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长相依固定设计下部分线性 EV 模型的小波估计的渐近性质

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摘 要: 本文主要内容是当随机误差为高斯随机变量的函数且长相依时. 利用小波估计的方法来研究固定设计下的部分线性 EV(errors-in-variables) 模型. 在一些合适的条件下, 推广了模型中参数估计量的渐近表示, 以及参数与非参数变量的渐近分布和弱相依速度.

关键词: 长相依; 部分线性 EV 模型; 小波估计; 渐近表示; 弱相依速度
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1 引言

部分线性 EV 模型如下

$$y_k = x_k\beta + g(t_k) + \varepsilon_k, X_k = x_k + u_k, k = 1, 2, \cdots, n, \tag{1.1}$$

这里 $\{y_k, k = 1, 2, \cdots, n\}$ 是观测值, β 是一维未知参数, $g(t)$ 是光滑的曲线, $\{t_k\}$ 为区间 $[0, 1]$ 内的序列. $\{\varepsilon_k, k = 1, 2, \cdots, n\}$ 表示随机误差, 变量 $\{x_k\}$ 不能直接被观测, 只能通过 $X_k = x_k + u_k$ 来观测, 其中 $\{u_k\}$ 是测量误差, 且 $E(u_k) = 0, \mathrm{var}(u_k) = \sigma_1^2$. 假设 $\{u_k\}$ 为已知独立同分布的随机变量, 而且对于每个 k 值, $\{u_k\}$ 与 $\{\varepsilon_k, k = 1, 2, \cdots, n\}$ 都是相互独立的. 假设 $E(\varepsilon_i) = 0$, $\mathrm{var}(\varepsilon_i) = 1$ (即假定已知). 则自协方差函数为

$$\rho(k) = E(\varepsilon_1\varepsilon_{k+1}) = k^{-\theta}L(k), k = 1, 2, \cdots, n, \tag{1.2}$$

这里 $0 < \theta < 1$ 是常数, $L(t), t \in (0, \infty)$ 是正的缓慢变化函数, 即 $\lim\limits_{k\to\infty} \frac{L(t_k)}{L(k)} = 1, t \in (0, \infty)$.

几十年来, 部分线性 EV 模型已经被广泛研究, 文献 [1–4] 用小波估计的方法研究了部分线性回归模型, 在误差序列 $\{\varepsilon_k\}$ 独立同分布时, 得到估计量的大样本性质; 若其误差为鞅差序列, 如参考文献 [5], 则使用近邻估计方法来研究了部分线性 EV 模型. 相依误差的一个重要的特殊情况就是误差为长相依的. 这种情况会出现在经济学, 时间序列分析和其他学科领域的运用中, 见文献 [6–8]. 使用小波估计对半参数模型的研究见文献 [3, 9]. 文献 [10] 是对固定设计下的半参数回归模型使用非参数权函数法和最小二乘法. 在一定的正则性条件下, 文献 [11] 中研究了部分线性模型中参数 β 和函数 $g(\cdot)$ 的估计的弱相合性, 均方相合性和收敛速

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度, 得到了这些估计的渐近表示和渐近分布; 文献 [12] 研究了长相依随机设计下的部分线性 EV 模型, 得到了参数估计量的渐近表示, 渐近分布和弱收敛速度. 本文是在这些文献的基础上, 运用小波估计研究了固定设计下的参数估计量的渐近表示, 渐近分布和弱收敛速度.

2 小波估计

本节中, 我们使用小波方法估计未知参数和非参数. 令 $A_i = [s_{i-1}, s_i]$ 表示区间 $[0,1]$ 的划分区间, 且 $t_i \in A_i, 1 \leqslant i \leqslant n$. 由 (1.1) 式, 得到

$$y_k = X_k\beta + g(t_k) + v_k, v_k = \varepsilon_k + \beta u_k. \tag{2.1}$$

假设 Schwartz 空间 S_l 中存在尺度函数 $\phi(x)$, 在伴随 Hilbert 空间中存在多分辨率分析 V_m, 它的再生核定义为 $E_m(t,s) = 2^m E_0(2^m t, 2^m s) = 2^m \sum\limits_{k\in Z} \phi(2^m t - k)\phi(2^m s - k)$. 将 (2.3) 式看成一般部分线性回归模型. 首先, 假设 β 已知, 定义 $g(t)$ 的估计量为

$$\widetilde{g}_0(t) = \widetilde{g}_0(t,\beta) = \sum_{i=1}^{n}(y_i - X_i\beta)\int_{A_i} E_m(t,s)ds. \tag{2.2}$$

然后, 通过最小化方法来定义小波估计量

$$\widehat{\beta}_n = \arg\min_{\beta}\sum_{i=1}^{n}(y_i - X_i\beta - \widetilde{g}_0(t_i,\beta))^2 = (\sum_{i=1}^{n}\widetilde{X}_i^2)^{-1}\sum_{i=1}^{n}\widetilde{X}_i\widetilde{y}_i,$$

这里 $\tilde{X}_k = X_k - \sum\limits_{j=1}^{n} X_j \displaystyle\int_{A_j} E_m(t_k,s)ds$, $\tilde{y}_k = y_k - \sum\limits_{j=1}^{n} y_j \int_{A_j} E_m(t_k,s)ds$.

