Vol. 34 (2014)
No. 4

数 学 杂 志
J. of Math. (PRC)

MINIMUM DISTANCE SPECTRAL RADIUS OF GRAPHS WITH GIVEN EDGE CONNECTIVITY

LI Xiao-xin [1,2], FAN Yi-zheng [2], WANG Yi [2]

(*1. Department of Mathematics and Computer Sciences, Chizhou University, Chizhou 247000, China*)
(*2.School of Mathematical Sciences, Anhui University, Hefei 230601, China*)

Abstract: In this paper we study the extremal graphs with minimum distance spectral radius among all connected graphs of order n and edge connectivity r. By using the combinatorial method, we determine that $K(n-1,r)$ is the unique extremal graph, where $K(n-1,r)$ is obtained from the complete graph K_{n-1} by adding a vertex v together with edges joining v to r vertices of K_{n-1}. All the above generalize the related results of the extremal graph theory.

Keywords: graph; distance matrix; spectral radius; edge connectivity

2010 MR Subject Classification: 05C50

Document code: A **Article ID:** 0255-7797(2014)04-0671-08

1 Introduction

Let G be a connected simple graph with vertex set $V(G)$ and edge set $E(G)$. The distance between two vertices u, v of G, denoted by dis_{uv}, is defined as the length of the shortest path between u and v in G. The distance matrix of G, denoted by $D(G)$, is defined by $D(G) = (\mathrm{dis}_{uv})_{u,v\in V(G)}$. Since $D(G)$ is symmetric, its eigenvalues are all real. In addition, as $D(G)$ is nonnegative and irreducible, by Perron-Frobenius theorem, the spectral radius $\rho(G)$ of $D(G)$ (called the distance spectral radius of G), is exactly the largest eigenvalue of $D(G)$ with multiplicity one; and there exists a unique (up to a multiple) positive eigenvector corresponding to this eigenvalue, usually referred to the Perron vector of $D(G)$.

The distance matrix is very useful in different fields, including the design of communication networks [1], graph embedding theory [2–4] as well as molecular stability [5, 6]. Balaban et al. [7] proposed the use of the distance spectral radius as a molecular descriptor. Gutman et al. [8] used the distance spectral radius to infer the extent of branching and model boiling points of an alkane. Therefore, maximizing or minimizing the distance spectral radius over a given class of graphs is of great interest and significance. Recently, the maximal or the

* **Received date:** 2013-04-13 **Accepted date:** 2013-06-13

Foundation item: Supported by National Natural Science Foundation of China (11071002); Program for New Century Excellent Talents in University, Key Project of Chinese Ministry of Education (210091); Natural Science Research Foundation of Department of Education of Anhui Province(KJ2013A196).

Biography: Li Xiaoxin (1976–), male, born at Huaining, Anhui, associate professor, major in algebra graph theory and chemical graph theory. E–mail:lxx@czu.edu.cn.

minimal distance spectral radius of a given class of graphs has been studied extensively (see e.g. [9–18]).

Recall that the edge connectivity of a connected graph is the minimum number of edges whose removal disconnects the graph. For convenience, denote by $\mathcal{G}_n^r$ the set of all connected graphs of order n and edge connectivity r. Clearly, $1 \le r \le n-1$, and $\mathcal{G}_n^{n-1}$ consists of the unique graph K_n, where K_n denotes a complete graph of order n. Let $K(p,q)(p \ge q \ge 1)$ be a graph obtained from K_p by adding a vertex together with edges joining this vertex to q vertices of K_p. Surely $K(n-1,r) \in \mathcal{G}_n^r$. In this paper we prove that $K(n-1,r)$ is the unique graph with minimum distance spectral radius in $\mathcal{G}_n^r$, where $1 \le r \le n-2$.

