OPTIMIZATION PROBLEM OF EXCESS-OF-LOSS REINSURANCE AND INVESTMENT WITH DELAY AND MISPRICING UNDER THE JUMP-DIFFUSION MODEL

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Abstract: In this paper, we study an optimization problem of excess-of-loss reinsurance and investment with delay and mispricing under the Jump-diffusion model. Using the stochastic control theory, the equilibrium reinsurance-investment strategy and the corresponding equilibrium value function are derived by solving an extended HJB system. Finally, some special cases of our model and results are presented, and some numerical examples for our results are provided.

Keywords: excess-of-loss reinsurance; Lévy insurance model; mispricing; stochastic differential delay equation; jump-diffusion model

1 Introduction

An insurer can control risks through a number of measures, such as investment and reinsurance. In recent years, the problem of the optimal investment and reinsurance has been widely investigated, which was considered in the literature [1–3] and so on.

With the deepening of research in the insurance field, some scholars point out that the risky asset's price process is represented by a jump-diffusion model, which is more consistent with the stock market. Ignoring jump risks on risky asset's price process have an important impact on the optimal problem (see [4,5]). A et al.[6] showed that the development of real-world systems depends not only on their current state but also on their previous history. If we believe that financial market exists bounded memory or the performance-related capital inflow (outflow), then the wealth process with delay must be considered (see [7]). In addition, due to the existence of frictions in markets which are not absolutely mature, insurers can make a profit by mispricing, that is, by exploiting the price difference between a pair of stocks, we can refer to [8,9].

On the basis of previous literature, we establish a class of generalized optimal investment and reinsurance risk model, we consider the optimization problem of excess-of-loss reinsurance and investment with delay and mispricing under the Jump-diffusion model, and the purpose is to obtain the equilibrium reinsurance-investment strategy and the corresponding equilibrium value function. In which we introduce the performance-related capital inflow (outflow) and the price processes of stocks are described by jump-diffusion models with mispricing. Moreover, referring to Li et al.[10], the claim process is described by a spectrally negative Lévy process.

The remainder of this paper is organized as follows. Section 2 gives the model framework. Section 3 derives the explicit expressions of the equilibrium reinsurance-investment strategy and the corresponding equilibrium value function, and provides two special cases of our model. Section 4 provides some numerical examples for sensitivity analysis.

2 The Model

Let $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}_{t\in[0,T]}, \mathbb{P})$ be a complete probability space that fulfills the usual condition, where $[0, T]$ is a fixed and finite time horizon; $\mathcal{F}_t$ is the information of the market available up to time t and $\mathbb{P}$ is a reference measure.

Following the idea suggested by Li et al.[10], without reinsurance and investment, the insurer's surplus process modeled by a spectrally negative Lévy process defined on this probability space with dynamics

$$dR_t = cdt + \sigma dB(t) - \int_0^\infty z N_1(dz, dt), \qquad (2.1)$$

where

(i) $N_1(dz, dt)$ is a Poisson random measure representing the number of insurance claims of size $(z, z + dz)$ within the time period $(t, t + dt)$.

(ii) c is the premium rate, according to the expected value principle, $c = (1+\theta)\int_0^\infty zv(dz)$, where $\theta > 0$ is the safety loading of the insurer, $\sigma > 0$ is the volatility rate, $B(t)$ is a standard Brownian motion.

Let $\tilde{N}_1(dz, dt) = N_1(dz, dt) - v(dz)dt$ represent the compensated measure of $N_1(dz, dt)$, where v is a Lévy measure and $\int_0^\infty zv(dz) < \infty$, $v(dz)$ represents the expected number of insurance claims of size $(z, z + dz)$ within a unit time interval.

ℓ_t is the size of the claim paid by the insurer when the claim equals Z_t at time $t \in [0, T]$, where $0 < \ell_t < Z_t$. And $Z_t - \ell_t$ is the residual part of Z_t that needs to be paid by the reinsurer. Let $\ell_t = \ell(Z_t, t)$ be a reinsurance strategy, in which we slightly abuse notation by using ℓ on both sides of this equation. So the premium rate of reinsurer is $(1 + \eta)\int_0^\infty z - \ell(z, t)v(dz)$, where $\eta > \theta > 0$ is the safety loading of the reinsurer.

In theory, we should first suppose that the reinsurance strategy relies on surplus. But in the following Theorem 3.2, we find that the equilibrium reinsurance strategy is independent of the surplus. Thus, for simplicity, we omit ℓ's possible dependency on the surplus. So the

surplus process can be described as

$$dR_t = \int_0^\infty ((\theta - \eta)z + \eta\ell(z,t))v(dz)dt - \int_0^\infty \ell(z,t)\tilde{N}_1(dz,dt) + \sigma dB(t). \tag{2.2}$$

We assume that the insurer is allowed to invest in a financial market composing of one risk-free asset, a market index and a pair of stocks with mispricing (see Gu et al. [8]). The risk-free asset's price process $S_0(t)$ is described by

$$\frac{dS_0(t)}{S_0(t)} = rdt, \tag{2.3}$$

where $r > 0$ represents the risk-free interest rate. The price process of the market index $P_m(t)$ follows as

$$\frac{dP_m(t)}{P_m(t)} = (r + \mu_m)dt + \sigma_m dZ_m(t), \tag{2.4}$$

where the market risk premium μ_m and the market volatility σ_m are positive constants, and $\{Z_m(t)\}$ is a standard Brownian motion. The price processes of the pair of stocks are described by

$$\frac{dS_1(t)}{S_1(t)} = (r + \mu)dt + \sigma_1 dZ_1(t) - k_1 X(t)dt + d\sum_{i=1}^{N_2(t)} Y_{1i}, \quad S_1(0) = S_{10} > 0, \tag{2.5}$$

$$\frac{dS_2(t)}{S_2(t)} = (r + \mu)dt + \sigma_1 dZ_2(t) + k_2 X(t)dt + d\sum_{i=1}^{N_2(t)} Y_{1i}, \quad S_2(0) = S_{20} > 0, \tag{2.6}$$

(i) μ, σ_1, k_1 and k_2 are positive constants, $\sigma_1 dZ_i(t)$ describes the risk of stock i in the financial market, $i = 1, 2$.

(ii) $\{N_2(t)\}_{t\in[0,T]}$ is homogeneous Poisson process with intensity β_1, which represents the number of the price jumps that occurred the first or second stock during time interval $[0, T]$.

(iii) Y_{1i} is the ith jump amplitude of the stock price, and Y_{1i}, $i = 1, 2, 3, \cdots$ are i.i.d.random variables. We assume their distribution is $G(y_1)$, and they have finite first-order moment μ_{Y_1} and second-order moment $\sigma_{Y_1}^2$.

(iv) Suppose that $\{Z_1(t)\}$, $\{Z_2(t)\}$, $\{Z_m(t)\}$, $\{B(t)\}$, $N_1(dz,dt)$, and $\sum_{i=1}^{N_2(t)} Y_{1i}$ are independent and $P(Y_{1i} \geq -1) = 1$, $i = 1, 2, 3, \cdots$ to guarantee that these two stocks's prices have always been positive.