由于在线性回归或在部分线性回归中, 由测量误差所导致的不一致性可由“衰减效应”所克服. 因此使用以下修正的最小二乘估计量

$$\widetilde{\beta}_n = (\sum_{i=1}^{n}\widetilde{X}_i^2 - n\sigma_1^2)^{-1}\sum_{i=1}^{n}\widetilde{X}_i\widetilde{y}_i. \tag{2.3}$$

最后, 定义 $g(t)$ 的小波估计量为

$$\widetilde{g}_n(t) = \widetilde{g}_0(t,\widetilde{\beta}_n) = \sum_{i=1}^{n}(y_i - X_i\widetilde{\beta}_n)\int_{A_i} E_m(t,s)ds. \tag{2.4}$$

3 主要结果

为了获得主要结果, 作如下假设条件

(1) 令 $x_j = f(t_j) + \nu_j, j = 1,2,\cdots,n$, 这里 $f(t_j)$ 是区间 $[0,1]$ 中的函数, 对于实数列 $\{\nu_i\}$, 有 $\max\limits_i |\nu_i| = O(n^{-\varphi}), 0 < \varphi < \frac{1}{2}$;

(2) $g(\cdot), f(\cdot) \in H^\alpha$(Sobolev 空间), $\alpha > 1/2$;

(3) $g(\cdot), f(\cdot)$ 都是阶数为 $\gamma > 0$ 的 Lipschitz 函数;

(4) $2^m = O(n^{-1}), 0 < \theta < \min\{2\gamma/\tilde{m}, 2\lambda/\tilde{m}, (1-\lambda)/\tilde{m}\}, 0 < \lambda < 1/2, \tilde{m}$ 的定义见附录.

注 1 条件 (2)–(4) 在小波估计中常被用到 (见文献 [4,10,17,19]). 条件 (1) 是研究部分线性模型时常用到的条件, 但是本文条件与他们有所不同.

$(1)'$ 文献 [9, 10, 13] 均对 $|\nu_i|^p (p \geq 2)$ 施加条件, 本文仅对 $|\nu_i|$ 施加条件;

$(2)'$ 文献 [9, 10, 13] 中对 ν_i 施加的条件较多, 本文对 ν_i 仅有一个条件;

$(3)'$ 虽然文献 [10, 13] 的条件及文献 [9] 中的条件 A1)i) 均可由本文中 ν_i 的条件 (条件 (1)) 推出 (本文中 ν_i 的条件比文献 [10, 13] 的条件及文献 [9] 中的条件 1 强), 但本文中 ν_i 的条件推不出文献 [9] 中的条件 A1)ii). 说明本文 ν_i 条件比文献 [9] 中的条件 A1) ii) 弱.

事实上, 将 $\max_i |\nu_i| = O(n^{-\varphi}), 0 < \varphi < \frac{1}{2}$. 代入文献 [9] 中的 A1) ii) 式, 得

$$\limsup_{n\to\infty} \frac{1}{\sqrt{n}\log n} n^{1-\varphi} = (\log n)^{-1} n^{\frac{1}{2}-\varphi} = \infty.$$

因此文献 [9] 中条件 A1)ii) 不成立.

注 2 与文献 [12] 类似, 文献 [12] 是随机变量, 本文是固定设计.

定理 1 假设条件 (1)–(4) 均成立, 则对任意的 $\alpha \geq 3/2$,

$$\mu_n(\tilde{\beta}_n - \beta) = \lambda_n \frac{J(\tilde{m})}{\tilde{m}!} \sum_{k=1}^{n} u_k H_{\tilde{m}}(e_k) + o_p(1) \tag{3.1}$$

且有

$$\tilde{\beta}_n - \beta = O_p(n^{-\tilde{m}\theta/2} L^{\tilde{m}/2}(n)), n \to \infty, \tag{3.2}$$

这里 $\mu_n = S_n \Gamma_n^{-1}, \lambda_n = (S_n \Gamma_n)^{-1}, \Gamma_n = n^{(1-\tilde{n}\theta)/2} L^{\tilde{m}/2}(n), \tilde{x}_k = x_k - \sum\limits_{j=1}^{n} x_j \int_{A_j} E_m(t_k, s) ds$, $S_n^2 = \sum\limits_{k=1}^{n} \tilde{x}_k^2$. 记号 $J(\cdot)$ 和 $H_{\tilde{m}}(\cdot)$ 见附录.

