Vol. 35 (2015)
No. 2

数 学 杂 志
J. of Math. (PRC)

THE EQUIVALENCE AND ESTIMATE OF SEVERAL AFFINE INVARIANT DISTANCES BETWEEN CONVEX BODIES

SHAO Yong-chong, GUO Qi

(*Department of Mathematics, Suzhou University of Science and Technology, Suzhou 215009, China*)

Abstract: In this paper, the equivalences of several different definitions of two types of Banach-Mazur distance between convex bodies are shown respectively, the conditions under which these two types of Banach-Mazur distance coincide are discussed, and the Banach-Mazur distances between polar bodies of special convex bodies are studied as well. The results obtained here will play some role in estimating the best upper bound of Banach-Mazur distances.

Keywords: affine invariant distance; convex body; Banach-Mazur distance; polar set

2010 MR Subject Classification: 52A38

Document code: A **Article ID:** 0255-7797(2015)02-0287-07

1 Introduction

Denote by $\mathcal{K}^n$ the family of all convex bodies (i.e. the convex sets with nonempty interior) in the Euclidean space $\mathbb{R}^n$. Other notation are referred to [15].

Denote by $\mathcal{A}ff(\mathbb{R}^n)(\mathrm{GL}(\mathbb{R}^n))$ the family of all affine (linear) maps from $\mathbb{R}^n$ to $\mathbb{R}^n$ and by $aff(\mathbb{R}^n)$ the family of all affine functionals on $\mathbb{R}^n$. As a rule, elements of $\mathbb{R}^n$ are denoted by lower-case letters, subsets by capitals and real numbers by small Greek letters. Given $C \in \mathcal{K}^n$, then by λC we mean the homothetic copy of C of ratio λ with the center at the origin o, and we write $\lambda_x C := \lambda(C - x) + x$.

In the well-known paper [9], John proved that for every centrally symmetric convex body $C \in \mathcal{K}^n$ with the origin as its center, there is a unique ellipsoid E (i.e. an affine image of the unit ball in $\mathbb{R}^n$) such that $E \subset C \subset \sqrt{n}E$, which in some sense describes the similarity between C and E. Later on, it was realized that the John's approach provided actually a way describing the differences between convex bodies, therefore, as a consequence, several (similarly, translation or affine invariant) distances between convex bodies, such as the so-called Banach-Mazur distance etc, were introduced and studied (see [1–3, 5, 8, 10–14]). It turns out that these distances defined for convex bodies play some roles in convex geometrical analysis and other related mathematics areas (cf. [4, 6, 7]).

* **Received date:** 2012-10-30 **Accepted date:** 2013-04-11

Foundation item: Supported partially by National Natural Science Foundation of China (11271282); partially by the GIF of Jiangsu (CXLX11_0960).

Biography: Shao Yongchong (1986–), male, born at Lankao, Henan, master, major in convex geometric analysis.

Corresponding author: Guo Qi.

In this article, we discuss some well-known (affine invariant) distances which appear different. Precisely, following distances will be discussed.

Definition 1 For $K, L \in \mathcal{K}^n$, four (affine invariant) distances of different forms are defined as follows (see [4–6, 9]).

i) $d_1(K,L) := \inf\{\alpha\beta \mid \alpha > 0, \beta > 0, (1/\beta)L_x \subset uK_z \subset \alpha L_x\}$ where L_x denotes $L - x$ and the infimum is taken over all applicable $z, x \in \mathbb{R}^n, u \in \mathrm{GL}(\mathbb{R}^n)$;

ii) $d_2(K,L) := \inf\{\lambda \geq 1 \mid L \subset \mathrm{T}K \subset \lambda_x L\}$;

iii) $d_3(K,L) := \inf\{\lambda \geq 1 \mid \mathrm{T}L \subset K \subset \lambda_x \mathrm{T}L\}$;

iv) $d_4(K,L) := \inf\{\lambda \geq 1 \mid \mathrm{T}_1 L \subset \mathrm{T}_2 K \subset \lambda_x \mathrm{T}_1 L\}$,

where the infimum is taken over all applicable $x \in \mathbb{R}^n, \mathrm{T}, \mathrm{T}_1, \mathrm{T}_2 \in \mathcal{A}ff(\mathbb{R}^n)$.