(v) The term $k_j X(t)dt$ shows the effect of mispricing on the jth stock's price, $j = 1, 2$. $X(t)$ is the pricing error or mispricing between two stocks, and is defined as

$$X(t) = \ln \frac{S_1(t)}{S_2(t)}.$$

Based on eqs.(2.5) and (2.6), using standard Itô's calculus, we find that the dynamics of the mispricing $X(t)$ satisfy the following equation

$$dX(t) = -(k_1 + k_2)X(t)dt + \sigma_1(dZ_1(t) - dZ_2(t)), \quad X(0) = x_0. \tag{2.7}$$

Let $\ell(Z_t, t)$ represent the retained level of the excess-of-loss at time t, $\pi_m(t)$ represent the money amount invested in the market index at time t, $\pi_1(t)$ and $\pi_2(t)$ represent the money amount invested in two stocks. $(W^u(t) - \pi_m(t) - \pi_1(t) - \pi_2(t))$ represents the money amount invested in the risk-free asset, where $W^u(t)$ is the wealth process at time t. We denote the whole reinsurance-investment strategy by $u = \{\ell(Z_t, t), \pi_m(t), \pi_1(t), \pi_2(t)\}$, $t \in [0, T]$.

In addition, we also consider that there exists capital inflow into or outflow from the insurer's current wealth. We can refer to A et al.[6]. Denote the average and pointwise performance of the wealth in the past horizon $[t - h, t]$ by $\bar{Y}(t)$ and $M(t)$, respectively, i.e.,

$$\bar{Y}(t) = \frac{\int_{-h}^0 e^{\delta s}W(t+s)ds}{\frac{1}{h}\int_{-h}^0 e^{\delta s}ds}, \quad M(t) = W(t-h), \quad \forall t \in [0, T], \tag{2.8}$$

where $\delta \geq 0$ is an average parameter and $h > 0$ is the delay parameter. Let $Y(t) = \int_{-h}^0 e^{\delta s}W(t+s)ds$, then $\bar{Y}(t) = \frac{Y(t)}{\frac{1}{h}\int_{-h}^0 e^{\delta s}ds}$. Let the function $g(t, W(t) - \bar{Y}(t), W(t) - M(t))$ represent the capital inflow (outflow) amount which is related to the past performance of the wealth. $W(t) - \bar{Y}(t)$ accounts for the average performance of the wealth between $t - h$ and t, and $W(t) - M(t)$ implies the absolute performance of the wealth in the time horizon $[t - h, t]$.

This capital inflow (outflow) may occur in a variety of situations, as described in A et al. [6]. We assume that the amount of the capital inflow (outflow) is proportional to the past performance of the insurer's wealth, i.e.,

$$g(t, W(t) - \bar{Y}(t), W(t) - M(t)) = b(W(t) - \bar{Y}(t)) + c(W(t) - M(t)), \tag{2.9}$$

where b and c are nonnegative constants.

Thus, the insurer's wealth process $\{W^u(t), t \in [0, T]\}$ is described by

$$
\begin{aligned}
dW^u(t) =& dR_t + \pi_m(t)\frac{dP_m(t)}{P_m(t)} + \pi_1(t)\frac{dS_1(t)}{S_1(t)} + \pi_2(t)\frac{dS_2(t)}{S_2(t)} + (W^u(t) - \pi_m(t) \\
& - \pi_1(t) - \pi_2(t))\frac{dS_0(t)}{S_0(t)} - g(t, W^u(t) - \bar{Y}(t), W^u(t) - M(t))dt \\
=& [lW^u(t) + \mu_m\pi_m + \mu(\pi_1 + \pi_2) + \int_0^\infty ((\theta - \eta)z + (1 + \eta)\ell(z, t))v(dz) - k_1\pi_1 x \\
& + k_2\pi_2 x + \bar{b}Y(t) + cM(t)]dt + \sigma dB(t) + \sigma_1\pi_1 dZ_1(t) + \sigma_1\pi_2 dZ_2(t) + \sigma_m\pi_m dZ_m(t) \\
& - \int_0^\infty \ell(z, t)N_1(dz, dt) + \int_{-1}^\infty \pi_1 y_1 N_2(dy_1, dt) + \int_{-1}^\infty \pi_2 y_1 N_2(dy_1, dt), \tag{2.10}
\end{aligned}
$$

where $l = r - b - c$ and $\bar{b} = \frac{b}{\frac{1}{h}\int_{-h}^0 e^{\delta s}ds}$, π_m, π_1 and π_2 are short for $\pi_m(t)$, $\pi_1(t)$ and $\pi_2(t)$. We suppose that $W^u(t) = w_0 > 0$, $\forall t \in [-h, 0]$, which implies that the insurer is endowed

with the initial wealth w_0 at time $-h$ and do not start the business until time 0. Then the initial value of the average performance wealth $Y(0)$ is $Y(0) = \frac{w_0(1-e^{-\delta h})}{\delta}$. Referring to Zeng et al.[2] and Li et al.[10], we give two definitions.

Definition 2.1 (Admissible Strategy) A strategy $u = \{\ell(Z_t, t), \pi_m(t), \pi_1(t), \pi_2(t)\}_{t \in [0.T]}$ is admissible if

(i) for all $t \in [0, T]$, $Z_t \geq 0$, $0 \leq \ell(Z_t, t) \leq Z_t$;

(ii) u is predictable w.r.t.$\{\mathcal{F}_t\}_{t \in [0,T]}$, and $E_{t,w,x,y}[\int_0^T \| v(t) \|^2 dt] < \infty$, where $\| v(t) \|^2 = (\ell(Z_t, t))^2 + (\pi_m(t))^2 + (\pi_1(t))^2 + (\pi_2(t))^2$;

(iii) $\forall (t, w, x, y) \in [0, T] \times R \times R \times [-1, \infty)$, Eq.(2.10) has a pathwise unique solution $\{W^u(t)\}_{t \in [0.T]}$ with $W(t) = w$, $X(t) = x$, $Y(t) = y$.

Let Π denote the set of all admissible strategies. In this paper, our main purpose is to research the reinsurance and investment problem for an insurer under mean-variance criterion, i.e., wishes to maximize $J^u(t, w, x, y)$, in which J^u is given by

$$J^u(t, w, x, y) = E_{t,w,x,y}[W_T^u] - \frac{\gamma}{2} Var_{t,w,x,y}[W_T^u], \quad (t, w, x, y) \in [0, T] \times R \times R \times [-1, \infty),$$

where $\gamma > 0$ represents a constant absolute risk aversion coefficient. We know that mean-variance criterion has the issue of time-inconsistency. But in many situations, time-consistency of strategies is a basic requirement for rational investors. So, we tackle the problem from a non-cooperative game point of view by defining an equilibrium strategy and its corresponding equilibrium value function (see [11]).