假设定理 1 的条件及结论成立, 则有以下推论, 其中推论 1 和推论 2 与文献 [12] 中推论相同.

推论 1 若 $\{e_k, k = 1, 2, \cdots\}$ 是独立随机变量, $G(s) = s$, 则对 $E|u_k|^{4+2\delta} < \infty$, $E|H_{\tilde{m}}(e_k)|^{4+2\delta} < \infty, \forall \delta > 0$, 有 $n^{1/2}(\tilde{\beta} - \beta) \xrightarrow{D} N(0, \sigma_1^2 J^2(\tilde{m})/\tilde{m}!)$, $n \to \infty$.

推论 2 在定理 1 的条件下, 若 $u_k = 0$, 则 $n^{1/2}(\tilde{\beta} - \beta) \xrightarrow{D} N(0, \nu_k^2 J^2(\tilde{m})/\tilde{m}!), n \to \infty$.

定理 2 若定理 1 的条件成立, 则对 $\lambda/\tilde{m} < \theta$,

$$n^{(\tilde{m}\theta-\lambda)/2} L^{-\tilde{m}/2} (\tilde{g}_n(t) - g(t)) \xrightarrow{D} C_{\tilde{m},\theta}(t) Z_{\tilde{m}} \tag{3.3}$$

且有

$$\tilde{g}_n(t) - g(t) = O_p(n^{(\lambda-\tilde{m})/2} L^{\tilde{m}/2}(n)),\ n \to \infty, \tag{3.4}$$

这里 $C_{\tilde{m},\theta}(t) = \int_{A_{\tilde{m}}} E_m(t,s) ds C_{\tilde{m},\theta}, C_{\tilde{m},\theta} = (2\Gamma(\theta)\cos(\pi\theta/2))^{-\tilde{m}/2} J(\tilde{m})/\tilde{m}!$.

4 主要结果证明所需的引理

为了证明主要结果, 首先介绍一些有用的结论及引理.

首先介绍 Hermite 秩的表示和 Hermite 级数的一些基本结论. 令 e 表示 $N(0,1)$ 上的随机变量, 且 $\mathbf{G}=\{G:E(G(e))=0,E(G^2(e))<\infty\}$, 则 G 是

$$\{L^2(R.\Phi(s)):\{G:\int_{-\infty}^{+\infty}G^2(s)\Phi(s)ds<\infty\}\}$$

的一个子集, 这里 $\Phi(s)=\exp(-s^2/2)/\sqrt{2\pi}$. Hermite 多项式

$$H_q(s)=(-1)^q\exp(s^2/2)\frac{d^q}{ds^q}\exp(-s^2/2),s\in R,q=0,1,2,\cdots$$

构成了 $L^2(R,\Phi(s))$ 中函数的完整正交系统, 并满足 $E(H_l(e)H_q(e))=\delta_{lq}q!$.

对任意的 $G\in\mathbf{G}$, 令 $J(q)=EG(e)H_q(e)$. 定义 $\tilde{m}=\min\limits_{q\geq0}\{q:J(q)\neq0\}$, 且为 G 的 Hermite 秩. 由于 $J(0)=E(G)=0$, 故 $\tilde{m}$ 通常是正数.

级数 $\sum\limits_{q=0}^{n}J(q)/q!\cdot H_q(s)$ 收敛到 $L^2(R,\Phi(s))$ 中的 $G(s)$ (见文献 [17]), 即秩为 $\tilde{m}$ 的 $G(s)$ 的 Hermite 展式为 $\sum\limits_{q=0}^{n}J(q)/q!\cdot H_q(s)$.

令 $\{e_k,k=1,2,\cdots,n\}$ 和 $\rho(k)$ 如上所述, $\tilde{m}>0$ 为固定常数, 则以下结果成立 [18].

(F_1) $\sum\limits_{i=1}^{n}\sum\limits_{j=1}^{n}|\rho(i-j)|^{\tilde{m}}=\begin{cases}O(n^{2-\tilde{m}\theta}L^{\tilde{m}}(n)), & \tilde{m}\theta<1,\\ O(nL_o(n)), & \tilde{m}\theta=1,\\ O(n), & \tilde{m}\theta>1,\end{cases}$ 其中 $L_o(n)$ 是缓慢变化函数.

