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GLOBAL STABILITY FOR A PREDATOR-PREY MODEL WITH DISEASE IN THE PREY

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Abstract: This paper describes a three dimensional eco-epidemiological model consisting of susceptible prey, infected prey and predator. We consider the positivity and boundedness of the solution first and the local stability of the equilibrium is discussed. Obviously, the interior equilibrium A^* is always locally asymptotically stable according to the Routh-Hurwitz criterion. At last, the geometric method of Li and Muldowney is used to investigate the global stability of the interior equilibrium.

Keywords: predator-prey; disease; geometric method; global stability

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1 Introduction

Since the seminal models for predator-prey interactions was put forward by Vito Voterra and Alfred James Lotka in the mid 1920s, mutualist and competitive mechanism received much attention from scientists [20, 21]. Mathematical ecology becomes an important factor along with the experimental ecology in the development of quantitative theory for interaction of predator and prey, see [24]. Similarly, after the pioneering work of Kermack-McKendrick on SIRS, epidemiological models were studied extensively in the recent years by researchers, see [22, 23]. Both the theoretical and experimental investigations of ecology and epidemiology progressed independently along the years, until the late eighties and early nineties, more and more people begin to pay their attention to merge these two important areas of research, i.e., eco-epidemiological system, see [8, 22–24], in these three papers the authors investigated the predator-prey system with disease in the prey species only. Obviously, we can find some models with disease in the predator only or both the populations affected by disease [25, 26]. In these papers, the boundedness, bifurcation and stability criterion of the different equilibrium points of the systems are analyzed. But most of them just investigate the local stability criterion of the equilibrium points because it is not so easy to get a Lyapunov function which is the well-known method to derive the global stability of the equilibrium. Then, a new criterion for the global stability of equilibrium point is derived for nonlinear

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autonomous ordinary differential equations by Muldowney [1] in 1996 which received much attention in the recent years. But most of them use the method in the epidemiological models, see [2–4], or in the ecological model, see [5].

In the present paper, we consider an eco-epidemiological system, i.e., a predator-prey model with disease in the prey, see [6]. Our paper is significantly different from the above papers as we use the new method to study the global stability of the interior equilibrium point. The organization is as follows: Section 2 introduces the model, giving the meaning of the parameters. Section 3 deals with some basic results, e.g. positivity, boundedness of the solutions. In Section 4, we analysis the local stability of the equilibrium points of the model. Lastly, the geometric approach is used to solve the global stability problem of the interior equilibrium point. For detailed calculations one can see Haque et al. [2], Bunomo et al. [3], Kar and Mondal [4].

2 The Model

We consider the following model:

$$\begin{aligned} \frac{dS}{dt} &= rS(1-\frac{S+I}{K})-\beta SI, \\ \frac{dI}{dt} &= -cI+\beta SI-pIY, \\ \frac{dY}{dt} &= -dY+qpIY \end{aligned} \tag{2.1}$$

with initial conditions:

$$S(0), I(0), Y(0) \geq 0, \tag{2.2}$$

where all the parameters are strictly positive constants. The state variables S, I and Y denote, respectively, the population of susceptible prey species, infected prey species and predator species at time t. The susceptible prey population grows according to the logistic with law intrinsic growth rate r, K is the environmental carrying capacity, β is the rate of transmission from susceptible prey species to infected prey species who have a natural death rate c, d is the death rate of predator species and the predation coefficients is p when the coefficients of conversing prey into predator is $q(0 < q \leq 1)$.

Remark 1 It is to be noted that the prey species is divided into two classes, namely, the susceptible prey (S) and the infected prey (I) , in the presence of disease. The disease is only spread among the prey species.

Remark 2 It is assumed that only the susceptible prey have the ability to reproduce with logistic law.

3 Positivity and Boundedness of Solution

In this section, we try to investigate the positivity and boundedness of the solutions of system (2.1).

Theorem 3.1 All the solutions of system (2.1) are positive.

Proof From the first equation of system (2.1) we can get $\frac{dS}{S} = [r(1 - \frac{S+I}{K}) - \beta I]dt$, which implies $\frac{dS}{S} = \phi(S, I)dt$, where $\phi(S, I) = r(1 - \frac{S+I}{K}) - \beta I$.

Now integrating above differential equation in the region $[0, t]$, we obtain

$$S(t) = S(0)e^{\int \phi(S,I)dt} > 0, \forall t \geq 0.$$

Again, from the second equation of system (2.1) we have $\frac{dI}{I} = (-c + \beta S - pY)dt$, which implies $\frac{dI}{I} = \psi(S, I, Y)dt$, where $\psi(S, I, Y) = (-c + \beta S - pY)$.

