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K-ORDER GENERALIZED DERIVATIONS OF WEIGHT λ ON δ JORDAN-LIE TRIPLE SYSTEMS

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Abstract: This paper deals with the k-order generalized derivations of weight λ on δ Jordan-Lie triple systems. By computing, we conclude that every k-order Jordan triple θ-derivation of weight λ on δ Jordan-Lie triple systems is a k-order θ-derivation of weight λ. Under the definitions, we give another equivalent form of k-order Jordan triple θ-derivation of weight λ. Meanwhile, We also establish the inheritance property of k-order generalized (θ,φ)-derivation of weight λ and Rota-Baxter operator of weight λ on Rota-Baxter δ Jordan-Lie triple systems. We obtain that every Rota-Baxter δ Jordan-Lie algebra can be seen as a Rota-Baxter δ Jordan-Lie triple system.

Keywords: δ Jordan-Lie triple systems; k-order (θ, φ)-derivations; k-order Jordan triple (θ, φ)-derivations; weight λ; Rota-Baxter δ Jordan-Lie triple systems of weight λ

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1 Introduction

Lie triple systems have played an extremely important role in mathematics and physics for a long time. The concept of Lie triple systems was first introduced by N.Jacobson [1],[2](see also [3]). As the generalization of Lie algebra, δ Jordan-Lie algebra was first introduced in [4]. Let R be a commutative ring with a unit. A δ Jordan-Lie algebra is a R-module L with a R-bilinear mapping $L \times L \ni (x, y) \longmapsto [x, y] \in L$ satisfying the following conditions:

$$[x, y] = -\delta[y, x], \tag{1.1}$$

$$[[x, y], z] + [[y, z], x] + [[z, x], y] = 0 \tag{1.2}$$

for all $x, y, z \in L$. $\delta = \pm 1$. For $\delta = 1$, these equations represent the Lie algebra [5]. Similarly, we can obtain the definition of δ Jordan-Lie triple system in [4]. A δ Jordan-Lie triple system is a R-module L with a R-trilinear mapping $L \times L \times L \ni (x, y, z) \longmapsto [x, y, z] \in L$ satisfying

$$[x, y, z] = -\delta[y, x, z], \tag{1.3}$$

$$[x, y, z] + [y, z, x] + [z, x, y] = 0, \tag{1.4}$$

$$[u, v, [x, y, z]] = [[u, v, x], y, z] + [x, [u, v, y], z] + \delta[x, y, [u, v, z]] \tag{1.5}$$

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for all $u, v, x, y, z \in L$. $\delta = \pm 1$. The case of $\delta = 1$ gives the Lie triple system [5]. Clearly, every δ Jordan-Lie algebra with product $[\cdot, \cdot]$ is a δ Jordan-Lie triple system with respect to $[x, y, z] := [[x, y], z]$.

As is well known, derivations and generalized derivation algebras are very important subjects both in the research of rings and Lie algebras. In the study of Levi factors in derivation algebras of nilpotent Lie algebras, the generalized derivations, quasiderivations, centroids and quasicentroids play key roles(see [6]). The most important and systematic research on the generalized derivative algebras of Lie algebras and their subalgebras were due to Leger and Luks. Much work have been done in this area, showing an interesting derivation and generalized derivative algebras(see [7–17]). In particular, some nice properties of the generalized derivation on Lie triple systems have been obtained in [16–17].

In [18–20], the concepts of Rota-Baxter 3-Lie algebras were introduced and the authors studied the inheritance property of Rota-Baxter 3-Lie algebras and Rota-Baxter Lie triple systems. They introduced the concepts of a Rota-Baxter operator and differential operator with weights on Lie triple system. Rota-Baxter operators on Lie algebras are operator forms of the classical Yang-Baxter equations and contribute to the study of integrable systems [6],[18–23],[25–29].

The paper is organized as follows. In section 2, we conclude that every k-order Jordan triple θ-derivation of weight λ on δ Jordan Lie triple system is a k-order θ-derivation of weight λ and we give another equivalent form of k-order Jordan triple θ-derivation of weight λ on δ Jordan-Lie triple systems. In section 3, we establish the inheritance property of k-order generalized (θ,φ)-derivation of weight λ on Rota-Baxter δ Jordan-Lie triple systems of weight λ and generalize some results in [18] to Rota-Baxter δ Jordan-Lie triple systems of weight λ.

2 On K-Order Generalized Derivation of Weight λ on δ Jordan-Lie Triple Systems

The purpose of this section is to study a k-order derivation and k-order generalized derivation of weight λ on δ Jordan-Lie triple system. In particular, we generalize some results in [16]-[17] to k-order (generalized) derivation of weight λ on δ Jordan-Lie triple systems. We first introduce the concepts of k-order (generalized) (θ,φ)-derivations of weight λ and k-order (generalized) Jordan triple (θ,φ)-derivations of weight λ on δ Jordan-Lie triple systems. Then we prove that every k-order generalized Jordan triple (θ,φ)-derivation of weight λ on Lie triple system is a k-order generalized (θ,φ)-derivation of weight λ under some conditions. In particular, we conclude that every k-order Jordan triple θ-derivation of weight λ on Lie triple systems is a k-order θ-derivation of weight λ. In the end we give another equivalent form of k-order Jordan triple θ-derivation of weight λ on δ Jordan-Lie triple systems.

Given an integer $n > 1$, a ring R is said to be n-torsion free, if for $x \in R, nx = 0$ implies that $x = 0$.

Definition 2.1 Let L be a δ Jordan-Lie triple system over ring R. Let $\theta, \varphi : L \to L$ be R-linear maps. A R-linear map $D : L \to L$ is called a k-order (θ, φ)-derivation of weight $\lambda \in R$ on L if

$$\begin{aligned} D([x, y, z]) &= \delta^k[D(x), \theta(y), \varphi(z)] + \delta^k[\theta(x), D(y), \varphi(z)] + \delta^k[\theta(x), \varphi(y), D(z)] \\ &\quad + \lambda\delta^k[D(x), D(y), \varphi(z)] + \lambda\delta^k[D(x), \theta(y), D(z)] \\ &\quad + \lambda\delta^k[\theta(x), D(y), D(z)] + \lambda^2\delta^k[D(x), D(y), D(z)] \end{aligned} \tag{2.1}$$

for all $x, y, z \in L, \lambda \in R$. $\delta = \pm 1$. If $\varphi = \theta$, a k-order (θ, φ)-derivation of weight λ is called a k-order θ-derivation of weight λ. If $\varphi = \theta = I_L$, where I_L is the identity map on L, a k-order (θ, φ)-derivation of weight λ is called a k-order derivation of weight λ.

Remark 2.2 If $\lambda = 0$ and $\delta = 1$, a k-order (θ, φ)-derivation of weight λ could be seen as a (θ, φ)-derivation (see[16]-[17]).

Definition 2.3 Let L be a δ Jordan-Lie triple system over ring R. Let $\theta, \varphi : L \to L$ be R-linear maps. A R-linear map $D : L \to L$ is called a k-order Jordan triple (θ, φ)-derivation of weight $\lambda \in R$ on L if

$$\begin{aligned} D([x, y, x]) &= \delta^k[D(x), \theta(y), \varphi(x)] + \delta^k[\theta(x), D(y), \varphi(x)] + \delta^k[\theta(x), \varphi(y), D(x)] \\ &\quad + \lambda\delta^k[D(x), D(y), \varphi(x)] + \lambda\delta^k[D(x), \theta(y), D(x)] \\ &\quad + \lambda\delta^k[\theta(x), D(y), D(x)] + \lambda^2\delta^k[D(x), D(y), D(x)] \end{aligned} \tag{2.2}$$

for all $x, y \in L, \lambda \in R$, $\delta = \pm 1$. If $\varphi = \theta$, a k-order Jordan triple (θ, φ)-derivation of weight λ is called a k-order Jordan triple θ-derivation of weight λ. If $\varphi = \theta = I_L$, where I_L is the identity map on L, a k-order Jordan triple (θ, φ)-derivation of weight λ is called a k-order Jordan triple derivation of weight λ.

Remark 2.4 If $\lambda = 0$ and $\delta = 1$, a k-order Jordan triple (θ, φ)-derivation of weight λ could be seen as a Jordan triple (θ, φ)-derivation (see[16,17]).

Definition 2.5 Let $\alpha : L \to L$ be a k-order (θ, φ)-derivation of weight $\lambda \in R$. A R-linear map $D : L \to L$ is called a k-order generalized (θ, φ)-derivation of weight λ with respect to α if

$$\begin{aligned} D([x, y, z]) &= \delta^k[\alpha(x), \theta(y), \varphi(z)] + \delta^k[\theta(x), \alpha(y), \varphi(z)] + \delta^k[\theta(x), \varphi(y), \alpha(z)] \\ &\quad + \lambda\delta^k[D(x), D(y), \varphi(z)] + \lambda\delta^k[\alpha(x), \theta(y), \alpha(z)] \\ &\quad + \lambda\delta^k[\theta(x), \alpha(y), \alpha(z)] + \lambda^2\delta^k[\alpha(x), \alpha(y), \alpha(z)] \end{aligned} \tag{2.3}$$

for all $x, y, z \in L, \lambda \in R$. $\delta = \pm 1$.

