{d}z + 2\theta\lambda\beta e^{-2\beta x}\int_0^x V(z)e^{2\beta z}\mathrm{d}z \le 0. \quad (3.17)$$

Plugging $V(x) = x + \frac{c}{\lambda+\delta}$ into the left of (3.17), by Lemma 3.7, we have

$$\begin{aligned} I(x) =& cV'(x) - (\lambda+\delta)V(x) + (1-\theta)\lambda\beta e^{-\beta x}\int_0^x V(z)e^{\beta z}\mathrm{d}z \\ &+ 2\theta\lambda\beta e^{-2\beta x}\int_0^x V(z)e^{2\beta z}\mathrm{d}z \le 0. \end{aligned}$$

If $(\lambda+\delta)^2 < (1+\theta)\lambda\beta c$. The facts $I(0) = 0$ and $I'(0) = \Big[(1+\theta)\frac{\lambda\beta c}{\lambda+\delta} - (\lambda+\delta)\Big] > 0$ imply that there exists some $x_1 > 0$ such that $I(x_1) > 0$, so the second part in the maximum of (3.1) is positive, hence $V(x) = x + \frac{c}{\lambda+\delta}$ does not solve (3.1).

As $V''(x) = \frac{g''(x)}{g'(x_0)} < 0$ for $x < x_0$, $V''(x_0-) = V''(x_0+) = \frac{g''(x_0-)}{g'(x_0)} = 0$, $V'(x_0-) = 1$ and $V''(x) = 0$ for $x > x_0$, we know that (3.11) is a twice continuously differentiable concave solution to (3.1).

In the end, by Theorem 3.8, we give a verification theorem which tells that the function $V(x)$ defined by (3.11) is the value function. We omit its proof because it is quite similar to Proposition 5 of Thonhauser and Albrecher [6].

Theorem 3.9 For every admissible dividend strategy L, $V(x) \ge V(x, L)$, where the function $V(x)$ is defined by (3.11). Let L_0 be the barrier strategy given by the barrier x_0, then $V(x) = V(x, L_0)$.

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具有混合指数索赔分布的经典复合泊松风险模型中的分红问题

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摘 要: 本文研究了具有某混合指数索赔分布的经典复合泊松风险模型中的分红问题. 利用随机控制理论, 在无界分红强度的假设下, 给出了值函数的显式表达式和相应的最优分红策略. 推广了文献[4] 的结果.

关键词: 分红; 混合指数分布; HJB方程

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