(F_2) 对任意 Hermite 秩为 $\tilde{m}<1/\theta$, 可测函数 $G\in L^2(R,\Phi(s))$,

$$\mathrm{Var}(\sum_{i=1}^{n}G(e_i))\approx\sum_{i=1}^{n}\sum_{j=1}^{n}|\rho(i-j)|^{\tilde{m}}\approx n^{2-\tilde{m}\theta}L^{\tilde{m}}(n).$$

特别的,

$$\mathrm{Var}(\sum_{i=1}^{n}\tilde{G}(e_i))\approx\sum_{i=1}^{n}\sum_{j=1}^{n}a_ia_j\rho^{\tilde{m}}(i-j)\approx\max_{1\leq i\leq n}a_i^2n^{2-\tilde{m}\theta}L^{\tilde{m}}(n).$$

(F_3) 对任意缓慢变化函数 $L(n)$ 而言, 有 $n^{\kappa}L(n)\stackrel{n\to\infty}{\longrightarrow}\begin{cases}\infty, & \kappa>0,\\ 0, & \kappa<0.\end{cases}$

(F_4) $\{H_q(e_i)\}$ 是 $L^2(R,\Phi(s))$ 中的一列随机变量, 且满足

$$H_o(x)=1,EH_q(e)=0(q\geq1),E(H_q(e_j)H_q(e_k))=q!\rho^q(j-k),\forall j,k.$$

引理 1 [3] 假设条件 (3) 成立, 则

(1) $\sup\limits_t|E_m(t,s)|=O(2^m)$; (2) $\sup\limits_t\int_0^1|E_m(t,s)|ds\leq C$; (3) $\int_0^1|E_m(t,s)|ds\leq1,n\to\infty$.

引理 2 假设条件 (1) 和 (3) 成立, 则 x_i 有界, $\tilde{x_i}$ 有界, 且 $S_n^2=\sum\limits_{i=1}^{n}\tilde{x}_i^2=O(n)$.

证 由于 $\{\nu_i\}$ 是实数列, $f(\cdot)$ 是阶数为 $\gamma>0$ 的 Lipschitz 函数, 故 $x_j=f(t_j)+\nu_j$, $j=1,2,\cdots,n$ 有界. 由 $\tilde{x}_i=x_i-\sum\limits_{j=1}^n x_j\int_{A_j}E_m(t_i,s)ds$ 和引理 1 (2), 有

$$\begin{aligned}|\tilde{x_i}|&=|x_i-\sum_{j=1}^n x_j\int_{A_j}E_m(t_i,s)ds|\le|x_i|+|\sum_{j=1}^n x_j\int_{A_j}E_m(t_i,s)ds|\\&\le|x_i|+\sum_{j=1}^n|x_j|\int_{A_j}|E_m(t_i,s)|ds\le|x_i|+\max_j|x_j|\sum_{j=1}^n\int_{A_j}|E_m(t_i,s)|ds\\&=|x_i|+\max_j|x_j|\int_0^1|E_m(t_i,s)|ds\le|x_i|+C\max_j|x_j|.\end{aligned}$$

即 $\tilde{x}_i$ 有界, 得证.

注意到 $S_n^2=\sum\limits_{i=1}^n\tilde{x}_i^2=O(n)$ 是显然的.

引理 3 令 $B_n=S_n^2(\sum\limits_{i=1}^n\tilde{X}_i{}^2-n\sigma_1^2)^{-1}$, 则有 $B_n\xrightarrow{p}1,n\to\infty$.

证 为了证明此引理, 只需证明 $\sum\limits_{i=1}^n\tilde{X}_i^2-n\sigma_1^2-\sum\limits_{i=1}^n\tilde{x}_i^2\xrightarrow{p}0\ \ (n\to\infty)$. 注意到

$$\sum_{i=1}^n\tilde{X}_i^2-n\sigma_1^2-\sum_{i=1}^n\tilde{x}_i^2=\sum_{i=1}^n(\tilde{u}_i^2-\sigma_1^2)+2\sum_{i=1}^n\tilde{x}_i\tilde{u}_i=D_1+2D_2.$$

由 C_r 不等式及引理 2, 有

$$\begin{aligned}ED_2^2&=E(\sum_{i=1}^n\tilde{x}_i\tilde{u}_i)^2=E(\sum_{i=1}^n\tilde{x}_i(u_i-\sum_{j=1}^n\int_{A_j}E_m(t_i,s)ds))^2\\&\le2\sum_{i=1}^n\tilde{x}_i^2Eu_i^2+2\sum_{i=1}^n\tilde{x}_i^2E(\sum_{j=1}^n u_j\int_{A_j}E_m(t_i,s)ds)^2\\&\le CS_n^2+C\sum_{i=1}^n\tilde{x}_i^2\sum_{j=1}^nEu_j^2(\int_{A_j}E_m(t_i,s)ds)^2\\&\le CS_n^2+CS_n^2n^{-1}2^m\le CS_n^2=O(n),\end{aligned}$$