Definition 2.2 For an admissible strategy $u^* = \{\ell^*(Z_t, t), \pi_m^*(t), \pi_1^*(t), \pi_2^*(t)\}_{t \in [0,T]}$ for $\varepsilon > 0$ with any fixed chosen initial state $(t, w, x, y) \in [0, T] \times R \times R \times [-1, \infty)$, define the strategy $u^{\varepsilon,t}$ by

$$u_s^{\varepsilon,t} = \begin{cases} (\bar{\ell}(z, s), \bar{\pi}_m(s), \bar{\pi}_1(s), \bar{\pi}_2(s)), & t \leq s < t + \varepsilon, \\ u_s^*, & t + \varepsilon \leq s \leq T, \end{cases} \tag{2.11}$$

where $\bar{\ell}(z, s)$ is an admissible reinsurance strategy and $(\bar{\pi}_m(t), \bar{\pi}_1(t), \bar{\pi}_2(t)) \in R \times R \times R$,

$$\liminf_{\varepsilon \to 0} \frac{J^{u^*}(t, w, x, y) - J^{u^{\varepsilon,t}}(t, w, x, y)}{\varepsilon} \geq 0.$$

Then u^* is an equilibrium strategy and $J^{u^*}(t, w, x, y)$ is the corresponding equilibrium value function. For $\forall (t, w, x, y) \in [0, T] \times R \times R \times [-1, \infty)$, $\forall \phi(t, w, x, y) \in C^{1,2,2,1}([0, T] \times R \times R \times [-1, \infty))$, we define a variational operator $\mathcal{A}^u$ as follows

$$\mathcal{A}^u \phi(t, w, x, y) = \phi_t + \phi_w[lw + \mu(\pi_1(t) + \pi_2(t)) + \int_0^\infty ((\theta - \eta)z + (1 + \eta)\ell(z, t))v(dz)$$

$$+ \mu_m \pi_m(t) - k_1 \pi_1(t)x + k_2 \pi_2(t)x + \bar{b}y + cm] - \phi_x(k_1 + k_2)x + \phi_y(w - \delta y - e^{-\delta h}m)$$

$$+ \frac{1}{2}\phi_{ww}[\sigma^2 + \sigma_1^2 \pi_1^2(t) + \sigma_1^2 \pi_2^2(t) + \sigma_m^2 \pi_m^2(t)] + \sigma_1^2 \phi_{xx} + \phi_{wx}(\sigma_1^2 \pi_1(t) - \sigma_1^2 \pi_2(t))$$

$$+ \int_0^\infty (\phi(t, w - \ell, x, y) - \phi(t, w, x, y))v(dz) + \beta_1 E[\phi(t, w + \pi_1(t)y_1, x, y)$$

$$- \phi(t, w, x, y)] + \beta_1 E[\phi(t, w + \pi_2(t)y_1, x, y) - \phi(t, w, x, y)]. \tag{2.12}$$

3 Optimization Problem and the Equilibrium Optimal Strategy

In this section, we consider the optimization problem and seek the optimal strategy, and then analyze two special cases. We first provide a verification theorem whose proof is similar to Theorem 1 of Kryger and Steffensen [12]. We omit it here.

Theorem 3.1 (Verification Theorem) Suppose there exist $V(t, w, x, y)$ and $g(t, w, x, y) \in C^{1,2,2,1}([0, T] \times R \times R \times [-1, \infty))$ satisfying the following conditions: for all $(t, w, x, y) \in ([0, T] \times R \times R \times [-1, \infty))$,

$$\sup_{u \in \Pi} \{\mathcal{A}^u V(t, w, x, y) - \frac{\gamma}{2}\mathcal{A}^u g^2(t, w, x, y) + \gamma g(t, w, x, y)\mathcal{A}^u g(t, w, x, y)\} = 0, \qquad (3.1)$$

$$V(T, w, x, y) = w, \qquad (3.2)$$

$$\mathcal{A}^{u^*} g(t, w, x, y) = 0, \quad g(T, w, x, y) = w, \qquad (3.3)$$

and

$$u^* := \arg\sup_{u \in \Pi}\{\mathcal{A}^u V(t, w, x, y) - \frac{\gamma}{2}\mathcal{A}^u g^2(t, w, x, y) + \gamma g(t, w, x, y)\mathcal{A}^u g(t, w, x, y)\}, \qquad (3.4)$$

then $J^{u^*}(t, w, x, y) = V(t, w, x, y)$, $E[W^{u^*}(T)] = g(t, w, x, y)$ and u^* is the equilibrium reinsurance-investment strategy.

For an admissible strategy $u^* = \{\ell^*(Z_t, t), \pi_m^*(t), \pi_1^*(t), \pi_2^*(t)\}_{t \in [0,T]}$, the HJB function (3.1) follows

$$\sup_{u \in \Pi}\left\{V_w\left[lw + \mu(\pi_1(t) + \pi_2(t)) - k_1\pi_1 x + k_2\pi_2 x + \int_0^\infty ((\theta - \eta)z + (1 + \eta)\ell(z, t))v(dz)\right.\right.$$

$$+\mu_m\pi_m(t) + \bar{b}y + cm\Big] + V_t - V_x(k_1 + k_2)x + V_y(w - \delta y - e^{-\delta h}m) + \frac{1}{2}(V_{ww} - \gamma g_w^2)[\sigma^2$$

$$+ \sigma_1^2\pi_1^2(t) + \sigma_1^2\pi_2^2(t) + \sigma_m^2\pi_m^2(t)] + \sigma_1^2(V_{xx} - \gamma g_x^2) + (V_{wx} - \gamma g_w g_x)(\sigma_1^2\pi_1(t) - \sigma_1^2\pi_2(t))$$

$$+ \int_0^\infty \left(V(t, w - \ell, x, y) - \frac{\gamma}{2}g^2(t, w - \ell, x, y) + \gamma g(t, w, x, y)g(t, w - \ell, x, y)\right)v(dz)$$

$$- \int_0^\infty \left(V(t, w, x, y) + \frac{\gamma}{2}g^2(t, w, x, y)\right)v(dz) - \beta_1 E\left[V(t, w, x, y) + \frac{\gamma}{2}g^2(t, w, x, y)\right]$$

$$+ \beta_1 E\left[V(t, w + \pi_1 y_1, x, y) - \frac{\gamma}{2}g^2(t, w + \pi_1 y_1, x, y) + \gamma g(t, w, x, y)g(t, w + \pi_1 y_1, x, y)\right]$$

$$+ \beta_1 E\left[V(t, w + \pi_2 y_1, x, y) - \frac{\gamma}{2}g^2(t, w + \pi_2 y_1, x, y) + \gamma g(t, w, x, y)g(t, w + \pi_2 y_1, x, y)\right]$$

$$-\beta_1 E\left[V(t, w, x, y) + \frac{\gamma}{2}g^2(t, w, x, y)\right]\right\} = 0. \qquad (3.5)$$

To solve eqs. (3.3) and (3.5), we try to conjecture the solutions in the following forms

$$V(t, w, x, y) = A(t)(w + \alpha y) + B(t)x^2 + C(t)x + P(t), \qquad (3.6)$$

$$g(t, w, x, y) = \bar{A}(t)(w + \alpha y) + \bar{B}(t)x^2 + \bar{C}(t)x + \bar{P}(t) \qquad (3.7)$$

with $A(T) = \bar{A}(T) = 1$, $B(T) = C(T) = \bar{B}(T) = \bar{C}(T) = P(T) = \bar{P}(T) = 0$. For a detailed introduction to $w + \alpha y$, see A et al.[6]. The partial derivatives are

$$V_t = A_t(w + \alpha y) + B_t x^2 + C_t x + P_t, \quad V_w = A, \quad V_x = 2Bx + C, \quad V_y = \alpha A, \tag{3.8}$$

$$V_{xx} = 2B, \quad V_{ww} = V_{wx} = 0, \quad g_{xx} = 2\bar{B}, \quad g_{ww} = g_{wx} = 0, \tag{3.9}$$

$$g_t = \bar{A}_t(w + \alpha y) + \bar{B}_t x^2 + \bar{C}_t x + \bar{P}_t, \quad g_w = \bar{A}, \quad g_x = 2\bar{B}x + \bar{C}, \quad g_y = \alpha\bar{A}, \tag{3.10}$$

where V, g, A, B, C, P, $\bar{A}$, $\bar{B}$, $\bar{C}$ and $\bar{P}$ are short for $V(t,w,x,y)$, $g(t,w,x,y)$, $A(t)$, $B(t)$, $C(t)$, $P(t)$, $\bar{A}(t)$, $\bar{B}(t)$, $\bar{C}(t)$ and $\bar{P}(t)$.