Remark 2.6 If $\lambda = 0$ and $\delta = 1$, a k-order generalized (θ, φ)-derivation of weight λ with respect to α is called a generalized (θ, φ)-derivation with respect to α (see [16]-[17]).

Definition 2.7 Let $\alpha : L \to L$ be a k-order Jordan triple (θ, φ)-derivation of weight $\lambda \in R$. A R-linear map $D : L \to L$ is called a k-order generalized Jordan triple (θ, φ)-

derivation of weight λ with respect to α if

$$\begin{aligned} D([x,y,x]) &= \delta^k[\alpha(x),\theta(y),\varphi(x)]+\delta^k[\theta(x),\alpha(y),\varphi(x)]+\delta^k[\theta(x),\varphi(y),\alpha(x)] \\ &\quad +\lambda\delta^k[D(x),D(y),\varphi(x)]+\lambda\delta^k[\alpha(x),\theta(y),\alpha(x)] \\ &\quad +\lambda\delta^k[\theta(x),\alpha(y),\alpha(x)]+\lambda^2\delta^k[\alpha(x),\alpha(y),\alpha(x)] \end{aligned} \tag{2.4}$$

for all $x,y\in L,\lambda\in R$. $\delta=\pm 1$.

Remark 2.8 If $\lambda=0$ and $\delta=1$, a k-order generalized Jordan triple (θ,φ)-derivation of weight λ with respect to α is called a generalized Jordan triple (θ,φ)-derivation with respect to α (see [16, 17]).

Throughout this paper $\theta,\varphi,D,\alpha: L\longrightarrow L$ are R-linear maps and $A^{\alpha,D}_{\theta,\varphi}(\lambda,k): L\times L\times L\longrightarrow L$ is a map defined by

$$\begin{aligned} A^{\alpha,D}_{\theta,\varphi}(\lambda,k)(x,y,z) &= \delta^k[\alpha(x),\theta(y),\varphi(z)]+\delta^k[\theta(x),\alpha(y),\varphi(z)]+\delta^k[\theta(x),\varphi(y),\alpha(z)] \\ &\quad +\lambda\delta^k[D(x),D(y),\varphi(z)]+\lambda\delta^k[\alpha(x),\theta(y),\alpha(z)] \\ &\quad +\lambda\delta^k[\theta(x),\alpha(y),\alpha(z)]+\lambda^2\delta^k[\alpha(x),\alpha(y),\alpha(z)] \end{aligned} \tag{2.5}$$

for all $x,y,z\in L,\lambda\in R$.

It is clear that the map $A^{\alpha,D}_{\theta,\varphi}(\lambda,k)$ is R-trilinear.

Proposition 2.9 Let R be a 3-torsion free ring and let L be a δ Jordan-Lie triple system over ring R. Let $D: L\to L$ be a k-order generalized Jordan triple (θ,φ)-derivation of weight λ with respect to α, where α is a k-order Jordan triple (θ,φ)-derivation of weight λ. If

$$\begin{aligned} &[D(x),D(y),\varphi(z)]+[D(y),D(z),\varphi(x)]+[D(z),D(x),\varphi(y)] \\ =\ &[\alpha(x),\alpha(y),\varphi(z)]+[\alpha(y),\alpha(z),\varphi(x)]+[\alpha(z),\alpha(x),\varphi(y)] \end{aligned} \tag{2.6}$$

for all $x,y,z\in L$, then

$$(D-\alpha)([x,y,z])=\lambda(\delta^k[D(x),D(y),\varphi(z)]-\delta^k[\alpha(x),\alpha(y),\varphi(z)]), \tag{2.7}$$

$$B(\lambda,k)(x,y,z)+B(\lambda,k)(y,z,x)+B(\lambda,k)(z,x,y)=0 \tag{2.8}$$

for all $x,y,z\in L$, where $B(\lambda,k)=A^{\alpha,D}_{\theta,\varphi}(\lambda,k)-A^{\alpha,\alpha}_{\theta,\varphi}(\lambda,k)$.

Proof By (2.2) and (2.4), we have

$$\begin{aligned} (D-\alpha)([x,y,x]) &= D([x,y,x])-\alpha([x,y,x]) \\ &= \delta^k[\alpha(x),\theta(y),\varphi(x)]+\delta^k[\theta(x),\alpha(y),\varphi(x)]+\delta^k[\theta(x),\varphi(y),\alpha(x)] \\ &\quad +\lambda\delta^k[D(x),D(y),\varphi(x)]+\lambda\delta^k[\alpha(x),\theta(y),\alpha(x)] \\ &\quad +\lambda\delta^k[\theta(x),\alpha(y),\alpha(x)]+\lambda^2\delta^k[\alpha(x),\alpha(y),\alpha(x)] \\ &\quad -\delta^k[\alpha(x),\theta(y),\varphi(x)]-\delta^k[\theta(x),\alpha(y),\varphi(x)]-\delta^k[\theta(x),\varphi(y),\alpha(x)] \\ &\quad -\lambda\delta^k[\alpha(x),\alpha(y),\varphi(x)]-\lambda\delta^k[\alpha(x),\theta(y),\alpha(x)] \\ &\quad -\lambda\delta^k[\theta(x),\alpha(y),\alpha(x)]-\lambda^2\delta^k[\alpha(x),\alpha(y),\alpha(x)] \\ &= \lambda(\delta^k[D(x),D(y),\varphi(x)]-\delta^k[\alpha(x),\alpha(y),\varphi(x)]) \end{aligned} \tag{2.9}$$

for all $x, y \in L$. From (1.3), we have

$$\begin{aligned}(D-\alpha)([y,x,x]) &= -\delta(D-\alpha)([x,y,x]) \\ &= -\delta\lambda(\delta^k[D(x),D(y),\varphi(x)] - \delta^k[\alpha(x),\alpha(y),\varphi(x)]) \\ &= \lambda(\delta^k[D(y),D(x),\varphi(x)] - \delta^k[\alpha(y),\alpha(x),\varphi(x)]) \qquad (2.10)\end{aligned}$$

for all $x, y \in L$. Since $[x,y,x]+[y,x,x]+[x,x,y]=0$, by (2.6) we have

$$\begin{aligned}(D-\alpha)([x,x,y]) &= -((D-\alpha)([x,y,x]) + (D-\alpha)([y,x,x])) \\ &= -\lambda(\delta^k[D(x),D(y),\varphi(x)] - \delta^k[\alpha(x),\alpha(y),\varphi(x)]) \\ &\quad -\lambda(\delta^k[D(y),D(x),\varphi(x)] - \delta^k[\alpha(y),\alpha(x),\varphi(x)]) \qquad (2.11) \\ &= \lambda(\delta^k[D(x),D(x),\varphi(y)] - \delta^k[\alpha(x),\alpha(x),\varphi(y)])\end{aligned}$$

for all $x, y \in L$.

We denote $T(x,y,z) := \lambda(\delta^k[D(x),D(y),\varphi(z)] - \delta^k[\alpha(x),\alpha(y),\varphi(z)])$. Obviously, the map $T : L \times L \times L \longrightarrow L$ is R-trilinear. By (2.6), we have

$$\begin{aligned}T(z,y,x)+T(x,z,y)+T(y,x,z) &= \lambda(\delta^k[D(z),D(y),\varphi(x)] - \delta^k[\alpha(z),\alpha(y),\varphi(x)]) \\ &\quad +\lambda(\delta^k[D(x),D(z),\varphi(y)] - \delta^k[\alpha(x),\alpha(z),\varphi(y)]) \\ &\quad +\lambda(\delta^k[D(y),D(x),\varphi(z)] - \delta^k[\alpha(y),\alpha(x),\varphi(z)]) \\ &= \lambda\delta^k[D(z),D(y),\varphi(x)] + \lambda\delta^k[D(x),D(z),\varphi(y)] \\ &\quad +\lambda\delta^k[D(y),D(x),\varphi(z)] - \lambda\delta^k[\alpha(z),\alpha(y),\varphi(x)] \\ &\quad -\lambda\delta^k[\alpha(x),\alpha(z),\varphi(y)] - \lambda\delta^k[\alpha(y),\alpha(x),\varphi(z)] \\ &= 0 \qquad (2.12)\end{aligned}$$

for all $x, y, z \in L$. It follows (2.9) that

$$(D-\alpha)([x+z,y,x+z]) = T(x+z,y,x+z)$$

for all $x, y, z \in L$. That is

$$\begin{aligned}&(D-\alpha)([x,y,z]) + (D-\alpha)([x,y,x]) + (D-\alpha)([z,y,x]) + (D-\alpha)([z,y,z]) \\ =\ &T(x,y,z)+T(x,y,x)+T(z,y,x)+T(z,y,z)\end{aligned}$$

for all $x, y, z \in L$. Thus, by (2.9)

$$(D-\alpha)([x,y,z]) + (D-\alpha)([z,y,x]) = T(x,y,z)+T(z,y,x). \qquad (2.13)$$

Since

$$(D-\alpha)([x,y+z,y+z]) = T(x,y+z,x+z)$$

similarly by(2.11), we have

$$(D-\alpha)([x,y,z]) + (D-\alpha)([x,z,y]) = T(x,y,z)+T(x,z,y). \qquad (2.14)$$

And by (2.10), we have

$$(D-\alpha)([x,y,z])+(D-\alpha)([y,x,z])=T(x,y,z)+T(y,x,z). \tag{2.15}$$

From (2.13),(2.14),(2.15), we have

$$\begin{aligned} & (D-\alpha)([x,y,z])+(D-\alpha)([z,y,x])+(D-\alpha)([x,y,z]) \\ + & (D-\alpha)([x,z,y])+(D-\alpha)([x,y,z])+(D-\alpha)([y,x,z]) \\ = & T(x,y,z)+T(z,y,x)+T(x,y,z)+T(x,z,y)+T(x,y,z)+T(y,x,z) \end{aligned}$$

for all $x,y,z\in L$. From (1.4) and (2.12), we obtain

$$3(D-\alpha)([x,y,z])=3T(x,y,z).$$

Since R is a 3-torsion free ring,

$$(D-\alpha)([x,y,z])=T(x,y,z)$$

for all $x,y,z\in L$. This proves (2.7).

