Vol. 34 (2014)
No. 4

数 学 杂 志
J. of Math. (PRC)

A SERIES OF COMPLEX SOLITARY SOLUTIONS FOR NONLINEAR JAULENT-MIODEK EQUATION USING EXP-FUNCTION METHOD IN RATIONAL FORM

LAN Chun-Xia, JIN Yun-Juan
(*College of Science, Lishui University, Lishui 323000, China*)

Abstract: In this article, we study the traveling wave solutions of nonlinear Jaulent-Miodek equations. By using the method of exponential function in rational form, a series new complex solitary wave solutions including a combination of triangular periodic function and rational function are obtained.

Keywords: exp-function method; Jaulent-Miodek equation; complex solitary solutions
2010 MR Subject Classification: 35C07; 35C08; 35C09
Document code: A **Article ID:** 0255-7797(2014)04-0679-05

1 Introduction

In this Letter, we consider the Jaulent-Miodek equation

$$\begin{aligned} &u_t + u_{xxx} + \frac{3}{2}vv_{xxx} + \frac{9}{2}v_xv_{xx} - 6uvv_x - \frac{3}{2}u_xv^2 = 0, \\ &v_t + v_{xxx} - 6u_xv - 6uv_x - \frac{15}{2}v_xv^2 = 0 \end{aligned} \tag{1.1}$$

without any initial conditions. There are many methods to solve eq.(1.1) with some initial conditions, such as variational iteration method [1], He's homotopy perturbation method [2], and homotopy analysis method [3]. These methods can only solve a special kind of solutions and obtain solutions which satisfy initial conditions. Recently, some researchers use extended tanh-method [4], generalized $(G\prime/G)$-expansion method [5], and Riccati equations method [6] to investigate the traveling wave solutions of eq. (1.1), and obtained some new types of complex solitary solutions, i.e.various combinations of trigonometric periodic functions and rational function solutions.

The purpose of this work is basing on exp-function method [7], use the exp-function method in rational form [8] to find the exact solutions of eq.(1.1).

2 Exp-Function Method in Rational Form

* **Received date:** 2013-10-09 **Accepted date:** 2014-02-20
Foundation item: Supported by National Natural Science Foundation of Zhejiang Province (Y6090681; LY12A01011); Natural Science Foundation of Zhejiang Lishui University (KZQ09023).
Biography: Lan Chunxia (1964–), female, She people, born at Lishui, Zhejiang, associate professor, major in partial differential equation.

To apply the exp-function method in rational form to eq.(1.1), we make use of the traveling wave transformation $\xi = kx + wt, u = u(\xi), v = v(\xi)$ where and are constants to be determined later. Then eq.(1.1) reduce to ordinary differential equations.

$$\begin{aligned} &wu' + k^3u''' + \frac{3}{2}k^3vv''' + \frac{9}{2}k^3v'v'' - 6kuvv' - \frac{3}{2}ku'v^2 = 0, \\ &wv' + k^3v''' - 6k(uv)' - \frac{15}{2}kv'v^2 = 0, \end{aligned} \tag{2.1}$$

where the prime denotes the derivative with respect to ξ.

The exp-function method in rational form is based on the as

$$u = \sum_{j=0}^{m} \frac{a_j}{(1+e^\xi)^j}, \tag{2.2}$$

$$v = \sum_{j=0}^{n} \frac{b_j}{(1+e^\xi)^j}, \tag{2.3}$$

where m and n are positive integers which are unknown to be further determined, a_j and b_j are unknown constants. In order to determine the value of m and n, we balance the linear term u''' with the nonlinear term vv''' in the first equation of (2.1), and the linear term v''' with the nonlinear term $v'v^2$ in the second equation of (2.1), by normal calculation, we have