Plugging the above derivatives into (3.5) and simplifying yields

$$\sup_{u \in \Pi} \left\{ A_t(w + \alpha y) + B_t x^2 + C_t x + P_t + A[lw + \mu(\pi_1(t) + \pi_2(t)) + \int_0^\infty ((\theta - \eta)z + \eta\ell(z,t))v(dz) \right.$$
$$+ \mu_m\pi_m(t) - k_1\pi_1(t)x + k_2\pi_2(t)x + \beta_1\mu_{Y_1}\pi_1 + \beta_1\mu_{Y_1}\pi_2 + \bar{b}y + cm] - (2Bx + C)(k_1 + k_2)x$$
$$+ \alpha A(w - \delta y - e^{-\delta h}m) - \frac{\gamma}{2}\bar{A}^2[\sigma^2 + \sigma_1^2\pi_1^2(t) + \sigma_1^2\pi_2^2(t) + \sigma_m^2\pi_m^2(t) + \beta_1\sigma_{Y_1}^2\pi_1^2 + \beta_1\sigma_{Y_1}^2\pi_2^2]$$
$$\left. + 2B\sigma_1^2 - \gamma\sigma_1^2(2\bar{B}x + \bar{C})^2 - \gamma\bar{A}(2\bar{B}x + \bar{C})(\sigma_1^2\pi_1(t) - \sigma_1^2\pi_2(t)) - \int_0^\infty \frac{\gamma}{2}\bar{A}^2\ell^2(z,t)v(dz) \right\}$$
$$= 0. \tag{3.11}$$

Consider the terms involving ℓ in (3.11), that is,

$$\int_0^\infty (A\eta\ell(z,t) - \frac{\gamma}{2}\bar{A}^2\ell^2(z,t))v(dz). \tag{3.12}$$

According to Li et al.[10], if we maximize the integrand in the integral in (3.12) z-by-z for a given $t \in [0, T]$, then we will maximize the integral itself. With respect to ℓ, the graph of $f(\ell) := A\eta\ell - \frac{\gamma}{2}\bar{A}^2\ell^2$, is a concave parabola that increases through the origin $(0, f(0)) = (0, 0)$; by the first-order condition w.r.t. ℓ, we have

$$A\eta - \gamma\bar{A}^2\ell = 0, \tag{3.13}$$

$$\ell^*(z,t) = \frac{\eta A}{\gamma\bar{A}^2} \wedge z. \tag{3.14}$$

By the first-order condition w.r.t. $\pi_m(t)$, $\pi_1(t)$ and $\pi_2(t)$, we have

$$\pi_m^*(t) = \frac{A\mu_m}{\gamma\bar{A}^2\sigma_m^2}, \tag{3.15}$$

$$\pi_1^*(t) = \frac{A\beta_1\mu_{Y_1} + A\mu - \gamma\bar{A}\bar{C}\sigma_1^2}{(\sigma_1^2 + \beta_1\sigma_{Y_1}^2)\gamma\bar{A}^2} - \frac{Ak_1 + 2\gamma\bar{A}\bar{B}\sigma_1^2}{(\sigma_1^2 + \beta_1\sigma_{Y_1}^2)\gamma\bar{A}^2}x, \tag{3.16}$$

$$\pi_2^*(t) = \frac{A\beta_1\mu_{Y_1} + A\mu + \gamma\bar{A}\bar{C}\sigma_1^2}{(\sigma_1^2 + \beta_1\sigma_{Y_1}^2)\gamma\bar{A}^2} + \frac{Ak_2 + 2\gamma\bar{A}\bar{B}\sigma_1^2}{(\sigma_1^2 + \beta_1\sigma_{Y_1}^2)\gamma\bar{A}^2}x, \tag{3.17}$$

we find that the amounts invested in the two stocks, $\pi_1^*(t)$ and $\pi_2^*(t)$, are functions of x. Plugging eqs. (3.14)–(3.17) into eq. (3.11) and eq. (3.3), we have

$$A_t(w + \alpha y) + A[(l + \alpha)w + (\bar{b} - \alpha\delta)y] + A(c - \alpha e^{-\delta h})m + B_t x^2 + C_t x + P_t$$

$$+ A \int_0^\infty ((\theta - \eta)m + \eta \ell^*(z,t))v(dz) - 2B(k_1 + k_2)x^2 - C(k_1 + k_2)x - \frac{\gamma}{2}\bar{A}^2\sigma^2$$

$$- \int_0^\infty \frac{\gamma}{2}\bar{A}^2(\ell^*(z,t))^2 v(dz) - 4\gamma\bar{B}^2\sigma_1^2 x^2 - 4\gamma\bar{B}\bar{C}\sigma_1^2 x - \gamma\sigma_1^2\bar{C}^2 + \frac{A^2\mu_m^2}{2\bar{A}^2\sigma_m^2\gamma}$$

$$+ \frac{(A\beta_1\mu_{Y_1} + A\mu - \gamma\bar{A}\bar{C}\sigma_1^2)^2}{2(\sigma_1^2 + \beta_1\sigma_{Y_1}^2)\gamma\bar{A}^2} + \frac{(Ak_1 + 2\gamma\bar{A}\bar{B}\sigma_1^2)^2 x^2}{2\gamma\bar{A}^2(\sigma_1^2 + \beta_1\sigma_{Y_1}^2)} + \frac{(Ak_2 + 2\gamma\bar{A}\bar{B}\sigma_1^2)^2 x^2}{2\gamma\bar{A}^2(\sigma_1^2 + \beta_1\sigma_{Y_1}^2)}$$

$$- \frac{(A\beta_1\mu_{Y_1} + A\mu - \gamma\bar{A}\bar{C}\sigma_1^2)(Ak_1 + 2\gamma\bar{A}\bar{B}\sigma_1^2)x}{(\sigma_1^2 + \beta_1\sigma_{Y_1}^2)\gamma\bar{A}^2} + \frac{(A\beta_1\mu_{Y_1} + A\mu + \gamma\bar{A}\bar{C}\sigma_1^2)^2}{2(\sigma_1^2 + \beta_1\sigma_{Y_1}^2)\gamma\bar{A}^2}$$

$$+ 2B\sigma_1^2 + \frac{(A\beta_1\mu_{Y_1} + A\mu + \gamma\bar{A}\bar{C}\sigma_1^2)(Ak_2 + 2\gamma\bar{A}\bar{B}\sigma_1^2)x}{(\sigma_1^2 + \beta_1\sigma_{Y_1}^2)\gamma\bar{A}^2} = 0, \tag{3.18}$$