$$\begin{aligned}ED_1^2=&E(\sum_{i=1}^n(u_i^2-\sigma_1^2)-2\sum_{i=1}^n u_i\sum_{j=1}^n u_j\int_{A_j}E_m(t_i,s)ds+\sum_{i=1}^n(\sum_{j=1}^n u_j\int_{A_j}E_m(t_i,s)ds)^2)^2\\\le&CE(\sum_{i=1}^n(u_i^2-\sigma_1^2))^2+CE(\sum_{i=1}^n u_i\sum_{j=1}^n u_j\int_{A_j}E_m(t_i,s)ds)^2\\&+CE(\sum_{i=1}^n(\sum_{j=1}^n u_j\int_{A_j}E_m(t_i,s)ds)^2)^2\\=&D_{11}+D_{12}+D_{13}.\end{aligned}$$

由 u_i 的独立性得

$$D_{11}=CE(\sum_{i=1}^n(u_i^2-\sigma_1^2))^2\le Cn.$$

由引理 1 及 u_i 的独立性得

$$\begin{aligned}
D_{12} =&CE(\sum_{1\leq i,k\leq n} u_iu_k(\sum_{j=1}^{n} u_j\int_{A_j} E_m(t_i,s)ds)(\sum_{j=1}^{n} u_j\int_{A_j} E_m(t_i,s)ds))\\
=&C\sum_{i=1}^{n} E(u_i^2\sum_{j=1}^{n} u_j^2\int_{A_j} E_m(t_i,s)ds^2)\\
&+C\sum_{i\neq k} E(u_iu_k\sum_{j\neq l} u_ju_l\int_{A_j} E_m(t_i,s)ds\int_{A_l} E_m(t_k,s)ds)\\
=&C\sum_{i=1}^{n}\sum_{j=1}^{n}\sigma_1^4(\int_{A_j} E_m(t_i,s)ds)^2\\
&+C\sum_{i\neq k} E(u_i^2u_k^2\int_{A_i} E_m(t_i,s)ds\int_{A_k} E_m(t_k,s)ds)\\
\leq&Cnn^{-1}2^m+Cnn^{-1}2^m=C2^m=Cn^{\lambda},\\
D_{13} =&CE(\sum_{1\leq i,k\leq n}(\sum_{j=1}^{n} u_j\int_{A_j} E_m(t_i,s)ds)^2(\sum_{j=1}^{n} u_j\int_{A_j} E_m(t_k,s)ds)^2)\\
\leq&C\sum_{1\leq i,k\leq n}(\sum_{j=1}^{n} Eu_j^4(\int_{A_j} E_m(t_i,s)ds)^2(\int_{A_j} E_m(t_k,s))^2\\
&+\sum_{j=1}^{n}\sigma^4(\int_{A_j} E_m(t_i,s)ds)^2(\int_{A_j} E_m(t_k,s)ds)^2)\\
\leq&C\sum_{1\leq i,k\leq n}(n^{-1}2^m)^3=n^{-1}2^{3m}=n^{3\lambda-1},
\end{aligned}$$

故 $ED_1^2=O(n)$.

综上, 由 Chebyshev 不等式可得 $B_n=S_n^2(\sum\limits_{i=1}^{n}\tilde{X}_i^{\,2}-n\sigma_1^2)^{-1}\xrightarrow{p}1, n\to\infty$.

引理 4[14] 假设条件 (1)–(4) 成立, 则

$$\sup_t|f(t)-\sum_{k=1}^{n}(\int_{A_k} E_m(t,s)ds)f(t_k)|=O(n^{-\gamma})+O(\tau_m),$$

且有 $\sup\limits_t|g(t)-\sum\limits_{k=1}^{n}(\int_{A_k} E_m(t,s)ds)g(t_k)|=O(n^{-\gamma})+O(\tau_m)$, 其中

$$\tau_m=\begin{cases}2^{-m(\theta-1/2)}, & 1/2<\theta<3/2,\\ \sqrt{m}\cdot 2^{-m}, & \theta=3/2,\\ 2^{-m}, & \theta>3/2.\end{cases}$$

引理 5[11] 假设条件 (4) 成立, 则对每个实数 $a_1,a_2,\cdots,a_n$, 下列分解成立

$$\sum_{j=1}^{n} a_j\varepsilon_j=\frac{J(\tilde{m})}{\tilde{m}!}\sum_{j=1}^{n} a_jH_{\tilde{m}}(e_j)+\sum_{j=1}^{n} a_j(G(e_j)-\frac{J(\tilde{m})}{\tilde{m}!}H_{\tilde{m}}(e_j)):=T_{l1}+T_{l2},$$