The following are some weaker version of the above distances

Definition 2 For $K, L \in \mathcal{K}^n$, we define (see [4–6, 9]).

i) $\widetilde{d}_1(K,L) := \inf\{|\alpha\beta| > 0 \mid (1/\beta)L_x \subset uK_z \subset \alpha L_x\}$, where the infimum is taken over all applicable $z, x \in \mathbb{R}^n, u \in \mathrm{GL}(\mathbb{R}^n)$;

ii) $\widetilde{d}_2(K,L) := \inf\{|\lambda| \geq 1 \mid L \subset \mathrm{T}K \subset \lambda_x L\}$;

iii) $\widetilde{d}_3(K,L) := \inf\{|\lambda| \geq 1 \mid \mathrm{T}L \subset K \subset \lambda_x \mathrm{T}L\}$;

iv) $\widetilde{d}_4(K,L) := \inf\{|\lambda| \geq 1 \mid \mathrm{T}_1 L \subset \mathrm{T}_2 K \subset \lambda_x \mathrm{T}_1 L\}$,

where the infimum is taken over all applicable $x \in \mathbb{R}^n, \mathrm{T}, \mathrm{T}_1, \mathrm{T}_2 \in \mathcal{A}ff(\mathbb{R}^n)$.

Remark All d_i's (resp. $\widetilde{d}$'s) are called (resp. absolute) Banach-Mazur distance (B-M distance for short) between K, L by different authors respectively, however, as far as we know, there seems no proofs available to show that they are indeed the same.

In next section, we will show that all d_i's (resp. $\widetilde{d}_i'$s) are indeed the same. Furthermore, we provide a sufficient condition for K and L under which $d_i(K,L) = \widetilde{d}_i(K,L)$.

2 The Equivalence of Distances of Different Forms

The first result in this section concerns the equivalence of all d_i's (resp. $\widetilde{d}_i$'s).

Theorem 1 For any convex bodies $K, L \in \mathcal{K}^n$, we have

i) $d_1(K,L) = d_2(K,L) = d_3(K,L) = d_4(K,L)$;

ii) $\widetilde{d}_1(K,L) = \widetilde{d}_2(K,L) = \widetilde{d}_3(K,L) = \widetilde{d}_4(K,L)$.

Proof i) First we prove $d_1(K,L) = d_2(K,L)$. For any λ and affine map $\mathrm{T} = u + x^*$ and $x^* \in \mathbb{R}^n$ (where $u \in \mathrm{GL}(\mathbb{R}^n)$) with $L \subset \mathrm{T}K = uK + x^* \subset \lambda_x L = \lambda(L - x) + x$, we have

$$L - x \subset uK + x^* - x = uK_z \subset \lambda(L - x),$$

where $z = u^{-1}(x - x^*)$. Thus $d_1(K,L) \leq d_2(K,L)$ (taking $\beta = 1$ and $\alpha = \lambda$!).

Conversely, if $(1/\beta)L_x \subset uK_z \subset \alpha L_x$, i.e. $L_x \subset \beta uK_z \subset \alpha\beta L_x$ or

$$L \subset \beta uK_z + x \subset \alpha\beta(L - x) + x,$$

then writing $\mathrm{T} := \beta u - \beta u(z) + x$ and $\lambda := \alpha\beta$, we get $L \subset \mathrm{T}K \subset \lambda_x L$, which clearly leads to $d_2(K,L) \leq d_1(K,L)$. So $d_1(K,L) = d_2(K,L)$.