To prove (2.8), from (2.7) we have

$$\begin{aligned} B(\lambda,k)(x,y,z) &= A^{\alpha,D}_{\theta,\varphi}(\lambda,k)(x,y,z)-A^{\alpha,\alpha}_{\theta,\varphi}(\lambda,k)(x,y,z) \\ &= \delta^k[\alpha(x),\theta(y),\varphi(z)]+\delta^k[\theta(x),\alpha(y),\varphi(z)]+\delta^k[\theta(x),\varphi(y),\alpha(z)] \\ &\quad +\lambda\delta^k[D(x),D(y),\varphi(z)]+\lambda\delta^k[\alpha(x),\theta(y),\alpha(z)]+\lambda\delta^k[\theta(x),\alpha(y),\alpha(z)] \\ &\quad +\lambda^2\delta^k[\alpha(x),\alpha(y),\alpha(z)]-\delta^k[\alpha(x),\theta(y),\varphi(z)]-\delta^k[\theta(x),\alpha(y),\varphi(z)] \\ &\quad -\delta^k[\theta(x),\varphi(y),\alpha(z)]-\lambda\delta^k[\alpha(x),\alpha(y),\varphi(z)]-\lambda\delta^k[\alpha(x),\theta(y),\alpha(z)] \\ &\quad -\lambda\delta^k[\theta(x),\alpha(y),\alpha(z)]-\lambda^2\delta^k[\alpha(x),\alpha(y),\alpha(z)] \\ &= \lambda\delta^k[D(x),D(y),\varphi(z)]-\lambda\delta^k[\alpha(x),\alpha(y),\varphi(z)] \\ &= (D-\alpha)([x,y,z]). \end{aligned}$$

Then

$$\begin{aligned} & B(\lambda,k)(x,y,z)+B(\lambda,k)(y,z,x)+B(\lambda,k)((z,x,y) \\ = & (D-\alpha)([x,y,z])+(D-\alpha)([y,z,x])+(D-\alpha)([z,x,y]) \\ = & (D-\alpha)([x,y,z]+[y,z,x]+[z,x,y]) \\ = & 0. \end{aligned}$$

Proposition 2.10 If $D: L\longrightarrow L$ is a k-order generalized Jordan triple (θ,φ)-derivation of weight λ with respect to α satisfying (2.7), where α is a k-order Jordan triple (θ,φ)-derivation of weight λ, then (2.6) holds.

Proof If $\lambda=0$, then $D([x,y,x])=\alpha([x,y,x])$. By (1.3) and (1.4)

$$D([y,x,x])=\alpha([y,x,x]),\quad D([x,x,y])=\alpha([x,x,y]).$$

Using the same method in Proposition 2.9, we have

$$D([x,y,z]) = \alpha([x,y,z])$$

for all $x, y, z \in L$. Therefore, (2.6) holds. If $\lambda \neq 0$, By (1.4) and (2.7), we have

$$\begin{aligned}
0 &= (D-\alpha)([x,y,z]+[y,z,x]+[z,x,y]) \\
&= (D-\alpha)([x,y,z])+(D-\alpha)([y,z,x])+(D-\alpha)([z,x,y]) \\
&= \lambda(\delta^k[D(x),D(y),\varphi(z)]+\delta^k[D(y),D(z),\varphi(x)]+\delta^k[D(z),D(x),\varphi(y)]) \\
&\quad -\lambda(\delta^k[\alpha(x),\alpha(y),\varphi(z)]+\delta^k[\alpha(y),\alpha(z),\varphi(x)]+\delta^k[\alpha(z),\alpha(x),\varphi(y)]).
\end{aligned}$$

So, we obtain that

$$\begin{aligned}
[D(x),D(y),\varphi(z)] &+ [D(y),D(z),\varphi(x)]+[D(z),D(x),\varphi(y)] \\
&= [\alpha(x),\alpha(y),\varphi(z)]+[\alpha(y),\alpha(z),\varphi(x)]+[\alpha(z),\alpha(x),\varphi(y)]
\end{aligned}$$

for all $x, y, z \in L$.

Theorem 2.11 Let R be a 3-torsion free ring and let $D : L \longrightarrow L$ be a k-order Jordan triple (θ,φ)-derivation of weight λ. Then D is a k-order (θ,φ)-derivation of weight λ if only if

$$[\theta(x),\varphi(y),D(z)] = [\varphi(x),\theta(y),D(z)]; \tag{2.16}$$

$$A^{D,D}_{\theta,\varphi}(\lambda,k)(x,y,z)+A^{D,D}_{\theta,\varphi}(\lambda,k)(y,z,x)+A^{D,D}_{\theta,\varphi}(\lambda,k)(z,x,y) = 0 \tag{2.17}$$

for all $x, y, z \in L$.

Proof Suppose D is a k-order (θ,φ)-derivation of weight λ. On the one hand,

$$\begin{aligned}
D([x,y,z]) &= \delta^k[D(x),\theta(y),\varphi(z)]+\delta^k[\theta(x),D(y),\varphi(z)]+\delta^k[\theta(x),\varphi(y),D(z)] \\
&\quad +\lambda\delta^k[D(x),D(y),\varphi(z)]+\lambda\delta^k[D(x),\theta(y),D(z)] \\
&\quad +\lambda\delta^k[\theta(x),D(y),D(z)]+\lambda^2\delta^k[D(x),D(y),D(z)]
\end{aligned}$$

for all $x, y, z \in L$. On the other hand,

$$\begin{aligned}
D([x,y,z]) &= -\delta D([y,x,z]) \\
&= -\delta(\delta^k[D(y),\theta(x),\varphi(z)]+\delta^k[\theta(y),D(x),\varphi(z)]+\delta^k[\theta(y),\varphi(x),D(z)] \\
&\quad +\lambda\delta^k[D(y),D(x),\varphi(z)]+\lambda\delta^k[D(y),\theta(x),D(z)] \\
&\quad +\lambda\delta^k[\theta(y),D(x),D(z)]+\lambda^2\delta^k[D(y),D(x),D(z)]) \\
&= \delta^k[D(x),\theta(y),\varphi(z)]+\delta^k[\theta(x),D(y),\varphi(z)]+\delta^k[\varphi(x),\theta(y),D(z)] \\
&\quad +\lambda\delta^k[D(x),D(y),\varphi(z)]+\lambda\delta^k[D(x),\theta(y),D(z)] \\
&\quad +\lambda\delta^k[\theta(x),D(y),D(z)]+\lambda^2\delta^k[D(x),D(y),D(z)]
\end{aligned}$$

for all $x, y, z \in L$. Therefore, $[\theta(x), \varphi(y), D(z)] = [\varphi(x), \theta(y), D(z)]$ for all $x, y, z \in L$. This proves (2.16).

Since $A^{D,D}_{\theta,\varphi}(\lambda, k)(x, y, z) = D([x, y, z])$, and by (1.4), we have

$$\begin{aligned} & A^{D,D}_{\theta,\varphi}(\lambda, k)(x, y, z) + A^{D,D}_{\theta,\varphi}(\lambda, k)(y, z, x) + A^{D,D}_{\theta,\varphi}(\lambda, k)(z, x, y) \\ = \quad & D([x, y, z] + [y, z, x] + [z, x, y]) = 0. \end{aligned}$$

This proves (2.17).

Conversely, we prove that D is a k-order (θ, φ)-derivation of weight λ if (2.16) and (2.17) hold. Since D is a k-order Jordan triple (θ, φ)-derivation of weight λ, and by (1.3) and (2.16), we have

$$\begin{aligned} D([y, x, x]) &= -\delta D([x, y, x]) \\ &= -\delta(\delta^k[D(x), \theta(y), \varphi(x)] + \delta^k[\theta(x), D(y), \varphi(x)] + \delta^k[\theta(x), \varphi(y), D(x)] \\ &\quad +\lambda\delta^k[D(x), D(y), \varphi(x)] + \lambda\delta^k[D(x), \theta(y), D(x)] \\ &\quad +\lambda\delta^k[\theta(x), D(y), D(x)] + \lambda^2\delta^k[D(x), D(y), D(x)]) \\ &= \delta^k[D(y), \theta(x), \varphi(x)] + \delta^k[\theta(y), D(x), \varphi(x)] + \delta^k[\varphi(y), \theta(x), D(x)] \\ &\quad +\lambda\delta^k[D(y), D(x), \varphi(x)] + \lambda\delta^k[D(y), \theta(x), D(x)] \\ &\quad +\lambda\delta^k[\theta(y), D(x), D(x)] + \lambda^2\delta^k[D(y), D(x), D(x)] \\ &= \delta^k[D(y), \theta(x), \varphi(x)] + \delta^k[\theta(y), D(x), \varphi(x)] + \delta^k[\theta(y), \varphi(x), D(x)] \\ &\quad +\lambda\delta^k[D(y), D(x), \varphi(x)] + \lambda\delta^k[D(y), \theta(x), D(x)] \\ &\quad +\lambda\delta^k[\theta(y), D(x), D(x)] + \lambda^2\delta^k[D(y), D(x), D(x)] \\ &= A^{D,D}_{\theta,\varphi}(\lambda, k)(y, x, x) \end{aligned}$$

for all $x, y, z \in L$. By (1.4) and (2.17), we have

$$\begin{aligned} D([x, x, y]) &= -(D([y, x, x]) + D([x, y, x])) \\ &= -(A^{D,D}_{\theta,\varphi}(\lambda, k)(y, x, x) + A^{D,D}_{\theta,\varphi}(\lambda, k)(x, y, x)) \\ &= A^{D,D}_{\theta,\varphi}(\lambda, k)(x, x, y). \end{aligned}$$

Using the same proof method as Proposition 2.9, we get

$$A^{D,D}_{\theta,\varphi}(\lambda, k)(x, y, z) = D([x, y, z])$$

for all $x, y, z \in L$.