这里 T_{l1},T_{l2} 满足

$$ET_{l1}^2\le\max_{1\le j\le n}|a_j|^2O(n^{2-\tilde m\theta}L^{\tilde m}(n)),\ ET_{l2}^2\le\max_{1\le j\le n}|a_j|^2O(n^{2-\tilde m\theta}L^{\tilde m}(n)).$$

引理 6[13] 令 $\tilde m$ 表示函数 $G\in\mathbf{G}$ 的 Hermite 秩, a_i 表示有界的非负实数, 则 $\tilde G(e_i)=a_iG(e_i)\in\mathbf{G}$, 它的 Hermite 秩也是 $\tilde m$, 且 $\tilde G(e_i)=\sum\limits_{q=\tilde m}^{\infty}\frac{J(\tilde q)}{\tilde q!}\cdot H_q(e_i),\tilde J(q)=a_iJ(q)$.

引理 7[15] 若 $G(e_i)\in\mathbf{G}$ 的 Hermite 秩是 $\tilde m$, 则

$$\sum_{j=1}^n G(e_j)/(\mathrm{Var}(\sum_{j=1}^n G(e_j)))^{1/2}\xrightarrow{D}C_{\tilde m,\theta}Z_{\tilde m}\quad(n\to\infty),$$

这里 $C_{\tilde m,\theta}=(2\Gamma(\theta)\cos(\pi\theta/2))^{-\tilde m/2}J(\tilde m)/\tilde m!$,

$$Z_m=\int\cdots\int\exp(i\sum_{j=1}^{\tilde m}t_j)((\exp(i\sum_{j=1}^{\tilde m}t_j)-1)/i\sum_{j=1}^{\tilde m}t_j)\prod_{j=1}^{\tilde m}|t_j|^{(\theta-1)/2}\cdot dW(t_1)\cdots dW(t_{\tilde m}).$$

5 主要结果的证明

定理 1 的证明 对 $\mu(\tilde\beta_n-\beta)$, 由引理 2 进行如下分解

$$\begin{aligned}
\mu(\tilde\beta_n-\beta)=&\lambda_n(S_n^2(\sum_{k=1}^n\tilde X_k^2-n\sigma_1^2)^{-1}\sum_{k=1}^n\tilde X_ky_k-S_n^2\beta)\\
=&\lambda_n(\sum_{k=1}^n\tilde X_ky_k-S_n^2\beta)\\
=&\lambda_n(\sum_{k=1}^n\tilde x_k\varepsilon_k-\sum_{k=1}^n\tilde x_k\bar\varepsilon_k+\sum_{k=1}^n\tilde x_k\tilde g_k+\sum_{k=1}^n\tilde u_k\varepsilon_k)\\
&+\lambda_n(-\sum_{k=1}^n\tilde u_k\bar\varepsilon_k+\sum_{k=1}^n\tilde u_k\tilde g_k+\beta\sum_{k=1}^n\tilde x_k\tilde u_k)\\
=&\lambda_n\frac{J(\tilde m)}{\tilde m!}\sum_{k=1}^n u_kH_{\tilde m}(e_k)+\lambda_n\frac{J(\tilde m)}{\tilde m!}\sum_{k=1}^n\nu_kH_{\tilde m}(e_k)\\
&+\lambda_n\frac{J(\tilde m)}{\tilde m!}\sum_{k=1}^n\tilde f_kH_{\tilde m}(e_k)+\lambda_n\sum_{k=1}^n\tilde x_k(\varepsilon_k-\frac{J(\tilde m)}{\tilde m!}H_{\tilde m}(e_k))\\
&-\lambda_n\sum_{k=1}^n\tilde x_k\bar\varepsilon_k+\lambda_n\sum_{k=1}^n\tilde x_k\tilde g_k-\lambda_n\frac{J(\tilde m)}{\tilde m!}\sum_{k=1}^n\bar u_kH_{\tilde m}(e_k)\\
&+\lambda_n\sum_{k=1}^n\tilde u_k(\varepsilon_k-\frac{J(\tilde m)}{\tilde m!}H_{\tilde m}(e_k))\\
&+\lambda_n(-\sum_{k=1}^n\tilde u_k\bar\varepsilon_k+\sum_{k=1}^n\tilde u_k\tilde g_k+\beta\sum_{k=1}^n\tilde x_k\tilde u_k)\\
:=&\lambda_n\frac{J(\tilde m)}{\tilde m!}\sum_{k=1}^n u_kH_{\tilde m}(e_k)+\sum_{l=1}^{10}T_{nl}+o_p(1).
\end{aligned}$$