2 Main Results

Given a graph G on n vertices, a vector $x \in \mathbb{R}^n$ is considered as a function defined on G, if there is a 1-1 map φ from $V(G)$ to the entries of x; simply written $x_u = \varphi(u)$ for each $u \in V(G)$. If x is an eigenvector of $D(G)$, then it is naturally defined on $V(G)$, i.e., x_u is the entry of x corresponding to the vertex u. One can find that

$$x^T D(G) x = \sum_{u,v \in V(G)} \mathrm{dis}_{uv} x_u x_v, \tag{2.1}$$

and λ is an eigenvalue of $D(G)$ corresponding to the eigenvector x if and only if $x \neq 0$ and

$$\lambda x_v = \sum_{u \in V(G)} \mathrm{dis}_{vu} x_u, \text{ for each vertex } v \in V(G). \tag{2.2}$$

In addition, for an arbitrary unit vector $x \in \mathbb{R}^n$,

$$x^T D(G) x \le \rho(G), \tag{2.3}$$

with the equality holds if and only if x is an eigenvector of $D(G)$ corresponding to $\rho(G)$.

The following lemma is an immediate consequence of Perron-Frobenius theorem.

Lemma 2.1 Let G be a connected graph with $u, v \in V(G)$. If $uv \notin E(G)$, then $\rho(G) > \rho(G+uv)$. If $uv \in E(G)$ and $G - uv$ is also connected, then $\rho(G) < \rho(G - uv)$.

By Lemma 2.1, for a connected graph G on n vertices, we have $\rho(G) \ge \rho(K_n) = n-1$, with equality holds if and only if $G = K_n$; and $\rho(G) \le \rho(T_G)$, with equality holds if and only if $G = T_G$, where T_G is a spanning tree of G.

Let G be a graph and let v be a vertex of G. Denote by $N(v)$ the set of neighbors of v in G, and by d_v the degree of v in G (i.e. the cardinality of $N(v)$).

Lemma 2.2 Let G be a connected graph containing two vertices u, v, and let x be a Perron vector of $D(G)$.

(1) If $N(u)\backslash\{v\} \subseteq N(v)\backslash\{u\}$, then $x_u \ge x_v$, with strict inequality if $N(u)\backslash\{v\} \subsetneq N(v)\backslash\{u\}$.

(2) If $N(u)\backslash\{v\} = N(v)\backslash\{u\}$, then $x_u = x_v$.

Proof The second assertion follows from the first or can be found in [11]. So we only prove assertion (1). From (2.2), we have

$$\rho(G)x_u = \mathrm{dis}_{uv}x_v + \sum_{w\in V(G)\backslash\{u,v\}} \mathrm{dis}_{uw}x_w, \tag{2.4}$$

$$\rho(G)x_v = \mathrm{dis}_{vu}x_u + \sum_{w\in V(G)\backslash\{u,v\}} \mathrm{dis}_{vw}x_w. \tag{2.5}$$

Since $N(u)\backslash\{v\} \subseteq N(v)\backslash\{u\}$, for each $w \in V(G)\backslash\{u,v\}$, we have

$$\mathrm{dis}_{uw} \geq \mathrm{dis}_{vw}, \tag{2.6}$$

and hence

$$\sum_{w\in V(G)\backslash\{u,v\}} \mathrm{dis}_{uw}x_w \geq \sum_{w\in V(G)\backslash\{u,v\}} \mathrm{dis}_{vw}x_w. \tag{2.7}$$

By (2.4), (2.5) and (2.7), we get

$$(\rho(G) + \mathrm{dis}_{uv})x_u \geq (\rho(G) + \mathrm{dis}_{uv})x_v.$$

So $x_u \geq x_v$.

If $N(u)\backslash\{v\} \subsetneq N(v)\backslash\{u\}$, then there exists a vertex $w \in (N(v)\backslash\{u\}) \setminus (N(u)\backslash\{v\})$ such that $\mathrm{dis}_{uw} > \mathrm{dis}_{vw} = 1$. So inequality (2.6) is strict for some vertex w and hence (2.7) holds strictly, which implies $x_u > x_v$.