Thus, D is a k-order (θ, φ)-derivation of weight λ on L.

Theorem 2.12 Let R be a 3-torsion free ring and let $D : L \longrightarrow L$ be a k-order generalized Jordan triple (θ, φ)-derivation of weight λ with respect to the k-order Jordan triple (θ, φ)-derivation α of weight λ satisfying (2.6). If

$$[\theta(x), \varphi(y), \alpha(z)] = [\varphi(x), \theta(y), \alpha(z)]; \tag{2.18}$$

$$A_{\theta,\varphi}^{\alpha,\alpha}(\lambda,k)(x,y,z)+A_{\theta,\varphi}^{\alpha,\alpha}(\lambda,k)(y,z,x)+A_{\theta,\varphi}^{\alpha,\alpha}(\lambda,k)(z,x,y)=0 \tag{2.19}$$

for all $x,y,z\in L$, then α is a k-order (θ,φ)-derivation of weight λ and D is a k-order generalized (θ,φ)-derivation of weight λ with respect to α.

Proof It follows from Theorem 2.11 that α is a k-order (θ,φ)-derivation of weight λ. Applying Proposition 2.9, we get from (2.6) that

$$B(\lambda,k)(x,y,z)+B(\lambda,k)(y,z,x)+B(\lambda,k)(z,x,y)=0.$$

Then by (2.19), we have

$$\begin{aligned}
& A_{\theta,\varphi}^{\alpha,D}(\lambda,k)(x,y,z)+A_{\theta,\varphi}^{\alpha,D}(\lambda,k)(y,z,x)+A_{\theta,\varphi}^{\alpha,D}(\lambda,k)(z,x,y)\\
= & A_{\theta,\varphi}^{\alpha,\alpha}(\lambda,k)(x,y,z)+A_{\theta,\varphi}^{\alpha,\alpha}(\lambda,k)(y,z,x)+A_{\theta,\varphi}^{\alpha,\alpha}(\lambda,k)(z,x,y)\\
= & 0
\end{aligned}$$

for all $x,y,z\in L$. The rest of the proof is similar to the proof of Theorem 2.11.

Corollary 2.13 Let R be a 3-torsion free ring and let $D:L\longrightarrow L$ be a k-order generalized Jordan triple θ-derivation of weight λ with respect to the k-order Jordan triple θ-derivation α of weight λ satisfying (2.6). Then α is a k-order θ-derivation of weight λ and D is a k-order generalized θ-derivation of weight λ with respect to α.

Proof It is clear that condition (2.18) of Theorem 2.12 is valid when $\theta=\varphi$. For condition (2.19) of Theorem 2.12, we have from (1.4) that

$$\begin{aligned}
& A_{\theta,\theta}^{\alpha,\alpha}(\lambda,k)(x,y,z)+A_{\theta,\theta}^{\alpha,\alpha}(\lambda,k)(y,z,x)+A_{\theta,\theta}^{\alpha,\alpha}(\lambda,k)(z,x,y)\\
= & \delta^k[\alpha(x),\theta(y),\theta(z)]+\delta^k[\theta(x),\alpha(y),\theta(z)]+\delta^k[\theta(x),\theta(y),\alpha(z)]\\
& +\lambda\delta^k[\alpha(x),\alpha(y),\theta(z)]+\lambda\delta^k[\alpha(x),\theta(y),\alpha(z)]+\lambda\delta^k[\theta(x),\alpha(y),\alpha(z)]\\
& +\lambda^2\delta^k[\alpha(x),\alpha(y),\alpha(z)]+\delta^k[\alpha(y),\theta(z),\theta(x)]+\delta^k[\theta(y),\alpha(z),\theta(x)]\\
& +\delta^k[\theta(y),\theta(z),\alpha(x)]+\lambda\delta^k[\alpha(y),\alpha(z),\theta(x)]+\lambda\delta^k[\alpha(y),\theta(z),\alpha(x)]\\
& +\lambda\delta^k[\theta(y),\alpha(z),\alpha(x)]+\lambda^2\delta^k[\alpha(y),\alpha(z),\alpha(x)]+\delta^k[\alpha(z),\theta(x),\theta(y)]\\
& +\delta^k[\theta(z),\alpha(x),\theta(y)]+\delta^k[\theta(z),\theta(x),\alpha(y)]+\lambda\delta^k[\alpha(z),\alpha(x),\theta(y)]\\
& +\lambda\delta^k[\alpha(z),\theta(x),\alpha(y)]+\lambda\delta^k[\theta(z),\alpha(x),\alpha(x)]+\lambda^2\delta^k[\alpha(z),\alpha(x),\alpha(y)]\\
= & \delta^k[\alpha(x),\theta(y),\theta(z)]+\delta^k[\theta(y),\theta(z),\alpha(x)]+\delta^k[\theta(z),\alpha(x),\theta(y)]\\
& +\delta^k[\theta(x),\alpha(y),\theta(z)]+\delta^k[\alpha(y),\theta(z),\theta(x)]+\delta^k[\theta(z),\theta(x),\alpha(y)]\\
& +\delta^k[\theta(x),\theta(y),\alpha(z)]+\delta^k[\theta(y),\alpha(z),\theta(x)]+\delta^k[\alpha(z),\theta(x),\theta(y)]\\
& +\lambda(\delta^k[\alpha(x),\alpha(y),\theta(z)]+\delta^k[\alpha(y),\theta(z),\alpha(x)]+\delta^k[\theta(z),\alpha(x),\alpha(y)])\\
& +\lambda(\delta^k[\alpha(x),\theta(y),\alpha(z)]+\delta^k[\theta(y),\alpha(z),\alpha(x)]+\delta^k[\alpha(z),\alpha(x),\theta(y)])\\
& +\lambda(\delta^k[\theta(x),\alpha(y),\alpha(z)]+\delta^k[\alpha(y),\alpha(z),\theta(x)]+\delta^k[\alpha(z),\theta(x),\alpha(y)])\\
& +\lambda^2(\delta^k[\alpha(x),\alpha(y),\alpha(z)]+\delta^k[\alpha(y),\alpha(z),\alpha(x)]+\delta^k[\alpha(z),\alpha(x),\alpha(y)])\\
= & 0+0+0+\lambda\cdot 0+\lambda\cdot 0+\lambda\cdot 0+\lambda^2\cdot 0=0.
\end{aligned}$$

So condition (2.19) of Theorem 2.12 is valid if $\varphi = \theta$. Hence α is a k-order θ-derivation of weight λ and D is a k-order generalized θ-derivation of weight λ with respect to α.

Corollary 2.14 Let R be a 3-torsion free ring. Then $D : L \longrightarrow L$ is a k-order Jordan triple θ-derivation of weight λ if and only if D is a k-order θ-derivation of weight λ.

Proof It is clear that condition (2.6) of Corollary 2.13 is valid when $D = \alpha$.

Corollary 2.15 Let R be a 3-torsion free ring. Then $D : L \longrightarrow L$ is a k-order Jordan triple derivation of weight λ if and only if D is a k-order derivation of weight λ.

Proof This is a special case that $\theta = I_L$ in Corollary 2.14, where I_L is the identity map on L.

Theorem 2.16 Let L be a δ Jordan-Lie triple system.Let $D : L \longrightarrow L$ be a k-order (θ, φ)-derivation of weight λ and A be a linear automorphism of L. If A, θ and φ satisfy any two of which are commutative, then ADA^{-1} is also a k-order (θ, φ)-derivation of weight λ.