由引理 6 及 (F4) 有

$$
\begin{aligned}
ET_{n1}^2 &= E(\lambda_n \frac{J(\tilde{m})}{\tilde{m}!}\sum_{k=1}^{n}\nu_k H_{\tilde{m}}(e_k))^2 = n^{\tilde{m}\theta-2}L^{-\tilde{m}}(n)\frac{J^2(\tilde{m})}{(\tilde{m}!)^2}(\sum_{i=1}^{n}\sum_{j=1}^{n}\nu_i\nu_j E(H_{\tilde{m}}(e_i)H_{\tilde{m}}(e_j)))\\
&\le n^{\tilde{m}\theta-2}L^{-\tilde{m}}(n)\frac{J^2(\tilde{m})}{(\tilde{m}!)^2}\sum_{i=1}^{n}\sum_{j=1}^{n}|\nu_i\nu_j||E(H_{\tilde{m}}(e_i)H_{\tilde{m}}(e_j))|\\
&\le n^{\tilde{m}\theta-2}L^{-\tilde{m}}(n)\frac{J^2(\tilde{m})}{\tilde{m}!}\max_{i,j}|\nu_i\nu_j|\sum_{i=1}^{n}\sum_{j=1}^{n}|\rho(i-j)|^{\tilde{m}}\\
&= n^{\tilde{m}\theta-2}L^{-\tilde{m}}(n)\frac{J^2(\tilde{m})}{\tilde{m}!}o(1)n^{2-\tilde{m}\theta}L^{\tilde{m}}(n) = o_p(1). \qquad (5.1)
\end{aligned}
$$

由引理 2 及 (F4) 有

$$
\begin{aligned}
ET_{n2}^2 &= E(\lambda_n \frac{J(\tilde{m})}{\tilde{m}!}\sum_{k=1}^{n}\tilde{f}_k H_{\tilde{m}}(e_k))^2 = n^{\tilde{m}\theta-2}L^{-\tilde{m}}(n)\frac{J^2(\tilde{m})}{(\tilde{m}!)^2}(\sum_{k=1}^{n}\sum_{j=1}^{n}\tilde{f}_k\tilde{f}_j E(H_{\tilde{m}}(e_k)H_{\tilde{m}}(e_j)))\\
&\le n^{\tilde{m}\theta-2}L^{-\tilde{m}}(n)\frac{J^2(\tilde{m})}{(\tilde{m}!)^2}\sum_{k=1}^{n}\sum_{j=1}^{n}|\tilde{f}_k\tilde{f}_j||E(H_{\tilde{m}}(e_i)H_{\tilde{m}}(e_j))|\\
&\le n^{\tilde{m}\theta-2}L^{-\tilde{m}}(n)\frac{J^2(\tilde{m})}{\tilde{m}!}\sup_{k,j}|\tilde{f}_k\tilde{f}_j|\sum_{k=1}^{n}\sum_{j=1}^{n}|\rho(i-j)|^{\tilde{m}}\\
&= n^{\tilde{m}\theta-2}L^{-\tilde{m}}(n)\frac{J^2(\tilde{m})}{\tilde{m}!}(O(n^{-\gamma})+O(\tau_m))n^{2-\tilde{m}\theta}L^{\tilde{m}}(n) = o_p(1). \qquad (5.2)
\end{aligned}
$$

对 T_{n3} 求二阶矩

$$
\begin{aligned}
ET_{n3}^2 &= E(\lambda_n\sum_{k=1}^{n}\tilde{x}_k\tilde{G}(e_k))^2 = E(\lambda_n\sum_{k=1}^{n}(\tilde{f}_k+\nu_k)\tilde{G}(e_k))^2\\
&\le C(E(\lambda_n^2(\sum_{k=1}^{n}\tilde{f}_k\tilde{G}(e_k)))^2 + E(\lambda_n^2(\sum_{k=1}^{n}\nu_k\tilde{G}(e_k))^2))\\
&:= T_{n3}^{(1)} + T_{n3}^{(2)},
\end{aligned}
$$