Let $G \in \mathcal{G}_n^r$. Then each vertex v of G holds $d_v \geq r$. If there exists some vertex v of G with $d_v = r$, we have the following result immediately.

Lemma 2.3 Let $G \in \mathcal{G}_n^r$ $(1 \leq r \leq n-2)$, which contains a vertex of degree r. Then $\rho(G) \geq \rho(K(n-1,r))$, with equality if and only if $G = K(n-1,r)$.

Proof Let v be a vertex of G such that $d_v = r$. Adding all possible edges within the subgraph of G induced by the vertices of $V(G) \setminus \{v\}$, we will arrive at a graph G', which is isomorphic to $K(n-1,r)$. If $G \neq G'$, then $\rho(G) > \rho(G') = \rho(K(n-1,r))$ by Lemma 2.1. The result follows.

In the following we discuss the graph $G \in \mathcal{G}_n^r$ each vertex of which has degree greater than r. We will formulate two lemmas about the behaviors of the distance spectral radius under some graph transformations, and then establish the main result of this paper.

Lemma 2.4 Let G be a graph obtained from $K_{n_1} \cup K_{n_2}$ by adding r (≥ 1) edges between u_1 and $v_1, v_2, \cdots, v_r$, where $V(K_{n_1}) = \{u_1, u_2, \cdots, u_{n_1}\}$, $V(K_{n_2}) = \{v_1, v_2, \cdots, v_{n_2}\}$, $\min\{n_1, n_2\} \geq r+2$. Let $\tilde{G}$ be the graph obtained from G by deleting the edges of K_{n_1} incident to u_1 and adding all possible edges between the vertices of $V(K_{n_1})\backslash\{u_1\}$ and those of $V(K_{n_2})$. Then $\rho(G) > \rho(\tilde{G})$.

Proof Arrange in order the vertices of $\tilde{G}$ as $u_1, u_2, \cdots, u_{n_1}, v_1, \cdots, v_r, v_{r+1}, \cdots, v_{n_2}$. Let x be the unit Perron vector of $D(\tilde{G})$. By Lemma 2.2, x may be written as

$$x = (x_1, \underbrace{x_2, \cdots, x_2}_{n_1-1}, \underbrace{x_3, \cdots, x_3}_{r}, \underbrace{x_2, \cdots, x_2}_{n_2-r})^T, \tag{2.8}$$

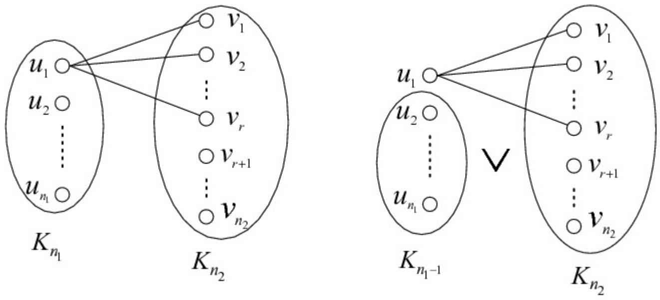

Fig. 2.1: The graphs G (left) and $\tilde{G}$ (right) in Lemma 2.4, where $\vee$ means joining each vertex of K_{n_1-1} and each of K_{n_2}

where $x_3 < x_2 < x_1$. Notice that the transformation from G to $\tilde{G}$ leads to the distance between u_1 and u_i $(i = 2, \cdots, n_1)$ increasing by 1, the distance between u_i $(i = 2, \cdots, n_1)$ and v_j $(j = 1, \cdots, r)$ decreasing by 1, and the distance between u_i $(i = 2, \cdots, n_1)$ and v_j $(j = r+1, \cdots, n_2)$ decreasing by 2, while the distance between any other two vertices having no change. Thus by (2.1) and (2.8),

$$\begin{aligned} x^T D(G)x - x^T D(\tilde{G})x &= -2 \sum_{i=2,\cdots,n_1} x_{u_1}x_{u_i} + 2 \sum_{\substack{i=2,\cdots,n_1,\\ j=1,\cdots,r}} x_{u_i}x_{v_j} + 4 \sum_{\substack{i=2,\cdots,n_1,\\ j=r+1,\cdots,n_2}} x_{u_i}x_{v_j} \\ &= -2(n_1-1)x_1x_2 + 2r(n_1-1)x_2x_3 + 4(n_1-1)(n_2-r)x_2^2 \\ &= 2(n_1-1)x_2[-x_1 + rx_3 + 2(n_2-r)x_2]. \end{aligned} \tag{2.9}$$