Proof Since D is a k-order (θ, φ)-derivation of weight λ and we have

$$\begin{aligned} D([x,y,z]) &= \delta^k[D(x), \theta(y), \varphi(z)] + \delta^k[\theta(x), D(y), \varphi(z)] + \delta^k[\theta(x), \varphi(y), D(z)] \\ &\quad + \lambda\delta^k[D(x), D(y), \varphi(z)] + \lambda\delta^k[D(x), \theta(y), D(z)] \\ &\quad + \lambda\delta^k[\theta(x), D(y), D(z)] + \lambda^2\delta^k[D(x), D(y), D(z)] \end{aligned}$$

for all $x, y, z \in L$. Since A is a linear automorphism of L, we have

$$\begin{aligned} A([x,y,z]) &= [A(x), A(y), A(z)] \\ A^{-1}([x,y,z]) &= [A^{-1}(x), A^{-1}(y), A^{-1}(z)] \end{aligned}$$

for all $x, y, z \in L$. Therefore,

$$\begin{aligned} ADA^{-1}([x,y,z]) &= AD([A^{-1}(x), A^{-1}(y), A^{-1}(z)]) \\ &= A(\delta^k[DA^{-1}(x), \theta(A^{-1}(y)), \varphi(A^{-1}(z))] \\ &\quad + \delta^k[\theta(A^{-1}(x)), D(A^{-1}(y)), \varphi(A^{-1}(z))] \\ &\quad + \delta^k[\theta(A^{-1}(x)), \varphi(A^{-1}(y)), D(A^{-1}(z))] \\ &\quad + \lambda\delta^k[D(A^{-1}(x)), D(A^{-1}(y)), \varphi(A^{-1}(z))] \\ &\quad + \lambda\delta^k[D(A^{-1}(x)), \theta(A^{-1}(y)), D(A^{-1}(z))] \\ &\quad + \lambda\delta^k[\theta(A^{-1}(x)), D(A^{-1}(y)), D(A^{-1}(z))] \\ &\quad + \lambda^2\delta^k[D(A^{-1}(x)), D(A^{-1}(y)), D(A^{-1}(z))]) \\ &= \delta^k[ADA^{-1}(x), \theta(y), \varphi(z)] + \delta^k[\theta(x), ADA^{-1}(y), \varphi(z)] \\ &\quad + \delta^k[\theta(x), \varphi(y), ADA^{-1}(z)] + \lambda\delta^k[ADA^{-1}(x), ADA^{-1}(y), \varphi(z)] \\ &\quad + \lambda\delta^k[ADA^{-1}(x), \theta(y), ADA^{-1}(z)] + \lambda\delta^k[\theta(x), ADA^{-1}(y), ADA^{-1}(z)] \\ &\quad + \lambda^2\delta^k[ADA^{-1}(x), ADA^{-1}(y), ADA^{-1}(z)] \end{aligned}$$

for all $x, y, z \in L$. So ADA^{-1} is a k-order (θ, φ)-derivation of weight λ on L.

Corollary 2.17 Let R be a 3-torsion free ring. Let A be a linear automorphism of L. Then $D : L \longrightarrow L$ is a k-order Jordan triple derivation of weight λ if and only if A^nDA^{-n} is a k-order derivation of weight λ for all positive integer n.

Proof If D is a k-order Jordan triple derivation of weight λ, it follows Corollary 2.15 that D is a k-order derivation of weight λ. And from Theorem 2.16, ADA^{-1} is also a k-order derivation of weight λ. By mathematical induction, A^nDA^{-n} is a k-order derivation of weight λ for all positive integer n. Conversely, we prove that D is a k-order Jordan triple derivation of weight λ if A^nDA^{-n} is a k-order derivation of weight λ for all positive integer n. Clearly, A^{-n} is a linear automorphism of L and from Theorem 2.16, $A^{-n}A^nDA^{-n}A^n = D$ is a k-order derivation of weight λ. Therefore, D is a k-order Jordan triple derivation of weight λ.

Theorem 2.18 If D is a k-order derivation of δ Jordan-Lie triple system L, $Z(L)$ is the center of L, then $D(Z(L)) \subseteq Z(L)$.

Proof For arbitrary element x in $Z(L)$ and for all $y, z \in L$, we have $[x, y, z] = 0$. Since D is a k-order derivation, we have

$$\delta^k[D(x), y, z] = D([x, y, z]) - \delta^k[x, D(y), z] - \delta^k[x, y, D(z)].$$

Therefore,

$$[D(x), y, z] = 0$$

for all $y, z \in L$. That is $D(Z(L)) \subseteq Z(L)$.

3 On K-Order Generalized Derivation of Weight λ on Rota-Baxter δ Jordan-Lie Triple Systems of Weight λ

In this section, firstly, we introduce the concepts of a Rota-Baxter δ Jordan-Lie algebra of weight λ and a Rota-Baxter δ Jordan-Lie triple system of weight λ. Afterwards, we associate some beautiful results in section 2 with the inheritance property in [18] on a Rota-Baxter δ Jordan-Lie triple system of weight λ. In the end, we obtain that every Rota-Baxter δ Jordan-Lie algebra can be seen as a Rota-Baxter δ Jordan-Lie triple system.

Definition 3.1 A Rota-Baxter δ Jordan-Lie algebra of weight λ is a δ Jordan-Lie algebra $(L, [\cdot,\cdot])$ with a R-trilinear map $p : L \longrightarrow L$ such that

$$[p(x), p(y)] \quad = \quad p([p(x), y] + [x, p(y)] + \lambda[x, y])$$

for all $x, y \in L$. D is called k-order derivation of weight λ on it if

$$D([x, y]) = \delta^k[D(x), y] + \delta^k[x, D(y)] + \lambda\delta^k[D(x), D(y)]$$

for all $x, y \in L$.

Definition 3.2 A Rota-Baxter δ Jordan-Lie triple system of weight λ is a δ Jordan-Lie triple system $(L, [,,])$ with a R-trilinear map $p : L \longrightarrow L$ such that

$$\begin{aligned}[p(x), p(y), p(z)] \quad = \quad & p([p(x), p(y), z] + [p(x), y, p(z)] + [x, p(y), p(z)] \\ & +\lambda[p(x), y, z] + \lambda[x, p(y), z] + \lambda[x, y, p(z)] + \lambda^2[x, y, z]) \qquad (3.1)\end{aligned}$$

for all $x, y, z \in L$. Furthermore, we call p is a Rota-Baxter operator of weight λ on L.

Let $(L, [, ,], p)$ be a Rota-Baxter δ Jordan-Lie triple system of weight λ. We define a ternary operation on L by

$$\begin{aligned} [x, y, z]_p &= [p(x), p(y), z] + [p(x), y, p(z)] + [x, p(y), p(z)] \\ &\quad +\lambda[p(x), y, z] + \lambda[x, p(y), z] + \lambda[x, y, p(z)] + \lambda^2[x, y, z] \end{aligned} \tag{3.2}$$

for all $x, y, z \in L$ (see[12]).

Theorem 3.3 Let $(L, [, ,], p)$ be a Rota-Baxter δ Jordan-Lie triple system of weight λ and T be a invertible linear map. Then T is a Rota-Baxter operator of weight λ if and only if T^{-1} is a 0-order derivation of weight λ.

Proof Since T is a invertible linear map, then for any $x_i \in L(i = 1, 2, 3)$, there exists $y_i \in L(i = 1, 2, 3)$ such that $x_i = Ty_i(i = 1, 2, 3)$. Suppose T is a Rota-Baxter operator of weight λ, then we have

$$\begin{aligned} T^{-1}([x_1, x_2, x_3]) &= T^{-1}([Ty_1, Ty_2, Ty_3]) \\ &= T^{-1}(T([Ty_1, Ty_2, y_3] + [Ty_1, y_2, Ty_3] + [y_1, Ty_2, Ty_3] \\ &\quad +\lambda[Ty_1, y_2, y_3] + \lambda[y_1, Ty_2, y_3] + \lambda[y_1, y_2, Ty_3] + \lambda^2[y_1, y_2, y_3])) \\ &= [x_1, x_2, T^{-1}x_3] + [x_1, T^{-1}x_2, x_3] + [T^{-1}x_1, x_2, x_3] \\ &\quad +\lambda[x_1, T^{-1}x_2, T^{-1}x_3] + \lambda[T^{-1}x_1, x_2, T^{-1}x_3] \\ &\quad +\lambda[T^{-1}x_1, T^{-1}x_2, x_3] + \lambda^2[T^{-1}x_1, T^{-1}x_2, T^{-1}x_3] \end{aligned}$$

for all $x_i \in L(i = 1, 2, 3)$. Thus, T^{-1} is a 0-order derivation of weight λ. Conversely, suppose T is a 0-order derivation of weight λ. Similarly, for all $x_i \in L(i = 1, 2, 3)$, we have

$$\begin{aligned} [T^{-1}x_1, T^{-1}x_2, T^{-1}x_3] &= T^{-1}(T([y_1, y_2, y_3])) \\ &= T^{-1}([Ty_1, y_2, y_3] + [y_1, y_2, Ty_3] + [y_1, Ty_2, y_3] \\ &\quad +\lambda[Ty_1, Ty_2, y_3] + \lambda[y_1, Ty_2, Ty_3] \\ &\quad +\lambda[Ty_1, y_2, Ty_3] + \lambda^2[Ty_1, Ty_2, Ty_3]) \\ &= T^{-1}([x_1, T^{-1}x_2, T^{-1}x_3] + [T^{-1}x_1, T^{-1}x_2, x_3] \\ &\quad +[T^{-1}x_1, x_2, T^{-1}x_3] + \lambda[x_1, x_2, T^{-1}x_3] + \lambda[T^{-1}x_1, x_2, x_3] \\ &\quad +\lambda[x_1, T^{-1}x_2, x_3] + \lambda^2[x_1, x_2, x_3]). \end{aligned}$$

Therefore, T^{-1} is a Rota-Baxter operator of weight λ.