Next, we prove $d_2(K,L) = d_4(K,L)$. It is obvious that $d_2(K,L) \geq d_4(K,L)$. On the other hand, set $d_4(K,L) = d^*$, then by the definition of d_4, for $\forall \varepsilon > 0$, there exist $\mathrm{T}_1^*, \mathrm{T}_2^* \in \mathcal{A}ff(\mathbb{R}^n)$ and $x \in \mathbb{R}^n$ such that

$$\mathrm{T}_1^* L \subset \mathrm{T}_2^* K \subset (d^* + \varepsilon)_x \mathrm{T}_1^* L$$

from which we get

$$d_2(\mathrm{T}_1^* L, K) = \inf\{\lambda \geq 1 \mid \mathrm{T}_1^* L \subset \mathrm{T} K \subset \lambda_x \mathrm{T}_1^* L\} \leq d^* + \varepsilon.$$

Thus by the affine invariant of d_2, we get $d_2(K,L) = d_2(\mathrm{T}_1^* L, K) \leq d^* + \varepsilon$ which, by the arbitrariness of ε, leads to $d_2(K,L) \leq d^* = d_4(K,L)$. So $d_2(K,L) = d_4(K,L)$.

The same argument works as well in showing $d_3(K,L) = d_4(K,L)$.

ii) The proof is similar to that for i).

Remark i) Since all d_i 's (resp. $\widetilde{d}_i$'s) are equal, we denote them uniformly by d_{BM} (resp. $\widetilde{d}_{BM}$). It may happen that $d_{BM}(K,L) > \widetilde{d}_{BM}(K,L)$ as shown by the example: in $\mathbb{R}^2$, suppose that K is a regular pentagon and L is a triangle, then it was shown by Lassak in [10] that $d_2(K,L) = 1 + \sqrt{5}/2 \approx 2.118$ while it was confirmed in [4] that $\widetilde{d}_2(K,L) \leq 2$ for all $K, L \in \mathcal{K}^2$.

ii) $\sup_{K,L \in \mathcal{K}^n} \widetilde{d}_{BM}(K,L) = n$ was confirmed in [4], however it is still a great challenge to find $\sup_{K,L \in \mathcal{K}^n} d_{BM}(K,L)$. A lot of efforts has been put on such an estimate, among which it is an applicable approach to find the relation between d_{BM} and $\widetilde{d}_{BM}$. Next theorem provides a sufficient condition for K and L under which $d_{BM}(K,L) = \widetilde{d}_{BM}(K,L)$ holds.

Theorem 2 Let $K, L \in \mathcal{K}^n$. Then $d_{BM}(K,L) = \widetilde{d}_{BM}(K,L)$ if one of K, L is centrally symmetric.

In order to prove Theorem 2, we need the following lemma.

Lemma 1 Let $K, L \in \mathcal{K}^n$. Then there are $\alpha, \beta \in \mathbb{R} \setminus \{0\}$, $x, z \in \mathbb{R}^n$ and $u \in \mathrm{GL}(\mathbb{R}^n)$ such that $(1/\beta)L_x \subset uK_z \subset \alpha L_x$ iff there exist $x_1, z_1 \in \mathbb{R}^n$ such that

$$(1/\beta)L_{x_1} \subset uK_{z_1} \subset \alpha L.$$

Proof If $(1/\beta)L_x \subset uK_z \subset \alpha L_x$, then $(1/\beta)L_x + \alpha x \subset uK_z + \alpha x \subset \alpha L$, i.e.,

$$(1/\beta)L_{(1-\alpha\beta)x} \subset uK_{z-u^{-1}(\alpha x)} \subset \alpha L.$$

Now the proof is done by taking $x_1 = (1-\alpha\beta)x$ and $z_1 = z - u^{-1}(\alpha x)$.