Considering (2.2) on the vertex u_1 of $\tilde{G}$, we get

$$\rho(\tilde{G})x_1 = \rho(\tilde{G})x_{u_1} = \sum_{i=1}^{r} x_{v_i} + \sum_{j=r+1}^{n_2} 2x_{v_j} + \sum_{k=2}^{n_1} 2x_{u_k} = rx_3 + 2(n_1+n_2-r-1)x_2. \tag{2.10}$$

Noting that $\rho(\tilde{G}) > n_1 + n_2 - 1$, from (2.10) we have

$$x_1 = \frac{1}{\rho(\tilde{G})}[rx_3 + 2(n_1+n_2-r-1)x_2] < rx_3 + 2(n_2-r)x_2. \tag{2.11}$$

By (2.9) and (2.11), we get $x^T D(G)x - x^T D(\tilde{G})x > 0$. So according to (2.3) we get

$$\rho(G) \geq x^T D(G)x > x^T D(\tilde{G})x = \rho(\tilde{G}).$$

Lemma 2.5 Let G be a graph obtained from $K_{n_1} \cup K_{n_2}$ by adding t (≥ 1) edges between u_1 and $v_1, v_2, \cdots, v_t$ and $r - t$ (≥ 1) edges between some vertices of $V(K_{n_1}) \setminus \{u_1\}$ and some vertices of $V(K_{n_2})$, where $V(K_{n_1}) = \{u_1, u_2, \cdots, u_{n_1}\}$, $V(K_{n_2}) = \{v_1, v_2, \cdots, u_{n_2}\}$,

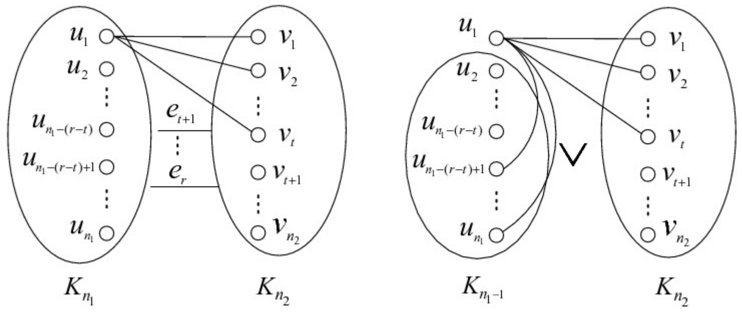

Fig. 2.2: The graphs G (left) and $\tilde{G}$ (right) in Lemma 2.5, where $\vee$ means joining each vertex of K_{n_1-1} and each of K_{n_2}

$\min\{n_1, n_2\} \geq r+2$. Let $\tilde{G}$ be a graph obtained from G by deleting $n_1-(r+1-t)$ edges of K_{n_1} between u_1 and u_i for $i=2,\cdots,n_1-(r-t)$, and adding all possible edges between the vertices of $V(K_{n_1})\backslash\{u_1\}$ and those of $V(K_{n_2})$. Then $\rho(G) > \rho(\tilde{G})$.