Remark 3.4 Obviously, the conclusion of Theorem 3.3 still holds on Rota-Baxter δ Jordan-Lie algebra of weight λ.

Theorem 3.5 Let $(L, [, ,], p)$ be a Rota-Baxter δ Jordan-Lie triple system of weight λ. Then $(L, [, ,]_p, p)$ is a Rota-Baxter δ Jordan-Lie triple system of weight λ.

Proof First of all, we need to prove $(L, [,,]_p, p)$ is a δ Jordan-Lie triple system. Clearly, $[,,]_p$ defined in (3.2) is multi-linear and we observe that

$$\begin{aligned}
[x,y,z]_p &= [p(x),p(y),z]+[p(x),y,p(z)]+[x,p(y),p(z)] \\
&\quad +\lambda[p(x),y,z]+\lambda[x,p(y),z]+\lambda[x,y,p(z)]+\lambda^2[x,y,z] \\
&= -\delta([p(y),p(x),z]+[y,p(x),p(z)]+[p(y),x,p(z)] \\
&\quad +\lambda[y,p(x),z]+\lambda[p(y),x,z]+\lambda[y,x,p(z)]+\lambda^2[y,x,z]) \\
&= -\delta[y,x,z]_p
\end{aligned}$$

for all $x,y,z \in L$. Therefore, $[,,]_p$ satisfies (1.3).It follows from (1.4) that

$$\begin{aligned}
&\quad [x,y,z]_p+[y,z,x]_p+[z,x,y]_p \\
&= [p(x),p(y),z]+[p(x),y,p(z)]+[x,p(y),p(z)] \\
&\quad +\lambda[p(x),y,z]+\lambda[x,p(y),z]+\lambda[x,y,p(z)]+\lambda^2[x,y,z] \\
&\quad +[p(y),p(z),x]+[p(y),z,p(x)]+[y,p(z),p(x)] \\
&\quad +\lambda[p(y),z,x]+\lambda[y,p(z),x]+\lambda[y,z,p(x)]+\lambda^2[y,z,x] \\
&\quad +[p(z),p(x),y]+[p(z),x,p(y)]+[z,p(x),p(y)] \\
&\quad +\lambda[p(z),x,y]+\lambda[z,p(x),y]+\lambda[z,x,p(y)]+\lambda^2[z,x,y] \\
&= [p(x),p(y),z]+[p(y),z,p(x)]+[z,p(x),p(y)] \\
&\quad +[p(x),y,p(z)]+[y,p(z),p(x)]+[p(z),p(x),y] \\
&\quad +[x,p(y),p(z)]+[p(y),p(z),x]+[p(z),x,p(y)] \\
&\quad +\lambda([p(x),y,z]+[y,z,p(x)]+[z,p(x),y]) \\
&\quad +\lambda([x,p(y),z]+[p(y),z,x]+[z,x,p(y)]) \\
&\quad +\lambda([x,y,p(z)]+[y,p(z),x]+[p(z),x,y]) \\
&\quad +\lambda^2([x,y,z]+[y,z,x]+[z,x,y]) \\
&= 0
\end{aligned}$$

for all $x,y,z \in L$. Therefore, $[,,]_p$ satisfies (1.4). To prove that $[,,]_p$ satisfies (1.5), we need to show

$$[x_1,x_2,[x_3,x_4,x_5]_p]_p = [[x_1,x_2,x_3]_p,x_4,x_5]_p+[x_3,[x_1,x_2,x_4]_p,x_5]_p+\delta[x_3,x_4,[x_1,x_2,x_5]_p]_p$$

for all $x_i \in L (i = 1,2,3,4,5)$. We notice that

$$\begin{aligned}
A &:= [x_1,x_2,[x_3,x_4,x_5]_p]_p \\
&= [p(x_1),p(x_2),[x_3,x_4,x_5]_p]+[p(x_1),x_2,p([x_3,x_4,x_5]_p)]+[x_1,p(x_2),p([x_3,x_4,x_5]_p)] \\
&\quad +\lambda[p(x_1),x_2,[x_3,x_4,x_5]_p]+\lambda[x_1,p(x_2),[x_3,x_4,x_5]_p]+\lambda[x_1,x_2,p([x_3,x_4,x_5]_p)] \\
&\quad +\lambda^2[x_1,x_2,[x_3,x_4,x_5]_p]
\end{aligned}$$

$$\begin{aligned}
= \; & [p(x_1), p(x_2), [p(x_3), p(x_4), x_5]] + [p(x_1), p(x_2), [p(x_3), x_4, p(x_5)]] \\
& + [p(x_1), p(x_2), [x_3, p(x_4), p(x_5)]] + \lambda[p(x_1), p(x_2), [p(x_3), x_4, x_5]] \\
& + \lambda[p(x_1), p(x_2), [x_3, p(x_4), x_5]] + \lambda[p(x_1), p(x_2), [x_3, x_4, p(x_5)]] \\
& + \lambda^2[p(x_1), p(x_2), [x_3, x_4, x_5]] + [p(x_1), x_2, [p(x_3), p(x_4), p(x_5)]] \\
& + [x_1, p(x_2), [p(x_3), p(x_4), p(x_5)]] + \lambda[p(x_1), x_2, [p(x_3), p(x_4), x_5]] \\
& + \lambda[p(x_1), x_2, [p(x_3), x_4, p(x_5)]] + \lambda[p(x_1), x_2, [x_3, p(x_4), p(x_5)]] \\
& + \lambda^2[p(x_1), x_2, [p(x_3), x_4, x_5]] + \lambda^2[p(x_1), x_2, [x_3, p(x_4), x_5]] \\
& + \lambda^2[p(x_1), x_2, [x_3, x_4, p(x_5)]] + \lambda^3[p(x_1), x_2, [x_3, x_4, x_5]] \\
& + [p(x_1), p(x_2), [p(x_3), p(x_4), x_5]] + [p(x_1), p(x_2), [p(x_3), x_4, p(x_5)]] \\
& + [p(x_1), p(x_2), [x_3, p(x_4), p(x_5)]] + \lambda[p(x_1), p(x_2), [p(x_3), x_4, x_5]] \\
& + \lambda[p(x_1), p(x_2), [x_3, p(x_4), x_5]] + \lambda[p(x_1), p(x_2), [x_3, x_4, p(x_5)]] \\
& + \lambda^2[p(x_1), p(x_2), [x_3, x_4, x_5]] + [p(x_1), x_2, [p(x_3), p(x_4), p(x_5)]] \\
& + [x_1, p(x_2), [p(x_3), p(x_4), p(x_5)]] + \lambda[p(x_1), x_2, [p(x_3), p(x_4), x_5]] \\
& + \lambda[p(x_1), x_2, [p(x_3), x_4, p(x_5)]] + \lambda[p(x_1), x_2, [x_3, p(x_4), p(x_5)]] \\
& + \lambda^2[p(x_1), x_2, [p(x_3), x_4, x_5]] + \lambda^2[p(x_1), x_2, [x_3, p(x_4), x_5]] \\
& + \lambda^2[p(x_1), x_2, [x_3, x_4, p(x_5)]] + \lambda^3[p(x_1), x_2, [x_3, x_4, x_5]] \\
& + \lambda[x_1, p(x_2), [p(x_3), p(x_4), x_5]] + \lambda[x_1, p(x_2), [p(x_3), x_4, p(x_5)]] \\
& + \lambda[x_1, p(x_2), [x_3, p(x_4), p(x_5)]] + \lambda^2[x_1, p(x_2), [p(x_3), x_4, x_5]] \\
& + \lambda^2[x_1, p(x_2), [x_3, p(x_4), x_5]] + \lambda^2[x_1, p(x_2), [x_3, x_4, p(x_5)]] \\
& + \lambda^3[x_1, p(x_2), [x_3, x_4, x_5]] + \lambda[x_1, x_2, [p(x_3), p(x_4), p(x_5)]] \\
& + \lambda^2[x_1, x_2, [p(x_3), p(x_4), x_5]] + \lambda^2[x_1, x_2, [p(x_3), x_4, p(x_5)]] \\
& + \lambda^2[x_1, x_2, [x_3, p(x_4), p(x_5)]] + \lambda^3[x_1, x_2, [p(x_3), x_4, x_5]] \\
& + \lambda^3[x_1, x_2, [x_3, p(x_4), x_5]] + \lambda^3[x_1, x_2, [x_3, x_4, p(x_5)]] \\
& + \lambda^3[x_1, x_2, [x_3, x_4, x_5]]
\end{aligned}$$

for all $x, y, z \in L$.

Similarly, we can compute $B := [[x_1, x_2, x_3]_p, x_4, x_5]_p$, $C := [x_3, [x_1, x_2, x_4]_p, x_5]_p$, $D := [x_3, x_4, [x_1, x_2, x_5]_p]_p$.