Now integrating above differential equation in the region $[0, t]$, we get

$$I(t) = I(0)e^{\int \psi(S,I,Y)dt} > 0, \forall t \geq 0.$$

Then from the third equation of system (2.1), we can write $\frac{dY}{Y} = (-d+qpI)dt$, which implies $\frac{dY}{Y} = \varphi(I, Y)dt$, where $\varphi(I, Y) = (-d + qpI)$.

Now integrating above differential equation in the region $[0, t]$ we have

$$Y(t) = Y(0)e^{\int \varphi(I,Y)dt} > 0, \forall t \geq 0.$$

Hence all the solutions of system (2.1) are positive. In the next theorem, we attempt to find some sufficient condition for which the solution of system (2.1) is bounded.

Theorem 3.2 All the solutions of system (2.1) are bounded above.

Proof From the third equation of system (2.1) we may conclude that $I \geq \frac{d}{qp}, \forall t$. Again, from the first equation of the system (2.1), we write $S+I \leq K, \forall t$ or $S \leq K-I \leq K - \frac{d}{qp}, \forall t$. Using above equation in the second equation we can get $Y \leq \beta K - \frac{\beta d}{qp} - c$. At last, we can obtain $I \leq K - S \leq K$. Thus, all the solutions of system (2.1) are bounded above.

4 Stability Analysis of the Model

In this section, we discuss the equilibrium points of system (2.1), consider the following equations

$$\begin{aligned} &S(r(1 - \frac{S+I}{K}) - \beta I) = 0, \\ &I(-c + \beta S - pY) = 0, \\ &Y(-d + p) = 0, \end{aligned} \tag{4.1}$$

obviously, it has three possible nonnegative equilibrium points:

(i) the boundary equilibria $A_0(K, 0, 0)$;

(ii) the predator free equilibria $A_1(S_1, I_1, 0)$ where $S_1 = \frac{c}{\beta}, I_1 = \frac{k\beta r - cr}{k\beta^2 + r\beta}$;

(iii) the interior equilibrium $A^*(S, I, Y)$ are the positive root of system $\dot{S} = \dot{I} = \dot{Y} = 0$, where

$$\begin{aligned} S &= K - \frac{d}{qp} - \frac{k\beta d}{qpr}, \\ I &= \frac{d}{qp}, \\ Y &= \frac{\beta K}{p} - \frac{\beta d}{qp^2} - \frac{k\beta^2 d}{qp^2 r} - \frac{c}{p}. \end{aligned}$$

At first, we analyze the stability criterion of system (2.1) at the boundary equilibria then we examine the local and global stability of the system followed by bifurcation analysis at its interior equilibrium $A^*(S, I, Y)$.

Theorem 4.1 The boundary equilibrium $A_0(K, 0, 0)$, is locally asymptotically when $\beta K < c$.

Proof The characteristic equation of system (2.1) at $A_0(K, 0, 0)$ can be written as $(\lambda + r)(\lambda + d)(\lambda + c - \beta K) = 0$. Therefore the eigenvalues to system (2.1) at $A_0(K, 0, 0)$ are given by

$$\lambda = -r, -d, -(\beta K - c).$$

Thus, if $c < \beta K$, then the boundary equilibrium $A_0(K, 0, 0)$ is stable otherwise it is unstable.

Theorem 4.2 The sufficient conditions for system (2.1) to be locally stable, at its predator free equilibrium $A_1(S_1, I_1, 0)$, are $qpI_1 < d$.

Proof The characteristic equation of system (2.1) at $A_1(S_1, I_1, 0)$ can be written as $(\lambda - qpI_1 + d)(\lambda^2 + \frac{rS_1}{K}\lambda + \frac{r\beta}{K}I_1 + \beta^2 S_1 I_1) = 0$. Clearly, if $qpI_1 < d$, then all the eigenvalues to system (2.1), at its predator free equilibrium is locally asymptotically stable. Hence the theorem.

Theorem 4.3 The interior equilibrium A^* is always locally asymptotically stable.