Proof Arrange in order the vertices of $V(\tilde{G})$ as $u_1, u_2, \cdots, u_{n_1-(r-t)}, u_{n_1-(r-t)+1}, \cdots, u_{n_1}$, $v_1, \cdots, v_t$, $v_{t+1}, \cdots, v_{n_2}$. Let x be the unit Perron vector of $D(\tilde{G})$. By Lemma 2.2, x may be written as

$$x = (x_1, \underbrace{x_2, \cdots, x_2}_{n_1-(r+1-t)}, \underbrace{x_3, \cdots, x_3}_{r-t}, \underbrace{x_3, \cdots, x_3}_{t}, \underbrace{x_2, \cdots, x_2}_{n_2-t})^T, \tag{2.12}$$

where $x_3 < x_2 < x_1$. Let

$$U_1 = \{u_2, \cdots, u_{n_1-(r-t)}\}, U_2 = \{u_{n_1-(r-t)+1}, \cdots, u_{n_1}\}, V_1 = \{v_1, \cdots, v_t\}, V_2 = \{v_{t+1}, \cdots, v_{n_2}\}.$$

Assume that in the graph G there are r_{ij} edges between U_i and V_j for $i,j=1,2$. Surely, $r_{11}+r_{12}+r_{21}+r_{22} = r-t$. Denote by F the set of order pairs (u,v) such that uv is an edge of G, where $u \in U_1 \cup U_2, v \in V_1 \cup V_2$, and by (U_i, V_j) the set of order pairs (u,v), where $u \in U_i, v \in V_j, i,j = 1,2$. Then by (2.1)

$$\begin{aligned}\tfrac{1}{2}[x^T D(\tilde{G})x - x^T D(G)x] \quad = \quad & \sum_{u\in U_1} x_{u_1}x_u - \sum_{(u,v)\in(U_1,V_1)\backslash F} x_u x_v - \sum_{(u,v)\in(U_1,V_2)\backslash F} \delta_{uv}x_u x_v \\ & - \sum_{(u,v)\in(U_2,V_1)\backslash F} x_u x_v - \sum_{(u,v)\in(U_2,V_2)\backslash F} \delta_{uv}x_u x_v,\end{aligned} \tag{2.13}$$

where $\delta_{uv} = 2$ if u,v has distance 3 in the graph G, and $\delta_{uv} = 1$ otherwise. By (2.12) and

(2.13), and taking $\delta_{uv}=1$, we have

$$\begin{aligned}
&\frac{1}{2}[x^TD(\tilde{G})x-x^TD(G)x]\le[n_1-1-(r-t)]x_1x_2-\{[n_1-1-(r-t)]t-r_{11}\}x_2x_3\\
&-\{[n_1-1-(r-t)](n_2-t)-r_{12}\}x_2^2-[(r-t)t-r_{21}]x_3^2\\
&-[(r-t)(n_2-t)-r_{22}]x_2x_3\\
=\ &[n_1-1-(r-t)]x_1x_2-\{[n_1-1-(r-t)]t+(r-t)(n_2-t)\}x_2x_3\\
&-\{[n_1-1-(r-t)](n_2-t)\}x_2^2-(r-t)tx_3^2\\
&+(r_{11}+r_{22})x_2x_3+r_{12}x_2^2+r_{21}x_3^2\\
\le\ &[n_1-1-(r-t)]x_1x_2-\{[n_1-1-(r-t)]t+(r-t)(n_2-t)\}x_2x_3\\
&-\{[n_1-1-(r-t)](n_2-t)\}x_2^2-(r-t)tx_3^2+(r-t)x_2^2\\
=\ &[n_1-1-(r-t)]x_1x_2-\{[n_1-1-(r-t)]t+(r-t)(n_2-t)\}x_2x_3\\
&-\{[n_1-1-(r-t)](n_2-t)-(r-t)\}x_2^2-(r-t)tx_3^2. \qquad (2.14)
\end{aligned}$$