It follows (1.5) that

$$A = B + C + \delta D.$$

Therefore, $[, ,]_p$ satisfies (1.5). Then $(L, [, ,]_p, p)$ is a δ Jordan-Lie triple system. Finally, we show that p satisfies (3.1) on $(L, [, ,]_p, p)$. From the definition of $[, ,]_p$, we have

$$\begin{aligned}
[p(x), p(y), p(z)]_p = \; & [p^2(x), p^2(y), p(z)] + [p^2(x), p(y), p^2(z)] + [p(x), p^2(y), p^2(z)] \\
& + \lambda[p^2(x), p(y), p(z)] + \lambda[p(x), p^2(y), p(z)] \\
& + \lambda[p(x), p(y), p^2(z)] + \lambda^2[p(x), p(y), p(z)].
\end{aligned}$$

Since p is a Rota-Baxter operator of weight λ on $(L, [,,], p)$, we have

$$\begin{aligned}
[p^2(x), p^2(y), p(z)] &= p([p^2(x), p^2(y), z] + [p^2(x), p(y), p(z)] + [p(x), p^2(y), p(z)] \\
&\quad +\lambda[p^2(x), p(y), z] + \lambda[p(x), p^2(y), z] + \lambda[p(x), p(y), p(z)] \\
&\quad +\lambda^2[p(x), p(y), z])
\end{aligned}$$

$$\begin{aligned}
[p^2(x), p(y), p^2(z)] &= p([p^2(x), p(y), p(z)] + [p^2(x), p(y), p^2(z)] + [p(x), p(y), p^2(z)] \\
&\quad +\lambda[p^2(x), y, p(z)] + \lambda[p(x), p(y), p(z)] + \lambda[p(x), y, p^2(z)] \\
&\quad +\lambda^2[p(x), y, p(z)])
\end{aligned}$$

$$\begin{aligned}
[p(x), p^2(y), p^2(z)] &= p([p(x), p^2(y), p(z)] + [p(x), p(y), p^2(z)] + [x, p^2(y), p^2(z)] \\
&\quad +\lambda[p(x), p(y), p(z)] + \lambda[x, p^2(y), p(z)] + \lambda[x, p(y), p^2(z)] \\
&\quad +\lambda^2[x, p(y), p(z)])
\end{aligned}$$

$$\begin{aligned}
[p^2(x), p(y), p(z)] &= p([p^2(x), p(y), z] + [p^2(x), y, p(z)] + [p(x), p(y), p(z)] \\
&\quad +\lambda[p^2(x), y, z] + \lambda[p(x), p(y), z] + \lambda[p(x), y, p(z)] \\
&\quad +\lambda^2[p(x), y, z])
\end{aligned}$$

$$\begin{aligned}
[p(x), p^2(y), p(z)] &= p([p(x), p^2(y), z] + [p(x), p(y), p(z)] + [x, p^2(y), p(z)] \\
&\quad +\lambda[p(x), p(y), z] + \lambda[x, p^2(y), z] + \lambda[x, p(y), p(z)] \\
&\quad +\lambda^2[x, p(y), z])
\end{aligned}$$

$$\begin{aligned}
[p(x), p(y), p^2(z)] &= p([p(x), p(y), p(z)] + [p(x), y, p^2(z)] + [x, p(y), p^2(z)] \\
&\quad +\lambda[p(x), y, p(z)] + \lambda[x, p(y), p(z)] + \lambda[x, y, p^2(z)] \\
&\quad +\lambda^2[x, y, p(z)])
\end{aligned}$$

$$\begin{aligned}
[p(x), p(y), p(z)] &= p([p(x), p(y), z] + [p(x), y, p(z)] + [x, p(y), p(z)] \\
&\quad +\lambda[p(x), y, z] + \lambda[x, p(y), z] + \lambda[x, y, p(z)] \\
&\quad +\lambda^2[x, y, z]).
\end{aligned}$$

By computing, we have

$$\begin{aligned}
[p(x), p(y), p(z)]_p &= p([p(x), p(y), z]_p + [p(x), y, p(z)]_p + [x, p(y), p(z)]_p \\
&\quad +\lambda[p(x), y, z]_p + \lambda[x, p(y), z]_p + \lambda[x, y, p(z)]_p \\
&\quad +\lambda^2[x, y, z]_p).
\end{aligned}$$

for all $x, y, z \in L$. This proves p is a Rota-Baxter operator of weight λ on $(L, [,,]_p, p)$.

Thus from the above sum, we conclude that $(L, [, ,]_p, p)$ is a Rota-Baxter δ Jordan-Lie triple system of weight λ.

Theorem 3.6 Let $(L, [, ,], p)$ be a Rota-Baxter δ Jordan-Lie triple system of weight λ. Let D be a k-order generalized (θ, φ)-derivation of weight λ with respect to α on L satisfying the relation that any two of $D, p, \alpha, \theta, \varphi$ are commutative. Then D is a k-order generalized (θ, φ)-derivation of weight λ with respect to α on the Rota-Baxter δ Jordan-Lie triple system of weight λ $(L, [, ,]_p, p)$, where α is a k-order (θ, φ)-derivation of weight λ.

Proof We have

$$\begin{aligned} D([p(x), p(y), z]) &= \delta^k[\alpha(p(x)), \theta(p(y)), \varphi(z)] + \delta^k[\theta(p(x)), \alpha(p(y)), \varphi(z)] \\ &\quad +\delta^k[\theta(p(x)), \varphi(p(y)), \alpha(z)] + \lambda\delta^k[D(p(x)), D(p(y)), \varphi(z)] \\ &\quad +\lambda\delta^k[\alpha(p(x)), \theta(p(y)), \alpha(z)] + \lambda\delta^k[\theta(p(x)), \alpha(p(y)), \alpha(z)] \\ &\quad +\lambda^2\delta^k[\alpha(p(x)), \alpha(p(y)), \alpha(z)]. \end{aligned} \tag{3.3}$$

$$\begin{aligned} D([p(x), y, p(z)]) &= \delta^k[\alpha(p(x)), \theta(y), \varphi(p(z))] + \delta^k[\theta(p(x)), \alpha(y), \varphi(p(z))] \\ &\quad +\delta^k[\theta(p(x)), \varphi(y), \alpha(p(z))] + \lambda\delta^k[D(p(x)), D(y), \varphi(p(z))] \\ &\quad +\lambda\delta^k[\alpha(p(x)), \theta(y), \alpha(p(z))] + \lambda\delta^k[\theta(p(x)), \alpha(y), \alpha(p(z))] \\ &\quad +\lambda^2\delta^k[\alpha(p(x)), \alpha(y), \alpha(p(z))]. \end{aligned} \tag{3.4}$$

$$\begin{aligned} D([x, p(y), p(z)]) &= \delta^k[\alpha(x), \theta(p(y)), \varphi(p(z))] + \delta^k[\theta(x), \alpha(p(y)), \varphi(p(z))] \\ &\quad +\delta^k[\theta(x), \varphi(p(y)), \alpha(p(z))] + \delta^k[D(x), D(p(y)), \varphi(p(z))] \\ &\quad +\lambda\delta^k[\alpha(x), \theta(p(y)), \alpha(p(z))] + \lambda\delta^k[\theta(x), \alpha(p(y)), \alpha(p(z))] \\ &\quad +\lambda^2\delta^k[\alpha(x), \alpha(p(y)), \alpha(p(z))]. \end{aligned} \tag{3.5}$$

$$\begin{aligned} D([p(x), y, z]) &= \delta^k[\alpha(p(x)), \theta(y), \varphi(z)] + \delta^k[\theta(p(x)), \alpha(y), \varphi(z)] \\ &\quad +\delta^k[\theta(p(x)), \varphi(y), \alpha(z)] + \lambda\delta^k[D(p(x)), D(y), \varphi(z)] \\ &\quad +\lambda\delta^k[\alpha(p(x)), \theta(y), \alpha(z)] + \lambda\delta^k[\theta(p(x)), \alpha(y), \alpha(z)] \\ &\quad +\lambda^2\delta^k[\alpha(p(x)), \alpha(y), \alpha(z)]. \end{aligned} \tag{3.6}$$

$$\begin{aligned} D([x, p(y), z]) &= \delta^k[\alpha(x), \theta(p(y)), \varphi(z)] + \delta^k[\theta(x), \alpha(p(y)), \varphi(z)] \\ &\quad +\delta^k[\theta(x), \varphi(p(y)), \alpha(z)] + \lambda\delta^k[D(x), D(p(y)), \varphi(z)] \\ &\quad +\lambda\delta^k[\alpha(x), \theta(p(y)), \alpha(z)] + \lambda\delta^k[\theta(x), \alpha(p(y)), \alpha(z)] \\ &\quad +\lambda^2\delta^k[\alpha(x), \alpha(p(y)), \alpha(z)]. \end{aligned} \tag{3.7}$$

$$\begin{aligned} D([x, y, p(z)]) &= \delta^k[\alpha(x), \theta(y), \varphi(p(z))] + \delta^k[\theta(x), \alpha(y), \varphi(p(z))] \\ &\quad +\delta^k[\theta(x), \varphi(y), \alpha(p(z))] + \lambda\delta^k[D(x), D(y)), \varphi(p(z))] \\ &\quad +\lambda\delta^k[\alpha(x), \theta(y), \alpha(p(z))] + \lambda\delta^k[\theta(x), \alpha(y), \alpha(p(z))] \\ &\quad +\lambda^2\delta^k[\alpha(x), \alpha(y), \alpha(p(z))]. \end{aligned} \tag{3.8}$$

$$
\begin{aligned}
D([x,y,z]) &= \delta^k[\alpha(x),\theta(y),\varphi(z)]+\delta^k[\theta(x),\alpha(y),\varphi(z)]\\
&\quad+\delta^k[\theta(x),\varphi(y),\alpha(z)]+\lambda\delta^k[D(x),D(y)),\varphi(z)]\\
&\quad+\lambda\delta^k[\alpha(x),\theta(y),\alpha(z)]+\lambda\delta^k[\theta(x),\alpha(y),\alpha(z)]\\
&\quad+\lambda^2\delta^k[\alpha(x),\alpha(y),\alpha(z)]. \qquad (3.9)
\end{aligned}
$$