$$\bar{A}_t(w + \alpha y) + \bar{A}[(l + \alpha)w + (\bar{b} - \alpha\delta)y] + \bar{A}(c - \alpha e^{-\delta h})m + \bar{B}_t x^2 + \bar{C}_t x + \bar{P}_t$$

$$+ \bar{A} \int_0^\infty ((\theta - \eta)z + \eta \ell^*(z,t))v(dz) + \frac{A\bar{A}\mu_m^2}{\bar{A}^2\sigma_m^2\gamma} - 2\bar{B}(k_1 + k_2)x^2 - \bar{C}(k_1 + k_2)x$$

$$+ \frac{(A\beta_1\mu_{Y_1} + A\mu + \gamma\bar{A}\bar{C}\sigma_1^2)\bar{A}k_2 x}{(\sigma_1^2 + \beta_1\sigma_{Y_1}^2)\gamma\bar{A}^2} - \frac{(A\beta_1\mu_{Y_1} + A\mu - \gamma\bar{A}\bar{C}\sigma_1^2)\bar{A}k_1 x}{(\sigma_1^2 + \beta_1\sigma_{Y_1}^2)\gamma\bar{A}^2} + 2\bar{B}\sigma_1^2$$

$$+ \frac{2A\bar{A}(\mu + \beta_1\mu_{Y_1})^2}{(\sigma_1^2 + \beta_1\sigma_{Y_1}^2)\gamma\bar{A}^2} + \frac{(\beta_1\mu_{Y_1} + \mu)\bar{A}A(k_2 - k_1)x}{(\sigma_1^2 + \beta_1\sigma_{Y_1}^2)\gamma\bar{A}^2} + \frac{\bar{A}k_1(Ak_1 + 2\gamma\bar{A}\bar{B}\sigma_1^2)x^2}{(\sigma_1^2 + \beta_1\sigma_{Y_1}^2)\gamma\bar{A}^2}$$

$$+ \frac{\bar{A}k_2(Ak_2 + 2\gamma\bar{A}\bar{B}\sigma_1^2)x^2}{(\sigma_1^2 + \beta_1\sigma_{Y_1}^2)\gamma\bar{A}^2} = 0. \tag{3.19}$$

To make the problem solvable, we assume the following conditions on parameters

$$c = \alpha e^{-\delta h}, \tag{3.20}$$

$$\bar{b}e^{-\delta h} = (\delta + l + \alpha)c. \tag{3.21}$$

So, we have $A(c - \alpha e^{-\delta h})m = 0$. In order to obtain the expressions of A, $\bar{A}$, B, $\bar{B}$, C, $\bar{C}$, P and $\bar{P}$, let A_t, A, $\bar{A}_t$ and $\bar{A}$ satisfy the following differential equations

$$A_t(w + \alpha y) + A[(l + \alpha)w + (\bar{b} - \alpha\delta)y] = 0, \tag{3.22}$$

$$\bar{A}_t(w + \alpha y) + \bar{A}[(l + \alpha)w + (\bar{b} - \alpha\delta)y] = 0 \tag{3.23}$$

with boundary condition $A(T) = \bar{A}(T) = 1$. Based on condition (3.21), we have $\bar{b} - \alpha\delta = (l + \alpha)\alpha$. So we have

$$A(t) = e^{(l+\alpha)(T-t)}, \quad \bar{A}(t) = e^{(l+\alpha)(T-t)}. \tag{3.24}$$

By separating the variables with and without x, x^2, we can derive the following equations

$$\bar{B}_t - \bar{B}\frac{2\beta_1\sigma_{Y_1}^2(k_1 + k_2)}{\sigma_1^2 + \beta_1\sigma_{Y_1}^2} + \frac{(k_1^2 + k_2^2)A\bar{A}}{(\sigma_1^2 + \beta_1\sigma_{Y_1}^2)\gamma\bar{A}^2} = 0,$$

$$\bar{C}_t - \bar{C}\frac{(k_1 + k_2)\beta_1\sigma_{Y_1}^2}{\sigma_1^2 + \beta_1\sigma_{Y_1}^2} + \frac{2(\mu + \beta_1\mu_{Y_1})(k_2 - k_1)A\bar{A}}{(\sigma_1^2 + \beta_1\sigma_{Y_1}^2)\gamma\bar{A}^2} = 0,$$

$$B_t - 2B(k_1 + k_2) - \bar{B}^2 \frac{4\gamma\sigma_1^2\beta_1\sigma_{Y_1}^2}{\sigma_1^2 + \beta_1\sigma_{Y_1}^2} + \bar{B}\frac{2(k_1 + k_2)\sigma_1^2 A\bar{A}}{\bar{A}^2(\sigma_1^2 + \beta_1\sigma_{Y_1}^2)} + \frac{(k_1^2 + k_2^2)A^2}{2(\sigma_1^2 + \beta_1\sigma_{Y_1}^2)\gamma\bar{A}^2} = 0,$$

$$C_t - C(k_1 + k_2) - \bar{B}\bar{C}\frac{4\gamma\sigma_1^2\beta_1\sigma_{Y_1}^2}{\sigma_1^2 + \beta_1\sigma_{Y_1}^2} - \frac{A^2(k_1 - k_2)(\mu + \beta_1\mu_{Y_1})}{(\sigma_1^2 + \beta_1\sigma_{Y_1}^2)\gamma\bar{A}^2} + \bar{C}\frac{(k_1 + k_2)\sigma_1^2 A\bar{A}}{(\sigma_1^2 + \beta_1\sigma_{Y_1}^2)\bar{A}^2} = 0,$$

$$\bar{P}_t + \bar{A}\int_0^\infty ((\theta - \eta)z + \eta\ell^*(z,t))v(dz) + 2\bar{B}\sigma_1^2 + \frac{A\bar{A}\mu_m^2}{\bar{A}^2\sigma_m^2\gamma} + \frac{2A\bar{A}(\mu + \beta_1\mu_{Y_1})^2}{(\sigma_1^2 + \beta_1\sigma_{Y_1}^2)\gamma\bar{A}^2} = 0,$$

$$P_t + A\int_0^\infty ((\theta - \eta)z + \eta\ell^*(z,t))v(dz) - \frac{\gamma}{2}\bar{A}^2\sigma^2 - \int_0^\infty \frac{\gamma}{2}\bar{A}^2(\ell^*(z,t))^2 v(dz) - \gamma\sigma_1^2\bar{C}^2$$

$$+ 2B\sigma_1^2 + \frac{A^2\mu_m^2}{2\bar{A}^2\sigma_m^2\gamma} + \frac{(A\beta_1\mu_{Y_1} + A\mu - \gamma\bar{A}\bar{C}\sigma_1^2)^2}{2(\sigma_1^2 + \beta_1\sigma_{Y_1}^2)\gamma\bar{A}^2} + \frac{(A\beta_1\mu_{Y_1} + A\mu + \gamma\bar{A}\bar{C}\sigma_1^2)^2}{2(\sigma_1^2 + \beta_1\sigma_{Y_1}^2)\gamma\bar{A}^2} = 0.$$