Conversely, suppose $(1/\beta)L_{x_1} \subset uK_{z_1} \subset \alpha L$. If $\alpha\beta = 1$, then $L_{x_1} \subset \beta u K_{z_1} \subset \alpha\beta L = L$ which implies obviously $x_1 = o$. Thus, we take $x = o$ and $z = z_1$. If $\alpha\beta \neq 1$, it is easy to check that $(1/\beta)L_x \subset uK_z \subset \alpha L_x$, where $x = (1/(1-\alpha\beta))x_1$ and $z = z_1 + u^{-1}((\alpha/(1-\alpha\beta))x_1)$.

Remark By similar arguments to that for Lemma 1, we can show that

$$d_{BM}(K,L) = \inf\{\alpha\beta \mid \alpha > 0, \beta > 0, (1/\beta)L_x \subset uK_z \subset \alpha L_y\},$$
$$\widetilde{d}_{BM}(K,L) = \inf\{|\alpha\beta| > 0 \mid (1/\beta)L_x \subset uK_z \subset \alpha L_y\},$$

which are actually the original definitions.

Proof of Theorem 2 Clearly we need only to show the equality for d_1. By definition, it is obvious that $d_1(K,L) \geq \widetilde{d}_1(K,L)$.

Now, without loss of generality, suppose that L is centrally symmetric with the origin as its center. It is a routine by a compactness argument to show that there are $\alpha^*, \beta^* \in \mathbb{R}\setminus\{0\}$, $x, z \in \mathbb{R}^n$ and $u \in \mathrm{GL}(\mathbb{R}^n)$ such that

$$(1/\beta^*)L_x \subset uK_z \subset \alpha^* L_x \text{ and } \widetilde{d}_1(K,L) = |\alpha^*\beta^*|. \tag{*}$$

If $\alpha^* > 0, \beta^* > 0$, then by definition we have clearly $d_1(K,L) \leq \alpha^*\beta^* = \widetilde{d}_1(K,L)$. If $\alpha^* < 0, \beta^* < 0$, then by (*) we have also

$$(1/(-\beta^*))L_x \subset (-u)K_z \subset (-\alpha^*)L_x,$$

which leads to $d_1(K,L) \leq (-\alpha^*)(-\beta^*) = \widetilde{d}_1(K,L)$ as well.

If $\alpha^* < 0$ and $\beta^* > 0$, by Lemma 1, there exist $x_1, z_1 \in \mathbb{R}^n$ such that

$$(1/\beta^*)L_{x_1} \subset uK_{z_1} \subset \alpha^* L = \alpha^*(-L) = (-\alpha^*)L.$$

Thus, by Lemma 1 again, we get $d_1(K,L) \leq (-\alpha^*)\beta^* = \widetilde{d}_1(K,L)$. Hence $d_1(K,L) = \widetilde{d}_1(K,L)$.

Remark A question related to Theorem 2 is : if $d_{BM}(K,L) = \widetilde{d}_{BM}(K,L)$ holds for all $L \in \mathcal{K}^n$, must K be centrally symmetric?

3 Banach-Mazur Distance Between Polar Bodies

As mentioned above, it is a long-standing open problem to get $\sup_{K,L\in\mathcal{K}^n} d_{BM}(K,L)$. There are many different approaches to tackling with such a problem, among which, besides relating d_{BM} to $\widetilde{d}_{BM}$, another method is to relate the B-M distance between convex bodies to that between their polar bodies (cf. [14]). In this section, we discuss the B-M distances between polar bodies.

For $K \in \mathcal{K}^n$ and $x \in \mathrm{int}K$, the interior of K, we write

$$K^x := \{z \in \mathbb{R}^n \mid \langle z, y - x\rangle \leq 1 \text{ for all } y \in K\}$$

called the polar set of K based on x. In particular, if $x = o \in \mathrm{int}K$, we use K^* in stead of K^o. It is obvious that if $x \in \mathrm{int}K \subset L$, then $K^x \supset L^x$. It is also easy to check that for any $x \in \mathrm{int}K$, $o \in \mathrm{int}K^x$; and $K^{**} = K$. Furthermore, K is symmetric (with center at o) iff so is K^*.