$$u''' = \frac{K_1}{(1+e^\xi)^{m+3}}, \tag{2.4}$$

$$vv''' = \frac{K_2}{(1+e^\xi)^{2n+3}}, \tag{2.5}$$

$$v''' = \frac{K_3}{(1+e^\xi)^{n+3}}, \tag{2.6}$$

$$v'v^2 = \frac{K_4}{(1+e^\xi)^{3n+1}}, \tag{2.7}$$

where K_1, K_2, K_3, and K_4 are determined coefficients only for simplicity. Balancing highest order of exp-function in equations (2.4) and (2.5), we have $m = 2n$. Similarly balancing equations (2.6) and (2.7), we obtain $n = 1$, so $m = 2$. Equations (2.2) and (2.3) become

$$u = a_0 + \frac{a_1}{1+e^\xi} + \frac{a_2}{(1+e^\xi)^2}, \tag{2.8}$$

$$v = b_0 + \frac{b_1}{1+e^\xi}. \tag{2.9}$$

Substituting equations (2.8) and (2.9) into equation (2.1), by help of maple16, we have

$$\frac{1}{A}[C_1e^{4\xi} + C_2e^{3\xi} + C_3e^{2\xi} + C_4e^\xi] = 0$$

and

$$\frac{1}{B}[D_1e^{3\xi} + D_2e^{2\xi} + D_3e^\xi] = 0,$$

where

$$
\begin{aligned}
A &= (1+e^{\xi})^5, B=(1+e^{\xi})^4,\\
C_1 &= -wa_1-\frac{3}{2}k^3b_1b_0+6ka_0a_1-k^3a_1+6kb_1a_0b_0+\frac{3}{2}ka_1b_0^2,\\
C_2 &= 18kb_1a_0b_0+18ka_0a_1+6ka_1^2-8k^3a_2+12ka_0a_2+9kb_1a_1b_0+3k^3a_1+\frac{9}{2}ka_1b_0^2+6kb_1^2a_0\\
&\quad +3ka_2b_0^2+\frac{9}{2}k^3b_1b_0-6k^3b_1^2-3wa_1-2wa_2,\\
C_3 &= -4wa_2+18kb_1a_0b_0+6ka_2b_0^2+\frac{15}{2}kb_1^2a_1+3k^3a_1+12kb_1a_2b_0+\frac{9}{2}k^3b_1b_0+24ka_0a_2\\
&\quad +18kb_1a_1b_0+\frac{9}{2}ka_1b_0^2+12ka_1^2+12kb_1^2a_0+\frac{21}{2}k^3b_1^2+18ka_0a_1+18ka_1a_2+14k^3a_2-3wa_1,\\
C_4 &= \frac{15}{2}kb_1^2a_1+9kb_1^2a_2+18ka_1a_2+12kb_1a_2b_0+6kb_1^2a_0+\frac{3}{2}ka_1b_0^2-\frac{3}{2}k^3b_1b_0+6ka_0a_1+3ka_2b_0^2\\
&\quad +12ka_2^2+6kb_1a_0b_0+12ka_0a_2-wa_1-2wa_2-k^3a_1+9kb_1a_1b_0-\frac{3}{2}k^3b_1^2+6ka_1^2-2k^3a_2,\\
D_1 &= -k^3b_1+6kb_1a_0-wb_1+\frac{15}{2}kb_1b_0^2+6ka_1b_0,\\
D_2 &= 12kb_1a_0+12kb_1a_1+12ka_1b_0+12ka_2b_0-2wb_1+4k^3b_1+15kb_1b_0^2+15kb_1^2b_0,\\
D_3 &= -wb_1+6kb_1a_0+6ka_1b_0+12ka_2b_0+\frac{15}{2}kb_1b_0^2-k^3b_1+15kb_1^2b_0+12kb_1a_1+\frac{15}{2}kb_1^3+18kb_1a_2.
\end{aligned}
$$

Equating the coefficients of all powers of $e^{n\xi}$ to be zero, we obtain

$$[C_1=0, C_2=0, C_3=0, C_4=0, D_1=0, D_2=0, D_3=0]. \tag{2.10}$$

Solving the system, equations (2.10), simultaneously, we get the following solution

$$a_0=-\frac{1}{4}b_0^2, a_1=-\frac{1}{2}k(k\pm b_0 i), a_2=\frac{3}{4}k^2, b_1=\pm ik, w=\pm 3k^2b_0i-k^3+3kb_0^2. \tag{2.11}$$