With boundary condition $B(T) = \bar{B}(T) = C(T) = \bar{C}(T) = P(T) = \bar{P}(T) = 0$, we have

$$B(t) = -e^{1-2(k_1+k_2)(T-t)}\int_t^T h_1(s)e^{-1+2(k_1+k_2)(T-s)}ds, \tag{3.25}$$

$$C(t) = -e^{1-(k_1+k_2)(T-t)}\int_t^T h_2(s)e^{-1+(k_1+k_2)(T-s)}ds, \tag{3.26}$$

$$\bar{B}(t) = \frac{\epsilon_2}{\epsilon_1}(1 - e^{-\epsilon_1(T-t)}), \quad \bar{C}(t) = \frac{\epsilon_4}{\epsilon_3}(1 - e^{-\epsilon_3(T-t)}), \tag{3.27}$$

$$P(t) = -\left(\frac{\mu_m^2}{2\gamma\sigma_m^2} + \frac{(\mu + \beta_1\mu_{Y_1})^2}{\gamma(\sigma_1^2 + \beta_1\sigma_{Y_1}^2)}\right)(T - t) - \frac{\gamma\sigma^2}{4(l + \alpha)}(1 - e^{2(l+\alpha)(T-t)})$$

$$- \int_t^T (h_3(s) + h_4(s))ds, \tag{3.28}$$

$$\bar{P}(t) = -\left(\frac{\mu_m^2}{\gamma\sigma_m^2} + \frac{2(\mu + \beta_1\mu_{Y_1})^2}{\gamma(\sigma_1^2 + \beta_1\sigma_{Y_1}^2)}\right)(T - t) - \int_t^T 2\sigma^2\bar{B}(s)ds$$

$$- \int_t^T e^{(l+\alpha)(T-s)}\int_0^\infty ((\theta - \eta)z + \eta\ell^*(z,s))v(dz)ds, \tag{3.29}$$

where

$$h_1(s) = \frac{4\gamma\sigma_1^2\beta_1\sigma_{Y_1}^2\bar{B}^2(s)}{\sigma_1^2 + \beta_1\sigma_{Y_1}^2} - \frac{2(k_1 + k_2)\sigma_1^2\bar{B}(s)}{\sigma_1^2 + \beta_1\sigma_{Y_1}^2} - \frac{k_1^2 + k_2^2}{2\gamma(\sigma_1^2 + \beta_1\sigma_{Y_1}^2)}, \tag{3.30}$$

$$h_2(s) = \frac{4\gamma\sigma_1^2\beta_1\sigma_{Y_1}^2}{\sigma_1^2 + \beta_1\sigma_{Y_1}^2}\bar{B}(s)\bar{C}(s) + \bar{C}(s)\frac{(k_1 + k_2)\sigma_1^2}{\gamma(\sigma_1^2 + \beta_1\sigma_{Y_1}^2)} + \frac{(k_1 - k_2)(\mu + \beta_1\mu_{Y_1})}{\gamma(\sigma_1^2 + \beta_1\sigma_{Y_1}^2)}, \tag{3.31}$$

$$h_3(s) = \int_0^\infty \left(e^{(l+\alpha)(T-s)}((\theta - \eta)z + \eta\ell^*(z,s)) - \frac{\gamma}{2}e^{2(l+\alpha)(T-s)}(\ell^*(z,s))^2\right)v(dz), \tag{3.32}$$

$$h_4(s) = \gamma\sigma_1^2\bar{C}(s) - \frac{\gamma\sigma_1^4}{\sigma_1^2 + \beta_1\sigma_{Y_1}^2}\bar{C}^2(s) - 2\sigma_1^2 B(s), \tag{3.33}$$

$$\epsilon_1 = \frac{2\beta_1\sigma_{Y_1}^2(k_1 + k_2)}{\sigma_1^2 + \beta_1\sigma_{Y_1}^2}, \quad \epsilon_2 = \frac{k_1^2 + k_2^2}{\gamma(\sigma_1^2 + \beta_1\sigma_{Y_1}^2)}, \tag{3.34}$$

$$\epsilon_3 = \frac{(k_1 + k_2)\beta_1\sigma_{Y_1}^2}{\sigma_1^2 + \beta_1\sigma_{Y_1}^2}, \quad \epsilon_4 = \frac{2(k_2 - k_1)(\mu + \beta_1\mu_{Y_1})}{\gamma(\sigma_1^2 + \beta_1\sigma_{Y_1}^2)}. \tag{3.35}$$

Theorem 3.2 According to the wealth process (2.10) and the reinsurance-investment problem, the equilibrium value function is

$$V(t,w,x) = e^{(l+\alpha)(T-t)}(w+\alpha y) - x^2 e^{1-2(k_1+k_2)(T-t)} \int_t^T h_1(s) e^{-1+2(k_1+k_2)(T-s)} ds$$

$$- xe^{1-(k_1+k_2)(T-t)} \int_t^T h_2(s) e^{-1+(k_1+k_2)(T-s)} ds - \int_t^T (h_3(s) + h_4(s)) ds$$

$$- \left(\frac{\mu_m^2}{2\gamma\sigma_m^2} + \frac{(\mu+\beta_1\mu_{Y_1})^2}{\gamma(\sigma_1^2+\beta_1\sigma_{Y_1}^2)} \right)(T-t) - \frac{\gamma\sigma^2}{4(l+\alpha)}(1 - e^{2(l+\alpha)(T-t)}), \quad (3.36)$$

where $h_1(s)$, $h_2(s)$, $h_3(s)$ and $h_4(s)$ are given by (3.30)–(3.33). The corresponding equilibrium strategy is given by

$$u^* = (\ell^*(z,t), \pi_m^*(t), \pi_1^*(t), \pi_2^*(t)), \quad (3.37)$$

where $\ell^*(z,t)$ is determined by (3.14), $\pi_m^*(t)$, $\pi_1^*(t)$ and $\pi_2^*(t)$ are given by (3.15)–(3.17).

In the following sections, we analyze two special cases of our model, i.e., without jump and without mispricing, and give the corresponding equilibrium strategies and equilibrium value functions.

Corollary 3.1 (Without Jump) We consider the optimal reinsurance-investment problem in which the price processes of these two stocks are represented by diffusion models. If we don't consider jump risk in our model, under the measure P, the wealth process becomes

$$dW^{\bar{u}}(t) = [lw + \mu_m\pi_m(t) + \mu(\pi_1(t) + \pi_2(t)) + \int_0^\infty ((\theta-\eta)z + (1+\eta)\ell(z,t))v(dz) - k_1\pi_1(t)x$$

$$+ k_2\pi_2(t)x + \bar{b}Y(t) + cM(t)]dt + \sigma dB(t) + \sigma_1\pi_1(t)dZ_1(t) + \sigma_1\pi_2(t)dZ_2(t)$$

$$+ \sigma_m\pi_m(t)dZ_m(t) - \int_0^\infty \ell(z,t)N_1(dz,dt). \quad (3.38)$$

The corresponding optimization problem becomes

$$\sup_{\bar{u}\in\Pi} \left\{ E_{t,w,x,y}[W_T^{\bar{u}}] - \frac{\gamma}{2}Var_{t,w,x,y}[W_T^{\bar{u}}] \right\}. \quad (3.39)$$