Proposition 1 Let $K \in \mathcal{K}^n$ and $o \in \mathrm{int}K$. Then $(\mathrm{T}K)^* = \mathrm{T}^{-\top}K^*$ for all invertible $\mathrm{T} \in \mathrm{GL}(\mathbb{R}^n)$, where $\mathrm{T}^{-\top} = (\mathrm{T}^\top)^{-1}$ and $\mathrm{T}^\top$ denotes the transpose of T. In particular, for $\lambda \neq 0$, $(\lambda K)^* = \frac{1}{\lambda}K^*$.

Proof By the definition of polar body,

$$
\begin{aligned}
&(\mathrm{T}K)^* = \{x \in \mathbb{R}^n \mid \langle x, \mathrm{T}y\rangle \leq 1 \text{ for all } y \in K\} \\
= \;& \{x \in \mathbb{R}^n \mid \langle \mathrm{T}^\top x, y\rangle \leq 1 \text{ for all } y \in K\} \\
= \;& \{\mathrm{T}^{-\top} z \in \mathbb{R}^n \mid \langle z, y\rangle \leq 1 \text{ for all } y \in K\} = \mathrm{T}^{-\top} K^*.
\end{aligned}
$$

The following theorem is natural.

Theorem 3 Let $K, L \in \mathcal{K}^n$ be symmetric with the origin o as their centers. Then $d_{BM}(K, L) = d_{BM}(K^*, L^*)$.

Proof Suppose $d_{BM}(K, L) = \lambda_0$. Then by the definition of $d_{BM}(\cdot, \cdot)$, for any $\varepsilon > 0$, there is $\mathrm{T}_0 \in \mathrm{GL}(\mathbb{R}^n)$ such that $K \subseteq \mathrm{T}_0 L \subseteq (\lambda_0 + \varepsilon)K$. Thus, by the property of polar bodys and Proposition 1, we have

$$
\begin{aligned}
& [(\lambda_0 + \varepsilon)K]^* \subseteq (\mathrm{T}_0 L)^* \subseteq K^* \\
\Leftrightarrow \;& \frac{1}{\lambda_0 + \varepsilon} K^* \subseteq \mathrm{T}_0^{-\top} L^* \subseteq K^* \\
\Leftrightarrow \;& K^* \subseteq (\lambda_0 + \varepsilon)\mathrm{T}_0^{-\top} L^* \subseteq (\lambda_0 + \varepsilon)K^*,
\end{aligned}
$$

i.e., $K^* \subseteq \mathrm{T}_1 L^* \subseteq (\lambda_0 + \varepsilon)K^*$, where $\mathrm{T}_1 := (\lambda_0 + \varepsilon)\mathrm{T}_0^{-\top} \in \mathrm{GL}(\mathbb{R}^n)$, which implies $d_{BM}(K^*, L^*) \leq \lambda_0 + \varepsilon$. So $d_{BM}(K, L) \geq d_{BM}(K^*, L^*)$ by the arbitrariness of ε.

Conversely, simply by substituting K with K^* in the above argument, we get

$$d_{BM}(K^*, L^*) \geq d_{BM}(K, L)$$

(using the fact that $K^{**} = K$). Thus $d_{BM}(K, L) = d_{BM}(K^*, L^*)$.

For non-symmetric cases, the situation becomes more complicated and of course more interesting. In general, given convex bodies $K, L \in \mathcal{K}^n$, we don't know if there exist $x \in \mathrm{int}K, y \in \mathrm{int}L$ such that $d_{BM}(K, L) = d_{BM}(K^x, L^y)$. In the following, we discuss in $\mathcal{K}^2$ a special case only where one of K and L is a triangle and the other is a quadrangle (observe that the polar sets of a triangle are still triangle).

Theorem 4 Let Q be a quadrangle. Then $d_{BM}(\triangle, Q) = d_{BM}(\triangle, Q^{x_0})$ for some $x_0 \in \mathrm{int}Q$.