Considering (2.2) on the vertex u_1 of $\tilde{G}$, we get

$$\rho(\tilde{G})x_1=rx_3+2(n_1+n_2-r-1)x_2.$$

So $x_1=\frac{1}{\rho(\tilde{G})}[rx_3+2(n_1+n_2-r-1)x_2]<\frac{1}{n_1+n_2-1}[rx_3+2(n_1+n_2-r-1)x_2]$. Therefore,

$$\begin{aligned}
\frac{1}{2}x^T[D(\tilde{G})-D(G)]x\ <\ &\frac{n_1-1-(r-t)}{n_1+n_2-1}rx_2x_3+\frac{[n_1-1-(r-t)]\cdot 2(n_1+n_2-r-1)}{n_1+n_2-1}x_2^2\\
&-\{[n_1-1-(r-t)]t+(r-t)(n_2-t)\}x_2x_3\\
&-\{[n_1-1-(r-t)](n_2-t)-(r-t)\}x_2^2-(r-t)tx_3^2. \qquad (2.15)
\end{aligned}$$

Let a,b,c be the coefficients of x_2^2,x_2x_3,x_3^2 in (2.15), respectively. Noting that $\min\{n_1,n_2\}\ge r+2$, we have

$$\begin{aligned}
a\ &=\ 2[n_1-1-(r-t)](1-\frac{r}{n_1+n_2-1})-\{[n_1-1-(r-t)](n_2-t)-(r-t)\}\\
&=\ 2[n_1-1-(r-t)](1-\frac{r}{n_1+n_2-1}-\frac{n_2-t}{2})+(r-t)\\
&<\ 2[n_1-1-(r-t)](1-\frac{n_2-t}{2})+(r-t)\\
&\le\ 2[n_1-1-(r-t)]\frac{2-n_2+t}{2}+(n_2-2-t)\\
&=\ -(n_2-t-2)[n_1-2-(r-t)]<0,\\
b\ &=\ [n_1-1-(r-t)](\frac{r}{n_1+n_2-1}-t)-(r-t)(n_2-t)<0,\\
c\ &=\ -(r-t)t<0.
\end{aligned}$$

Thus $x^TD(\tilde{G})x-x^TD(G)x<0$, and hence $\rho(G)\ge x^TD(G)x>x^TD(\tilde{G})x=\rho(\tilde{G})$.

Lemma 2.6 Let G be a connected graph, and let E_c be an edge cut set of G of size r (≥ 1) such that $G-E_c=K_{n_1}\cup K_{n_2}$, where $n_1+n_2=n$. If $d_v>r$ for each vertex $v\in V(G)$, then $n_1\ge r+2, n_2\ge r+2$.

Proof If $n_1 \leq r$, then there exists a vertex u of K_{n_1} such that

$$d(u) \leq n_1 - 1 + \frac{r}{n_1} \leq (n_1 - 1)\frac{r}{n_1} + \frac{r}{n_1} = r,$$

a contradiction. If $n_1 = r + 1$, then there exists a vertex w not incident with any edges of E_c, which implies $d(u) = r$, also a contradiction. The discussion for the assertion on n_2 is similar.

Theorem 2.7 For each $r = 1, 2, \cdots, n-2$, the graph $K(n-1, r)$ is the unique graph with minimum distance spectral radius in $\mathcal{G}_n^r$.

Proof Let G be a graph that attains the minimum distance spectral radius in $\mathcal{G}_n^r$. Note that each vertex of G has degree not less than r. If there exists a vertex u of G with degree r, by Lemma 2.3, $\rho(G) \geq \rho(K(n-1, r))$, with equality if and only if $G = K(n-1, r)$. So the result follows in this case.

Next we assume all vertices of G have degrees greater than r. Let E_c be an edge cut set of G containing r edges, and let G_1, G_2 be two components of $G - E_c$ with order n_1, n_2 respectively. We assert $G_1 = K_{n_1}$ and $G_2 = K_{n_2}$; otherwise adding all possible edges within G_1, G_2 we would get a graph with smaller distance spectral radius by Lemma 2.1. By Lemma 2.6, $n_1 \geq r + 2, n_2 \geq r + 2$. Let u_1 be a vertex of G_1 such that u_1 joins t vertices of G_2, where $1 \leq t \leq r$. If $t = r$, by Lemma 2.4 there exists a graph $\tilde{G} \cong K(n-1, r)$ such that $\rho(G) > \rho(\tilde{G})$. If $1 \leq t < r$, by Lemma 2.5 there also exists a graph $\tilde{G} \cong K(n-1, r)$, such that $\rho(G) > \rho(\tilde{G})$. This completes the proof.