Proof Clearly, the Jacobian matrix of system (2.1) at A^* is

$$J(A^*) = \begin{pmatrix} -\frac{rS}{K} & -\frac{rS}{K} - \beta S & 0 \\ \beta I & 0 & -pI \\ 0 & qpY & 0 \end{pmatrix}. \tag{4.2}$$

The characteristic equation of the system (2.1) around its interior equilibrium point can be written as

$$\lambda^3 + A\lambda^2 + B\lambda + C = 0, \tag{4.3}$$

where

$$A = \frac{rS}{K},\ B = \frac{r\beta}{K}SI + \beta^2 SI + qp^2 IY,\ C = \frac{rqp^2}{K}SIY.$$

Then

$$AB - C = \frac{r^2\beta S^2 I}{K^2} + \frac{r\beta^2 S^2 I}{K}. \tag{4.4}$$

Obviously, $A > 0, AB - C > 0$. The theorem is trivially proved according to the Routh-Hurwitz criterion.

5 Global Stability

Here, we illustrate the general method described by Li and Muldowney [1] to show an n-dimensional autonomous dynamical system $f : D \longrightarrow R^n, D \subset R^n$, an open and simply connected set and $f \in C^1(D)$, where the dynamical system is given by

$$\frac{dx}{dt} = f(x) \tag{5.1}$$

is global stable under certain parametric conditions. For detailed calculations one can see Haque et al. [2], Bunomo et al. [3], Kar and Mondal [4].

Now, we assume the following condition.

(A_1) The autonomous dynamical system (2.1) has a unique interior equilibrium point x^* in D.

(A_2) The domain D is simply connected.

(A_3) There is a compact absorbing set $\Omega \subset D$.

Definition 5.1 (Li and Muldowney) The unique equilibrium point x^* of the dynamical system (2.1) is global asymptotically stable in the domain D if it is locally asymptotically stable and all the trajectories in D converge to its interior equilibrium point x^*.

Let $J = (J_{ij})_n$ be the variational matrix of system (2.1) and $J^{|2|}$ be the second additive compound matrix with order ${}^nC_2 \times^n C_2$.

In particular, for n=3 we can write

$$J^{|2|} = \frac{\partial f^{|2|}}{\partial x} = \begin{pmatrix} V_{11} + V_{22} & V_{23} & -V_{13} \\ V_{32} & V_{11} + V_{33} & V_{12} \\ -V_{31} & V_{21} & V_{22} + V_{33} \end{pmatrix}. \tag{5.2}$$

Let $M(x)$ in $C^{1(D)}$, Li and Muldowney [1], be the ${}^nC_2 \times^n C_2$ matrix valued function.

Moreover, we also consider B a matrix such that $B = M_f M^{-1} + M J^{|2|} M^{-1}$ where the matrix M_f is represented by

$$(M_{ij}(x))_f = (\frac{\partial M_{ij}}{\partial x})^T \cdot f(x) = \nabla M_{ij} \cdot f(x).$$

Again, we consider the Lozinskii measure Γ of B (Martin [15]) with respect to a vector norm $| \cdot |$ in $R^N, N =^n C_2$, then we have

$$\Gamma(B) = \lim_{h \to 0^+} \frac{|l + hb| - 1}{h}.$$

If (A_1)–(A_3) hold then Li and Muldowney [1] show that

$$\limsup_{t \to \infty} \sup_{x_0 \in D} \frac{1}{t} \int_0^t \Gamma(B(x(s, x_0)))ds < 0. \tag{5.3}$$

Condition (5) ensures that there are no orbits (i.e., homoclinic orbits, heteroclinic cycles and periodic orbits) which give rise to a simple closed rectifiable curve in D, invariant for system (2.1). It is also a robust Bendixson criterion.

Now, we use the above discussion to show that our system (2.1) is globally stable around its interior equilibrium.

The autonomous system (2.1) can be written in the following form

$$\frac{dX}{dt} = f(X), \tag{5.4}$$

where $f(X) = \begin{pmatrix} rS(1-\frac{S+I}{K}) - \beta SI \\ -cI + \beta SI - pIY \\ -dY + qpIY \end{pmatrix}$ and $X = \begin{pmatrix} S \\ I \\ Y \end{pmatrix}$. Then the variational matrix $V(x, y, z)$ of system (2.1) can be written as

$$V = \frac{\partial f}{\partial X} = \begin{pmatrix} r - \frac{2rS+rI}{K} - \beta I & -\frac{rS}{K} - \beta S & 0 \\ \beta I & -c + \beta S - pY & -pI \\ 0 & qpY & -d + qpI \end{pmatrix}. \tag{5.5}$$