Then, by some similar calculations, the equilibrium reinsurance-investment strategy $\bar{u}^* = (\bar{\ell}^*(z,t), \bar{\pi}_m^*(t), \bar{\pi}_1^*(t), \bar{\pi}_2^*(t))$, $t \in [0.T]$, is given by

$$\bar{\ell}^*(z,t) = \frac{\eta A}{\gamma\bar{A}^2} \wedge z = \ell^*(z,t), \quad \bar{\pi}_1^*(t) = \frac{A\mu - \gamma\bar{A}\bar{C}\sigma_1^2}{\sigma_1^2\gamma\bar{A}^2} - \frac{Ak_1 + 2\gamma\bar{A}\bar{B}\sigma_1^2}{\sigma_1^2\gamma\bar{A}^2}x, \quad (3.40)$$

$$\bar{\pi}_m^*(t) = \frac{A\mu_m}{\gamma\bar{A}^2\sigma_m^2} = \pi_m^*(t), \quad \bar{\pi}_2^*(t) = \frac{A\mu + \gamma\bar{A}\bar{C}\sigma_1^2}{\sigma_1^2\gamma\bar{A}^2} + \frac{Ak_2 + 2\gamma\bar{A}\bar{B}\sigma_1^2}{\sigma_1^2\gamma\bar{A}^2}x, \quad (3.41)$$

and the corresponding equilibrium value function $\bar{V}(t,w,x,y)$ is

$$\bar{V}(t,w,x,y) = e^{(l+\alpha)(T-t)}(w+\alpha y) - x^2 e^{1-2(k_1+k_2)(T-t)} \int_t^T \bar{h}_1(s) e^{-1+2(k_1+k_2)(T-s)} ds$$

$$
- xe^{1-(k_1+k_2)(T-t)} \int_t^T \bar{h}_2(s)e^{-1+(k_1+k_2)(T-s)}ds - \left(\frac{\mu_m^2}{2\gamma\sigma_m^2} + \frac{\mu^2}{\gamma\sigma_1^2}\right)(T-t)
$$

$$
- \frac{\gamma\sigma^2}{4(l+\alpha)}(1 - e^{2(l+\alpha)(T-t)}) - \int_t^T (h_3(s) + \bar{h}_4(s))ds, \tag{3.42}
$$

where

$$
\bar{h}_1(s) = -2\bar{B}(s)(k_1 + k_2) - \frac{k_1^2 + k_2^2}{2\gamma\sigma_1^2}, \quad \bar{B}(t) = \frac{k_1^2 + k_2^2}{\gamma\sigma_1^2}(T-t), \tag{3.43}
$$

$$
\bar{h}_4(s) = \gamma\sigma_1^2\bar{C}(s) - \gamma\sigma_1^2\bar{C}^2(s) - 2\sigma_1^2 B(s), \quad \bar{C}(t) = \frac{2(k_2 - k_1)\mu}{\gamma\sigma_1^2}(T-t), \tag{3.44}
$$

$$
\bar{h}_2(s) = \frac{k_1 + k_2}{\gamma}\bar{C}(s) + \frac{(k_1 - k_2)\mu}{\gamma\sigma_1^2}. \tag{3.45}
$$

When Gu et al.[8] ignores mean reversion, we find that the equilibrium investment strategies given in Eqs.(3.40) and (3.41) are similar to that in Gu et al.[8], which considers the robust portfolio selection with the utility maximization.

Corollary 3.2 (Without Mispricing)   In this case, we assume that the insurer ignores the mispricing between stock 1 and stock 2 in the market. If we don't consider mispricing in our model, under the measure P, the wealth process becomes

$$
dW^{\tilde{u}}(t) = [lw + \mu_m\pi_m + \mu(\pi_1 + \pi_2) + \int_0^\infty ((\theta - \eta)z + (1 + \eta)\ell(z,t))v(dz) + \bar{b}y + cm]dt
$$

$$
+ \sigma dB(t) + \sigma_1\pi_1 dZ_1(t) + \sigma_1\pi_2 dZ_2(t) + \sigma_m\pi_m dZ_m(t) - \int_0^\infty \ell(z,t)N_1(dz,dt)
$$

$$
+ \int_{-1}^\infty \pi_1(t)y_1 N_2(dt,dy_1) + \int_{-1}^\infty \pi_2(t)y_1 N_2(dt,dy_1). \tag{3.46}
$$

The corresponding optimization problem becomes

$$
\sup_{\tilde{u}\in\Pi} \left\{ E_{t,w,y}[W_T^{\tilde{u}}] - \frac{\gamma}{2}Var_{t,w,y}[W_T^{\tilde{u}}] \right\}. \tag{3.47}
$$

Then, by some similar calculations, the equilibrium reinsurance-investment strategy $\tilde{u}^* = (\tilde{\ell}^*(z,t), \tilde{\pi}_m^*(t), \tilde{\pi}_1^*(t), \tilde{\pi}_2^*(t))$, $t \in [0.T]$, is given by

$$
\tilde{\ell}^*(z,t) = \frac{\eta A}{\gamma\bar{A}^2} \wedge z = \ell^*(z,t), \quad \tilde{\pi}_m^*(t) = \frac{A\mu_m}{\gamma\bar{A}^2\sigma_m^2} = \pi_m^*(t), \tag{3.48}
$$

$$
\tilde{\pi}_1^*(t) = \frac{A\beta_1\mu_{Y_1} + A\mu}{(\sigma_1^2 + \beta_1\sigma_{Y_1}^2)\gamma\bar{A}^2} = \tilde{\pi}_2^*(t). \tag{3.49}
$$

and the corresponding equilibrium value function $V_2(t,w,y)$ is

$$
\tilde{V}(t,w,y) = e^{(l+\alpha)(T-t)}(w + \alpha y) - \frac{\mu_m^2}{2\gamma\sigma_m^2}(T-t) - \frac{\gamma\sigma^2}{4(l+\alpha)}(1 - e^{2(l+\alpha)(T-t)}) - \int_t^T h_3(s)ds
$$

$$
- \frac{(\beta_1\mu_{Y_1} + \mu)^2}{(\gamma\sigma_1^2 + \gamma\beta_1\sigma_{Y_1}^2)}(T-t). \tag{3.50}
$$

We find that the equilibrium investment strategies given in eqs.(3.48) and (3.49) are similar to that in Zeng et al. [2], if Zeng et al. [2] considers the impact of delay on the optimal strategies.

4 Numerical Simulations

In this section, we supply some numerical examples to explain the effects of model parameters on the equilibrium investment strategy and utility losses from ignoring jump risk and mispricing. We suppose that the jump size Y_{1i} follow exponential distribution with parameter λ_{Y_1}, i.e., the density functions of Y_{1i} is given by $g(y_1) = \lambda_{Y_1} \exp\{-\lambda_{Y_1}(y_1 + 1)\}$, $y_1 \geq -1$. Throughout the numerical analyses, unless otherwise stated, the basic parameters are given by $\beta_1 = \lambda_{Y_1} = 1$, $r = 0.03$, $\mu = 0.05$, $\sigma_1 = 0.3$, $\theta = 0.2$, $\eta = 0.1$, $\gamma = 1$, $h = 0.5$, $\delta = 1.5$ $k_1 = 0.2$, $k_2 = 0.6$, $w = 1$, $T = 4$, $t = 0$.