References

[1] Graham R L, Pollak H O. On the addressing problem for loop switching[J]. Bell Sys. Tech. J., 1971, 50: 2495–2519.

[2] Edelberg M, Garey M R, Graham R L. On the distance matrix of a tree[J]. Discrete Math., 1976, 14: 23–29.

[3] Graham R L, Lovasz L. Distance matrix polynomials of trees[J]. Adv. Math., 1978, 29: 60–88.

[4] Graham R L, Pollak H O. On embedding graphs in squashed cubes[M]. Graph Theory and Applications, Berlin: Springer, 1973.

[5] Hosoya H, Murakami M, Gotoh M. Distance polynomial and characterization of a graph[J]. Natur. Sci. Rep. Ochanomizu Univ., 1973, 24: 27–34.

[6] Rouvray D H. The search for useful topological indices in chemistry[J]. Amer. Scientist, 1973,61: 729–735.

[7] Balaban A T, Ciubotariu D, Medeleanu M. Topological indices and real number vertex invariants based on graph eigenvalues or eigenvectors[J]. J. Chem. Inf. Comput. Sci., 1991, 31: 517–523.

[8] Gutman I, Medeleanu M. On structure-dependence of the largest eigenvalue of the distance matrix of an alkane[J]. Indian J. Chem. A, 1998, 37: 569–573.

[9] Bose S S, Nath M, Paul S. Distance spectral radius of graphs with r pendent vertices[J]. Linear Algebra Appl., 2011, 435: 2826–2836.

[10] Ilić A. Distance spectral radius of trees with given matching number[J]. Discrete Appl. Math., 2010, 158: 1799–1806.

[11] Liu Zhongzhu. On the spectral radius of the distance matrix[J]. Appl. Anal. Discrete Math., 2010, 4(2): 269–277.

[12] Nath M, Paul S. On the distance spectral radius of bipartite graphs[J]. Linear Algebra Appl., 2012, 436: 1285–1296.

[13] Stevanović D, Ilić A. Distance spectral radius of trees with fixed maximum degree[J]. Electron. J. Linear Algebra, 2010, 20: 168–179.

[14] Yu Guanglong, Jia Huicai, Zhang Hailian, et al. Some graft transformations and its application on a distance spectrum[J]. Appl. Math. Lett., 2012, 25: 315–319.

[15] Yu Guanglong, Wu Yarong, Shu Jinglong. Some graft transformations and its application on a distance spectrum[J]. Discrete Math., 2011, 311: 2117–2123.

[16] Zhang Xiaoling, Godsil C. Connectivity and minimal distance spectral radius[J]. Linear Multilinear Algebra, 2011, 59: 745–754.

[17] Zhou Bo. On the largest eigenvalue of the distance matrix of a tree[J]. MATCH Commun. Math. Comput. Chem., 2007, 58: 657–662.

[18] Zhou Bo, Trinajistić N. On the largest eigenvalue of the distance matrix of a connected graph[J]. Chem. Phys. Lett., 2007, 447: 384–387.

给定边连通度的图的最小距离谱半径

李小新[1,2], 范益政[2], 汪 毅[2]

(1.池州学院数学与计算机科学系, 安徽 池州 247000)

(2.安徽大学数学科学学院, 安徽 合肥 230601)

摘要: 本文研究了边连通度为r的n阶连通图中距离谱半径最小的极图问题. 利用组合的方法, 确定了$K(n-1,r)$为唯一的极图, 其中$K(n-1,r)$是由完全图K_{n-1}添加一个顶点v以及连接v与K_{n-1}中r个顶点的边所构成. 上述结论推广了极图理论中的相关结果.

关键词: 图; 距离矩阵; 谱半径; 边连通度

MR(2010)**主题分类号**: 05C50 **中图分类号**: O157.5