If $V^{|2|}$ be second additive compound matrix of V then $V^{|2|}$, Bunomo et al. [3], can be expressed as

$$V^{|2|} = \begin{pmatrix} r - \frac{2rS+rI}{K} - \beta I - c + \beta S - pY & -pI & 0 \\ qpY & r - \frac{2rS+rI}{K} - \beta I - d + qpI & -\frac{rS}{K} - \beta S \\ 0 & \beta I & -c + \beta S - pY - d + qpI \end{pmatrix}. \tag{5.6}$$

We consider $M(x)$ in $C^1(D)$ in such a way that $M = \operatorname{diag}\{\frac{S}{Y}, \frac{S}{Y}, \frac{S}{Y}\}$. Then we have

$$M^{-1} = \operatorname{diag}\{\frac{Y}{S}, \frac{Y}{S}, \frac{Y}{S}\}$$

and

$$M_f = \frac{dM}{dX} = \operatorname{diag}\{\frac{\dot{S}}{Y} - \frac{S}{Y^2}\dot{Y}, \frac{\dot{S}}{Y} - \frac{S}{Y^2}\dot{Y}, \frac{\dot{S}}{Y} - \frac{S}{Y^2}\dot{Y}\}.$$

Thus it is easy to show that

$$M_f M^{-1} = \operatorname{diag}\{\frac{\dot{S}}{S} - \frac{\dot{Y}}{Y}, \frac{\dot{S}}{S} - \frac{\dot{Y}}{Y}, \frac{\dot{S}}{S} - \frac{\dot{Y}}{Y}\}$$

and $MV^{|2|}M^{-1} = V^{|2|}$. We have

$$B = M_f M^{-1} + MV^{|2|}M^{-1} = \begin{pmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \end{pmatrix},$$

where $B_{11} = \frac{\dot{S}}{S} - \frac{\dot{Y}}{Y} + r - \frac{2rS+rI}{K} - \beta I - c + \beta S - pY$, $B_{12} = \begin{pmatrix} -pI & 0 \end{pmatrix}$, $B_{21} = \begin{pmatrix} qpY & 0 \end{pmatrix}^T$, and

$$B_{22} = \begin{pmatrix} \frac{\dot{S}}{S} - \frac{\dot{Y}}{Y} + r - \frac{2rS+rI}{K} - \beta I - d + qpI & -\frac{rS}{K} - \beta S \\ \beta I & \frac{\dot{S}}{S} - \frac{\dot{Y}}{Y} - c + \beta S - pY - d + qpI \end{pmatrix}.$$

Now let us define the following vector norm in $\Re^3$ as

$$|(u,v,w)| = \max\{|u|, |v|+|w|\},$$

where (u,v,w) is the vector in $\Re^3$ and it is denoted by Γ, the Lozinskii measure with respect to this norm. Therefore, $\Gamma(B) \leq \sup\{g_1, g_2\}$, where $g_i = \Gamma_1(B_{ii}) + |B_{ij}|$ for $i = 1,2$ and $i \neq j$, where $|B_{12}|, |B_{21}|$ are matrix norms with respect to the L^1 vector norm and Γ_1 is the Lozinskii measure with respect to that norm.

Consequently, we can obtain the following terms

$$\begin{aligned}
\Gamma_1(B_{11}) =& \frac{\dot{S}}{S} - \frac{\dot{Y}}{Y} + r - \frac{2rS+rI}{K} - \beta I - c + \beta S - pY.\\
|B_{12}| =& pI,\\
|B_{21}| =& qpY,\\
\Gamma_1(B_{22}) =& \frac{\dot{S}}{S} - \frac{\dot{Y}}{Y} + \max\{r - \frac{2rS+rI}{K} - d + qpI,\\
& -c + 2\beta S - pY - d + qpI + \frac{rS}{K}\}.
\end{aligned}$$

From system (2.1),

$$\begin{aligned}
\frac{\dot{Y}}{Y} &= -d + qpI,\\
g_1 &= \Gamma_1(B_{11}) + |B_{12}| = \frac{\dot{S}}{S} + d - c - qpI + r - \frac{2rS+rI}{K} - \beta I + \beta S - pY,\\
g_2 &= \Gamma_1(B_{22}) + |B_{21}| = \frac{\dot{S}}{S} + d - qpI + qpY\\
&\quad + \max\{r - \frac{2rS+rI}{K} - d + qpI, -c + 2\beta S - pY - d + qpI + \frac{rS}{K}\}\\
&= \frac{\dot{S}}{S} + d - c + \max\{r - \frac{2rS+rI}{K} - d + qpY + c, 2\beta S - pY - d + qpY + \frac{rS}{K}\}.
\end{aligned}$$