It is a straightforward computation. By (3.3)–(3.9), we obtain

$$
\begin{aligned}
D([x,y,z]_p) &= D([p(x),p(y),z])+D([p(x),y,p(z)])+D([x,p(y),p(z)])\\
&\quad+\lambda D([p(x),y,z])+\lambda D([x,p(y),z])\\
&\quad+\lambda D([x,y,p(z)]+\lambda^2 D([x,y,z])\\
&= \delta^k[\alpha(x),\theta(y),\varphi(z)]_p+\delta^k[\theta(x),\alpha(y),\varphi(z)]_p+\delta^k[\theta(x),\varphi(y),\alpha(z)]_p\\
&\quad+\lambda\delta^k[D(x),D(y),\varphi(z)]_p+\lambda\delta^k[\alpha(x),\theta(y),\alpha(z)]_p\\
&\quad+\lambda\delta^k[\theta(x),\alpha(y),\alpha(z)]_p+\lambda^2\delta^k[\alpha(x),\alpha(y),\alpha(z)]_p.
\end{aligned}
$$

Therefore, D is a k-order generalized (θ,φ)-derivation of weight λ with respect to α on the Rota-Baxter δ Jordan-Lie triple system of weight λ $(L,[,,]_p,p)$.

Corollary 3.7 Let $(L,[,,],p)$ be a Rota-Baxter δ Jordan-Lie triple system of weight λ. Let D be a k-order (θ,φ)-derivation of weight λ on L satisfying the relation any two of $D,p,\alpha,\theta,\varphi$ are commutative. Then D is a k-order (θ,φ)-derivation of weight λ on the Rota-Baxter δ Jordan-Lie triple system of weight λ $(L,[,,]_p,p)$.

Proof It is the direct results of Theorem 3.6.

Corollary 3.8 Let $(L,[,,])$ be a δ Jordan-Lie triple system. d is a invertible 0-order derivation of weight λ on L, then $(L,[,,]_{d^{-1}})$ with $[,,]$ defined in (3.2) is also a δ Jordan-Lie triple system. Furthermore

$$[x,y,z]_{d^{-1}}=d([d^{-1}(x),d^{-1}(y),d^{-1}(z)])$$

for all $x,y,z\in L$. d is a 0-order derivation of weight λ on $(L,[,,]_{d^{-1}})$.

Proof By Theorem 3.3, d^{-1} is a Rota-Baxter operator of weight λ on $(L,[,,])$. And by Theorem 3.5, $(L,[,,]_{d^{-1}})$ is a δ Jordan-Lie triple system. From Corollary 3.7, d is a 0-order derivation of weight λ on $(L,[,,]_{d^{-1}})$. Thus, d^{-1} is a Rota-Baxter operator of weight λ on $(L,[,,]_{d^{-1}})$. Clearly, we have

$$[x,y,z]_{d^{-1}}=d([d^{-1}(x),d^{-1}(y),d^{-1}(z)])$$

for all $x,y,z\in L$.

Theorem 3.9 Let $(L,[,,],p)$ be a Rota-Baxter δ Jordan-Lie triple system of weight λ and A be a linear automorphism on $(L,[,,],p)$. If A and p are commutative, then A is also a linear automorphism on $(L,[,,]_p,p)$, where $[,,]_p$ is defined as (3.2).

Proof We need to certify $A([x,y,z]_p)=[A(x),A(y),A(z)]_p$, for all $x,y,z\in L$. From the definition of $(L,[,,]_p,p)$, we have

$$\begin{aligned}
A([x,y,z]_p) &= A([p(x),p(y),z]+[p(x),y,p(z)]+[x,p(y),p(z)]\\
&\quad +\lambda[p(x),y,z]+\lambda[x,p(y),z]+\lambda[x,y,p(z)]+\lambda^2[x,y,z])\\
&= [p(A(x)),p(A(y)),A(z)]+[p(A(x)),A(y),p(A(z))]\\
&\quad +[A(x),p(A(y)),p(A(z))]+\lambda[p(A(x)),A(y),A(z)]\\
&\quad +\lambda[A(x),p(A(y)),A(z)]+\lambda[A(x),A(y),p(A(z))]+\lambda^2[A(x),A(y),A(z)]\\
&= [A(x),A(y),A(z)]_p.
\end{aligned}$$

So A is also a linear automorphism on $(L,[,,]_p,p)$.

Corollary 3.10 Let $(L,[,,],p)$ be a Rota-Baxter δ Jordan-Lie triple system of weight λ. Let D be a k-order Jordan triple θ-derivation of weight λ on $(L,[,,],p)$ and A be a linear automorphism of $(L,[,,],p)$. If A,p,θ,D satisfy the relation any two of which are commutative, then A^nDA^{-n} is a k-order θ-derivation of weight λ on $(L,[,,]_p,p)$ for all positive integer n, where $[,,]_p$ is defined as (3.2).

Proof Since D is a k-order Jordan triple θ-derivation of weight λ on $(L,[,,],p)$, and from Corollary 2.15, we have that D is a k-order θ-derivation of weight λ on $(L,[,,],p)$. From Corollary 3.7, D is a k-order θ-derivation of weight λ on $(L,[,,]_p,p)$. Since A is a linear automorphism of $(L,[,,],p)$, and from Theorem 3.9, we know that A is a linear automorphism of $(L,[,,]_p,p)$, and from Theorem 2.17, ADA^{-1} is a k-order θ-derivation of weight λ on $(L,[,,]_p,p)$. By mathematical induction, A^nDA^{-n} is a k-order θ-derivation of weight λ on $(L,[,,]_p,p)$ for all positive integer n.

Theorem 3.11 Let $(L,[\cdot,\cdot])$ be a δ Jordan-Lie algebra and D be a 0-order derivation of weight λ on it. Then D is also a 0-order derivation of weight λ on δ Jordan-Lie triple system $(L,[,,])$, where $[,,]$ is defined by $[x,y,z]:=[[x,y],z]]$ for all $x,y,z\in L$.

Proof Suppose D is a 0-order derivation of weight λ on $(L,[\cdot,\cdot])$, then

$$\begin{aligned}
D([x,y,z]) &= D([[x,y],z]])\\
&= [D([x,y]),z]+[[x,y],D(z)]+\lambda[D([x,y]),D(z)]\\
&= [[D(x),y],z]+[[x,D(y)],z]+\lambda[[D(x),D(y)],z]\\
&\quad +[[x,y],D(z)]+\lambda[[D(x),y],D(z)]\\
&\quad +\lambda[[x,D(y)],D(z)]+\lambda^2[[D(x),D(y)],D(z)]\\
&= [D(x),y,z]+[x,D(y),z]+[x,y,D(z)]\\
&\quad +\lambda[x,D(y),D(z)]+\lambda[D(x),D(y),z]\\
&\quad +\lambda[D(x),y,D(z)]+\lambda^2[D(x),D(y),D(z)]
\end{aligned}$$

for all $x,y,z\in L$.

Therefore, D is also a 0-order derivation of weight λ on $(L,[,,])$.

Corollary 3.12 Let $(L, [\cdot, \cdot], p)$ be a Rota-Baxter δ Jordan-Lie algebra of weight λ. Then $(L, [, ,], p)$ be a Rota-Baxter δ Jordan-Lie triple system of weight λ, where we assume p is invertible and $[, ,]$ is defined as above.

Proof It just need to prove that p is a Rota-Baxter operator of weight λ on $(L, [, ,], p)$. By Remark 3.4, p^{-1} is a 0-order derivation of weight λ on $(L, [\cdot, \cdot], p)$. From Theorem 3.11, p^{-1} is a 0-order derivation of weight λ on $(L, [, ,], p)$. By Theorem 3.3, we obtain that p is a Rota-Baxter operator of weight λ on $(L, [, ,], p)$.

Remark 3.13 Actually, the conclusion of Corollary 3.12 still holds when p is not invertible. It just need to use the same proof method as Theorem 3.11 to prove that p is a Rota-Baxter operator of weight λ on $(L, [, ,], p)$.

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δ Jordan-李三系上带有权λ的k-阶广义导子

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摘要: 本文研究了δ Jordan-李三系上带有权λ的k-阶广义导子的相关问题. 通过计算, 得到了每一个δ Jordan-李三系上带有权λ的k-阶Jordan 三角θ-导子都是一个带有权λ的k-阶θ-导子. 在定义下, 给出了带有权λ的k-阶Jordan 三角θ-导子的另一种等价形式. 同时, 建立了带有权λ的k-阶广义(θ,φ)-导子和Rota-Baxter δ Jordan- 李三系上带有权λ的Rota-Baxter 算子的遗传性质, 得到了每一个Rota-Baxter δ Jordan-李代数能看成一个Rota-Baxter δ Jordan-李三系的结论.

关键词: δ Jordan-李三系; k-阶(θ,φ)-导子; k-阶Jordan 三角(θ,φ)-导子; 权λ; 权λ的Rota-Baxter δ Jordan-李三系

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