To prove Theorem 4, we need the following lemmas.

Lemma 2 Let Q be a quadrangle, then there exists $\bar{x} \in \mathrm{int}Q$ such that $Q^{\bar{x}}$ is a parallelogram.

Proof Let e_i ($i = 1, \cdots, 4$) be the vertices of Q (indexed anti-o'clockwise) and $\bar{x}$ be the intersect point of diagonals of Q. Then we have first

$$Q^{\bar{x}} = \{y \mid \langle y, e_i - \bar{x}\rangle \leq 1, i = 1, \cdots, 4\} =: Q_1.$$

In fact, $Q^{\bar{x}} \subseteq Q_1$ obviously. Conversely, observing that for any $z \in Q$, $z = \sum\limits_{i=1}^{4} \lambda_i e_i$ for some

$\lambda_i \geq 0$ with $\sum\limits_{i=1}^{4} \lambda_i = 1$, we have then, for any

$$y \in Q_1, \langle y, z - \bar{x} \rangle = \langle y, \sum_{i=1}^{4} \lambda_i e_i - \bar{x} \rangle = \sum_{i=1}^{4} \lambda_i \langle y, e_i - \bar{x} \rangle \leq 1,$$

that is, $y \in Q^{\bar{x}}$. Now, since $(e_1 - \bar{x}) \parallel (e_3 - \bar{x})$ and $(e_2 - \bar{x}) \parallel (e_4 - \bar{x})$, It is easy to see Q_1 is a parallelogram. The proof is completed.

Lemma 3 Let P be an n-polygon and P' an m-polygon, and e_i, e'_j the vertices of P and P' respectively, $1 \leq i \leq n, 1 \leq j \leq m$. Then

$$\delta(P, P') \leq \max\{\max_{1\leq i\leq n} \min_{1\leq j\leq m} |e_i - e'_j|, \max_{1\leq j\leq m} \min_{1\leq i\leq n} |e_i - e'_j|\},$$

where $\delta(\cdot, \cdot)$ denotes the Hausdorff metric.

The proof is straightforward.

Lemma 4 Let $x_n, x \in \mathrm{int}Q$ and $x_n \to x$, then $Q^{x_n} \to Q^x$ with respect to the Hausdorff metric.

Proof Write $F'_i := \{y \mid \langle y, e_i - x_n \rangle = 1\}$, $F''_i := \{y \mid \langle y, e_i - x \rangle = 1\}$ and $e'_i = F'_i \cap F'_{i+1}, e''_i = F''_i \cap F''_{i+1}$, $i = 1, \cdots, 4$ (if $i = 4$, then $i + 1 := 1$), then e'_i, e''_i are the vertices of Q^{x_n} and Q^x respectively. It is easy to show that $|e'_i - e''_i| \to 0$ as $x_n \to x$. So, $\delta(Q^{x_n}, Q^x) \leq \max\limits_{1\leq i\leq 4} |e'_i - e''_i| \to 0$ as $x_n \to x$.

Remark With exactly the same argument, one can show that Lemma 4 holds for all polygons, then further, by the density of polygons in $\mathcal{K}^2$, holds for all convex bodies in $\mathcal{K}^2$.

By Lemma 4 (and Remark after Lemma 4) and the continuity of the B-M distance with respect to the Hausdorff metric, we have the following corollary.

Corollary 1 Given $K, L \in \mathcal{K}^2$, the function $p(x) := d_{BM}(K, L^x)$ is continuous in $\mathrm{int}L$.