Using the transformations

$$\begin{cases} \xi = i\zeta = Kx+Wt,\\ e^{\xi}=\cos(\zeta)+i\sin(\zeta),\end{cases}$$

and $k=iK, w=iW$, where K and W are real number, eq.(2.11) becomes

$$a_0=-\frac{1}{4}b_0^2, a_1=\frac{1}{2}K^2\pm\frac{1}{2}Kb_0, a_2=-\frac{3}{4}K^2, b_1=\mp K, w=\mp 3K^2b_0-K^3+3Kb_0^2. \tag{2.12}$$

Inserting equation (2.12) into equations (2.8) and (2.9) yields the following exact solution

$$
\begin{aligned}
u =& -\frac{1}{4}b_0^2+\frac{\frac{1}{2}K^2\pm\frac{1}{2}Kb_0}{1+e^{i\zeta}}-\frac{-\frac{3}{4}K^2}{(1+e^{i\zeta})^2}\\
=&\frac{-\frac{3}{2}b_0^2+\frac{3}{4}K^2\pm\frac{3}{2}b_0K+(\pm 2b_0K-2b_0^2+\frac{1}{2}K^2)\cos(\zeta)+(\pm\frac{1}{2}b_0K-\frac{1}{2}b_0^2-\frac{1}{4}K^2)\cos(2\zeta)}{6+8\cos(\zeta)+2\cos(2\zeta)}\\
&+i\frac{(b_0K\mp\frac{1}{2}K^2)\sin(\zeta)+(\frac{1}{2}b_0K\mp\frac{1}{4}K^2)\sin(2\zeta)}{6+8\cos(\zeta)+2\cos(2\zeta)}.
\end{aligned}
\tag{2.13}
$$

If we search for a periodic solution or compaction-like solution, the imaginary part of eq.(2.13) must be zero, that requires

$$(b_0K \mp \frac{1}{2}K^2)\sin(\zeta) + (\frac{1}{2}b_0K \mp \frac{1}{4}K^2)\sin(2\zeta) = 0. \tag{2.14}$$

Solving eq.(2.14), we obtain

$$b_0 = \pm\frac{1}{2}K. \tag{2.15}$$

Substituting eq.(2.15) into eq.(2.12), then eq.(2.12) becomes

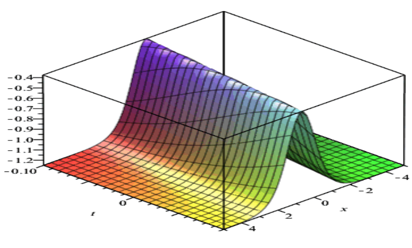
$$a_0 = -\frac{1}{16}K^2, a_1 = \frac{3}{4}K^2, a_2 = -\frac{3}{4}K^2, b_1 = \mp K, W = -\frac{7}{4}K^3. \tag{2.16}$$

Inserting equation (2.16) into the real part of equations (2.13) and (2.9) yields the following exact solution

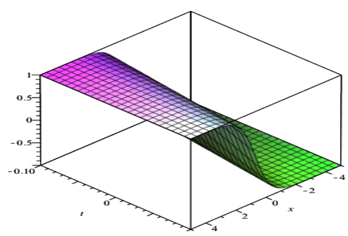
$$\begin{aligned}
u_1 &= \frac{-\frac{3}{8}K^2 - \frac{1}{2}K^2\cos(i(Kx - \frac{7}{4}K^3t)) - \frac{5}{8}K^2\cos(2i(Kx - \frac{7}{4}K^3t))}{6 + 8\cos(i(Kx - \frac{7}{4}K^3t)) + 2\cos(2i(Kx - \frac{7}{4}K^3t))},\\
v_1 &= \frac{-iK\sin(i(Kx - \frac{7}{4}K^3t)}{2 + 2\cos(i(Kx - \frac{7}{4}K^3t)},\\
u_2 &= \frac{\frac{9}{8}K^2 + K^2\cos(i(Kx - \frac{7}{4}K^3t)) - \frac{1}{8}K^2\cos(2i(Kx - \frac{7}{4}K^3t))}{6 + 8\cos(i(Kx - \frac{7}{4}K^3t)) + 2\cos(2i(Kx - \frac{7}{4}K^3t))},\\
v_2 &= \frac{iK\sin(i(Kx - \frac{7}{4}K^3t)}{2 + 2\cos(i(Kx - \frac{7}{4}K^3t)},
\end{aligned} \tag{2.17}$$