Hence

$$\begin{aligned}
\Gamma(B) &\leq \frac{\dot{S}}{S} + d - c + \max\{-qpI + r - \frac{2rS+rI}{K} - \beta I + \beta S - pY, r - \frac{2rS+rI}{K} - d + qpY + c,\\
&\quad 2\beta S - pY - d + qpY + \frac{rS}{K}\},
\end{aligned}$$

i.e.,

$$\begin{aligned}
\Gamma(B) &\leq \frac{\dot{S}}{S} + d - c - \min\{qpI - r + \frac{2rS+rI}{K} + \beta I - \beta S + pY, -r + \frac{2rS+rI}{K} + d - qpY - c,\\
&\quad -2\beta S + pY + d - qpY - \frac{rS}{K}\}.
\end{aligned}$$

It is assumed that there exists a positive $\mu_1 \in \Re$ and $t_1 > 0$, such that

$$\mu_1 = \{\inf_{t\geq t_1} S(t), \inf_{t\geq t_1} I(t), \inf_{t\geq t_1} Y(t)\}.$$

Also, we take

$$\mu_2 = \min\{qp\mu_1 - r + \frac{2r\mu_1 + r\mu_1}{K} + \beta\mu_1 - \beta\mu_1 + p\mu_1, -r + \frac{2r\mu_1 + r\mu_1}{K} + d - qp\mu_1 - c,$$
$$-2\beta\mu_1 + p\mu_1 + d - qp\mu_1 - \frac{r\mu_1}{K}\},$$
$$\mu_2 = \min\{qp\mu_1 - r + \frac{3r\mu_1}{K} + p\mu_1, -r + \frac{2r\mu_1 + r\mu_1}{K} + d - qp\mu_1 - c,$$
$$-2\beta\mu_1 + p\mu_1 + d - qp\mu_1 - \frac{r\mu_1}{K}\},$$
$$\Gamma(B) \le \frac{\dot{S}}{S} + d - c - \mu_2,$$
$$\Gamma(B) \le \frac{\dot{S}}{S} - (c + \mu_2 - d),$$
$$\frac{1}{t}\int_0^t \Gamma(B)ds \le \frac{1}{t}\frac{S(t)}{S(0)} - (c + \mu_2 - d),$$
$$\limsup_{t\to\infty}\sup_{x\in D}\frac{1}{t}\int_0^t \Gamma(B(x(s,x)))ds < -(c + \mu_2 - d) < 0.$$

In consequence to the above analysis we have reached to state the following theorem.

Theorem 5.1 System (2.1)is globally asymptotically stable around its interior equilibrium if $c + \mu_2 > d$, where

$$\mu_2 = \min\{qp\mu_1 - r + \frac{3r\mu_1}{K} + p\mu_1, -r + \frac{2r\mu_1 + r\mu_1}{K} + d - qp\mu_1 - c, -2\beta\mu_1 + p\mu_1 + d - qp\mu_1 - \frac{r\mu_1}{K}\}$$

with $\mu_1 \in \Re^+$ such that for $t_1 > 0$ we have $\mu_1 = \{\inf_{t\ge t_1} S(t), \inf_{t\ge t_1} I(t), \inf_{t\ge t_1} Y(t)\}$.

6 Conclusions

In this paper, we deal with a three dimensional eco-epidemiological model consisting of susceptible prey, infected prey and predator. It is assumed that the disease only spreads among the prey species and just the susceptible prey has the ability to reproduce with logistic law. The dynamics of the system such as, boundedness of the solutions, existence of nonnegative equilibria and the local stability of the equilibrium points are analyzed. Then, we use the geometric method of Li and Muldowney to investigate the global stability of system (2.1). We expect that this approach can be applied to solve global stability problems in many other models.

Lastly, two future directions of work are mentioned to extend the present paper:

i) One can consider the case in which incidence is nonlinear.

ii) One can consider the case in which the predation coefficient is nonlinear.

ii) One can considers the delay effect incurred in contacts between susceptible and infected populations.

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食饵带疾病的捕食模型的全局稳定性

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摘 要: 本文研究了一类三维生态传染病模型的正解性和边界性, 并分析了系统平衡点的局部稳定性. 利用一种新的几何方法, 获得了内平衡点的全稳定性, 推广了Li和Muldowney[1]提出的这种方法的应用, 这种方法避免了寻找Lyapunov的困难.

关 键 词: 捕食者与被捕食者; 疾病; Hopf分支; 全局稳定

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