4.1 Sensitivity Analysis of the Equilibrium Investment Strategy

Figure 1 provides a sensitivity analysis of the mispricing x, the delayed parameter h, jump intensity β_1 and parameter λ_{Y_1} of the jump's distribution function of these two stocks's price processes for the equilibrium investment strategy $\pi_i^*(t)$, $i = 1, 2$.

In parts (a) of Figure 1, we find that $\pi_1^*(t)$ decreases w.r.t. β_1 and increases w.r.t. λ_{Y_1}. This is because when β_1 becomes larger, the intensity of the jump in the first stock's price process becomes stronger and the first stock becomes higher, so the money is invested in the first stock becomes less. At the same time, when λ_{Y_1} becomes larger, the mean and variance of Y_{1i} become smaller. Therefore, under the same risk tolerance, the insurer will invest more in the first stock. In parts (b) of Figure 1, for $\pi_2^*(t)$, the analysis about it is similar to $\pi_1^*(t)$.

In part (d) of Figure 1, we find that when the retention level $\pi_2(t)$ becomes smaller, the delayed horizon h becomes smaller. In part (c) of Figure 1, we find that $\pi_1^*(t)$ decreases w.r.t. x and $\pi_2^*(t)$ increases w.r.t. x. In other words, as mispricing increases, the insurer will reduce their investment in stock 1 and increase their investment in stock 2.

4.2 Sensitivity Analysis of the Utility Loss Functions

In this subsection, we discuss the utility loss that can be caused when jump risks and mispricing are ignored for the insurer.

For equity but without loss of generality, we assume that the appreciation and volatility rates of stocks without jumps are the same as those with jumps, i.e., $\mu_{Y_{1i}}|_{\text{no jump}} = \mu + \beta_1 E[Y_{1i}]$ and $\sigma_{Y_{1i}}^2|_{\text{no jump}} = \sigma^2 + \beta_1 E[(Y_{1i})^2]$.

So, the utility loss that ignores jump risks is defined as

$$H_1(t) = 1 - \frac{\bar{V}(t, w, x, y)}{V(t, w, x, y)}. \tag{4.1}$$

As shown in (a) of Figure 2, we find that the utility loss increases w.r.t. the remaining time $T - t$ and the effect of the remaining time $T - t$ on the utility loss $H_1(t)$ is significant.
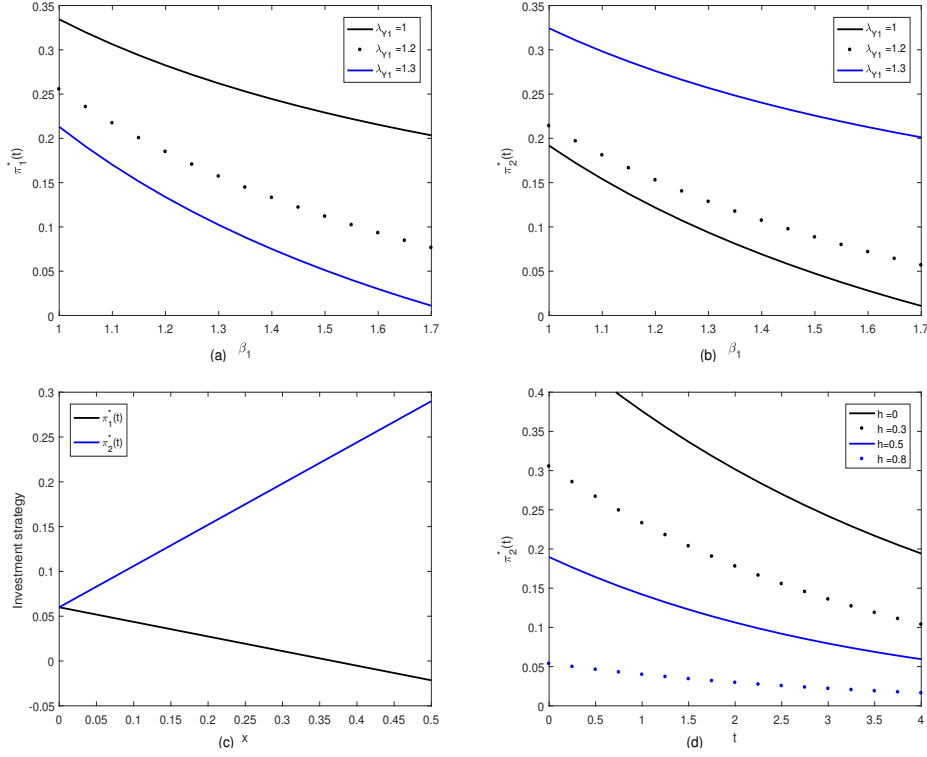
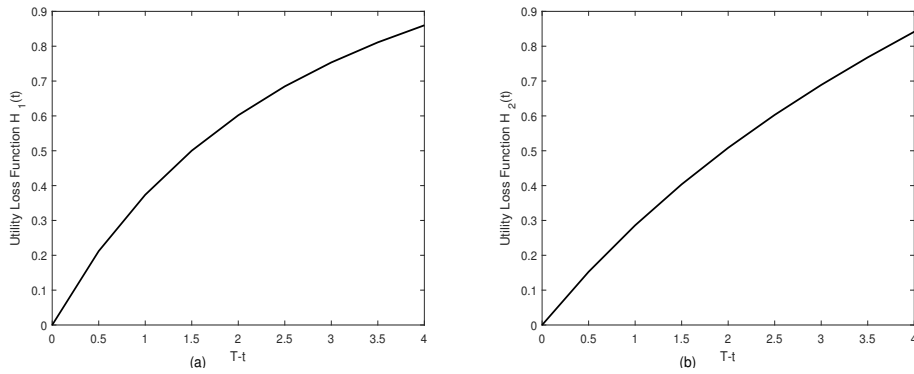Figure 1   The effects of parameters on the equilibrium investment strategy

Figure 2   The effects of parameters on the utility loss functions

Then, we discuss the utility loss that can be caused when the mispricing is ignored for the insurer. The utility loss that ignores mispricing is defined as

$$H_2(t) = 1 - \frac{\tilde{V}(t,w,y)}{V(t,w,y)}. \tag{4.2}$$

From (b) of Figure 2, we can see that when the remaining time $T - t$ increases, the

utility loss $H_2(t)$ will also increase. And when $T - t = 0.3$, we can find that the loss utility is less than 10%, however, when $T - t = 4$, we can find that the loss utility is more than 80%. This means that taking advantage of mispricing is more important for long-horizon investors than that for short-horizon investors.

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在跳跃扩散模型下带延迟和错误定价的超额损失再保险和投资的最优化问题

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摘要: 本文研究了在跳跃扩散模型下带延迟和错误定价的超额损失再保险和投资的最优化问题. 利用随机控制理论, 求解扩展的HJB方程, 推导出均衡再保险投资策略和相应的均衡值函数. 最后, 介绍模型和结果的一些特殊情况, 并为其结果提供了一些数值分析.

关键词: 超额损失再保险; Lévy保险模型; 错误定价; 随机微分延迟方程; 跳跃–扩散模型

MR(2010)主题分类号: 62P05; 91B30; 91B70　　　中图分类号: O211.6; O29