Proof of Theorem 4 Notice that, for any quadrangle Q, $1 < d_{BM}(\triangle, Q) \leq 2$ (assume that $[e_1, e_3]$ is a diagonal of Q, then one of e_2, e_4, say e_2, is further from $[e_1, e_3]$ than e_4. Thus, it is easy to see that $\Delta_1 \subset Q \subset 2_{e_2}\Delta_1$, where $\Delta_1 = \mathrm{cov}\{e_1, e_2, e_3\}$). Hence $p(\mathrm{int}Q) = (1, 2]$ by the fact (see [5]) that $d_{BM}(\triangle, Q^{\bar{x}}) = 2$ (where $Q^{\bar{x}}$ is as in Lemma 2) and that $d_{BM}(\triangle, Q^x) \to 1$ as $x \to \partial Q$, the boundary of Q.

Now by the continuity of $d_{BM}(\triangle, Q^x)$ as shown in Corollary 1, there is some $x_0 \in \mathrm{int}Q$ such that $d_{BM}(\triangle, Q) = d_{BM}(\triangle, Q^{x_0})$.

References

[1] Ball K. An elementary introdution to modern convex geometry[J]. Flavors of Geometry, Mathematical Sciences Research Institute Publications, 1997, 31: 1–58.

[2] Gluskin E D. Diameter of the Minkowski compactum is approximately equal to n[J]. Functional Analysis and Its Applications, 1981, 15(1): 57–58.

[3] Gluskin E D. Norms of random matrices and diameters of finite-dimensional sets[J]. Mat. Sbornik, 1983, 120(2): 180–189.

[4] Gordon Y, Litvak A E, Meyer M, Pajor A. John's decomposition in the general case and applications[J]. Journal of Differential Geometry, 2004, 68(1): 99–119.

[5] Grünbaum B. Measures of symmetry of convex sets[J]. Convexity, Proceedings of Symposia in Pure Mathematics, 1963, 7: 233–270.

[6] Guo Qi, Kaijser S. On the distance between convex bodies[J]. Northeast. Math. J. 1999, 15(3): 323–331.

[7] Guo Qi, Kaijser S. Approximations of convex bodies by convex bodies[J]. Northeasten Math. J, 2003, 19(4): 323–332.

[8] Jiménez C H, Naszódi M. On the extremal distance between two convex bodies[J]. Israel Journal of Mathematics, 2011, 183(1): 103–115.

[9] John F. Extremum problems with inequalities as subsidiary conditions[J]. Courant Anniversary Volume, 1948, 187–204 .

[10] Lassk M. Approximation of convex bodies by centrally symmetric bodies[J]. Geometriae Dedicata, 1998, 72(1): 63–68.

[11] Makeev V V. Estimating the diameter of the space of plane convex figures with respect to an affine-invareant metric[J]. Journal of Mathematical Sciences, 2007, 140(4): 529–534.

[12] Novotny P. Approximation of convex bodies by simplices[J]. Geom. Dedicata, 1994, 50(1): 53–55.

[13] Palmon O. The only convex body with extremal distance from the ball is the simplex[J]. Israel Journal of Mathematics, 1992, 80(3): 337–349.

[14] Rudelson M. Distance between non-symmetric convex bodies and the MM^*-estimate[J]. Positivity, 2000, 4(2): 161–178.

[15] Schneider R. Convex bodies: The Brunn-Minkowski theory[M]. Cambridge: Cambridge Univ. Press, 1993.

凸体间几种仿射不变距离的等价性与估计

邵永冲, 国 起

(苏州科技学院数学系, 江苏 苏州 215009)

摘要: 本文研究了凸体间(绝对)Banach-Mazur距离各种不同的定义, 证明了它们的等价性; 给出了Banach-Mazur距离与绝对Banach-Mazur距离相等的一个充分条件; 最后研究了凸体极体间的Banach-Mazur距离, 并对特殊凸体对证明了其Banach-Mazur距离与其某一对极体间的Banach-Mazur距离相等. 文中结果为Banach-Mazur距离最佳上界的估计提供了进一步研究的基础.

关键词: 仿射不变距离; 凸体; Banach-Mazur距离; 极体

MR(2010)**主题分类号**: 52A38 **中图分类号**: O184; O177.99