where K is free parameter.

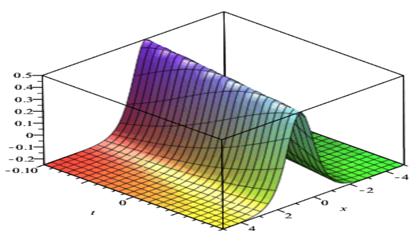
The graph of the solution (2.17) is given in the following figures with $K = 2$.



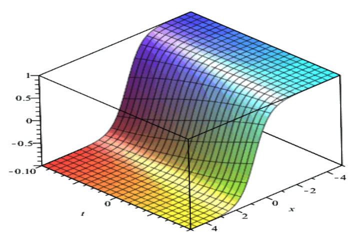
u_1



v_1



u_2



v_2

3 Conclusions

In this study, exp-function method in rational form with a computerized symbolic computation has been successfully applied to find generalized complex solitary solutions of Jaulent-Miodek equation without any initial conditions, so the solutions are more general. The results are simpler with the fewest free parameters and can be shown graphically.

References

[1] Zhu Jiamin, Lu Zhiming, Liu Yulu. Doubly periodic wave solutions of Jaulent-Miodek equations using variational iteration method combined with Jacobian-function method[J]. Commun.Theor. Phys., 2008, 49(6): 1403–1406.

[2] Yildirim A, Kelleci A. Numerical simulation of the Jaulent-miodek equation by He's homotopy perturbation method[J]. World Applied Sciences Journal 7 (Special Issue for Applied Math), 2009: 84–89.

[3] Biazar J, Eslami M. Homotopy analysis method for nonlinear Jaulent-Miodek equation[J]. Journal of Information and Computing Science, 2010, 5(2): 83–88.

[4] Zayed E M E, Rahman H M A. The extended tanh-method for finding traveling wave solutions of nonlinear evolution equations[J]. Appl. Math. E-Notes, 2010, 10: 235–245.

[5] Lü Hailing, Liu Xiqiang, Niu Lei. A generalized $(G\prime/G)$-expansion method and its applications to nonlinear evolution equations[J]. Appl. Math. Comput., 2010, 215(11): 3811–3816.

[6] Zayed E M E, Gepreel K A. A series of complexiton soliton solutions for nonlinear Jaulent-Miodek PDEs using the Riccati equations method[J]. Proceedings of the Royal Society of Edinburgh: Section A Mathematics, 2011, 141: 1001–1015.

[7] He Jihuan, Zhang Lina. Generalized solitary solution and compacton-like solutionof the Jaulent-Miodek equations using the exp-function method[J]. Physics Letters A, 2008, 372: 1044–1047.

[8] Abdul-Zahra K A. Extended exponential function method in rational form for exact solution of coupled Burgers equation[J]. Journal of Basrah Researches (Sciences), 2012, 38A(1): 72–78.

利用有理形式的指数函数法解决非线性Jaulent-Miodek方程的一系列复孤波解

兰春霞，金云娟

(丽水学院理学院, 浙江 丽水 323000)

摘要: 本文研究了非线性Jaulent-Miodek方程的行波解. 利用有理形式的指数函数法, 得到了一系列包括由三角周期函数和有理函数组合而成的新复孤波解.

关键词: 指数函数法; Jaulent-Miodek 方程; 复孤波解

MR(2010)**主题分类号**: 35C07; 35C08; 35C09 **中图分类号**: O175.2