由引理 2 及 (F2) 有

$$
\begin{aligned}
T_{n3}^{(1)} &= CE(\lambda_n\sum_{k=1}^{n}\tilde{f}_k\tilde{G}(e_k))^2\\
&= Cn^{\tilde{m}\theta-2}L^{-\tilde{m}}(n)E(\sum_{k=1}^{n}\sum_{j=1}^{n}\tilde{f}_k\tilde{f}_j\tilde{G}(e_k)\tilde{G}(e_j))\\
&\le Cn^{\tilde{m}\theta-2}L^{-\tilde{m}}(n)\sup_{k,j}|\tilde{f}_k\tilde{f}_j|E(\sum_{k=1}^{n}\sum_{j=1}^{n}\tilde{G}(e_k)\tilde{G}(e_j))\\
&\le Cn^{\tilde{m}\theta-2}L^{-\tilde{m}}(n)\frac{J^2(\tilde{m})}{\tilde{m}!}(O(n^{-2\gamma})+O(\tau_m^2))Var(\sum_{j=1}^{n}\tilde{G}(e_j))\\
&\le Cn^{\tilde{m}\theta-2}L^{-\tilde{m}}(n)\frac{J^2(\tilde{m})}{\tilde{m}!}(O(n^{-2\gamma})+O(\tau_m^2))n^{2-\tilde{m}\theta}L^{\tilde{m}}(n) = o_p(1). \qquad (5.3)
\end{aligned}
$$

由引理 6 及 (F2) 有

$$\begin{aligned}
T_{n3}^{(2)} &\leq CE(\lambda_n^2)E(\sum_{k=1}^{n}\nu_k\tilde{G}(e_k))^2 = n^{\tilde{m}\theta-2}L^{-\tilde{m}}(n)E(\sum_{j=1}^{n}\sum_{k=1}^{n}\nu_j\nu_k\tilde{G}(e_j)\tilde{G}(e_k)) \\
&\leq n^{\tilde{m}\theta-2}L^{-\tilde{m}}(n)\max_{j,k}|\nu_j\nu_k|E(\sum_{j=1}^{n}\sum_{k=1}^{n}\tilde{G}(e_j)\tilde{G}(e_k)) \\
&\leq n^{\tilde{m}\theta-2}L^{-\tilde{m}}(n)\max_{j,k}|\nu_j\nu_k|\mathrm{Var}(\sum_{j=1}^{n}\tilde{G}(e_j)) \\
&\leq n^{\tilde{m}\theta-2}L^{-\tilde{m}}(n)o(1)n^{2-\tilde{m}\theta}L^{\tilde{m}}(n) = o_p(1). \qquad (5.4)
\end{aligned}$$

由 (5.3)–(5.4) 式可得

$$ET_{n3}^2 \leq o_p(1). \qquad (5.5)$$

对 T_{n4} 求二阶矩

$$\begin{aligned}
ET_{n4}^2 &= E(\lambda_n\sum_{k=1}^{n}\tilde{x}_k\bar{\varepsilon}_k)^2 = E(\lambda_n\sum_{k=1}^{n}(\tilde{f}_k+\nu_k)\bar{\varepsilon}_k)^2 \\
&= E(\lambda_n^2(\sum_{k=1}^{n}\tilde{f}_k\bar{\varepsilon}_k)^2 + \lambda_n^2(\sum_{k=1}^{n}\nu_k\bar{\varepsilon}_k)^2) := T_{n4}^{(1)} + T_{n4}^{(2)}.
\end{aligned}$$

由引理 2 及 (F2) 有

$$\begin{aligned}
T_{n4}^{(1)} &\leq n^{\tilde{m}\theta-2}L^{-\tilde{m}}(n)\sup_k|\tilde{f}_k|^2E(\sum_{k=1}^{n}\bar{\varepsilon}_k^2) \\
&\leq n^{\tilde{m}\theta-2}L^{-\tilde{m}}(n)\sup_k|\tilde{f}_k|^2 n\max_k E(\sum_{k=1}^{n}\int_{A_j}E_m(t_k,s)dsG(e_j))^2 \\
&\leq n^{\tilde{m}\theta-2}L^{-\tilde{m}}(n)\sup_k|\tilde{f}_k|^2 nn^{2\lambda-2}\max_k E(\sum_{k=1}^{n}\sum_{j=1}^{n}G(e_k)G(e_j)) \\
&= n^{2\lambda-3+\tilde{m}\theta}L^{-\tilde{m}}(n)(O(n^{-2\gamma})+O(\tau_m^2))\mathrm{Var}(\sum_{k=1}^{n}G(e_k)) = n^{2\lambda-1}o(1) = o_p(1). \qquad (5.6)
\end{aligned}$$

由引理 6 及 (F2) 有

$$\begin{aligned}
T_{n4}^{(2)} &\leq n^{\tilde{m}\theta-2}L^{-\tilde{m}}(n)\max_{k,j}|\nu_k\nu_j|E(\sum_{k=1}^{n}\bar{\varepsilon}_k^2) \\
&\leq n^{\tilde{m}\theta-2}L^{-\tilde{m}}(n)\max_{k,j}|\nu_k\nu_j|n\max_k E(\sum_{k=1}^{n}\int_{A_j}E_m(t_k,s)dsG(e_j))^2 \\
&\leq n^{\tilde{m}\theta-2}L^{-\tilde{m}}(n)\max_{k,j}|\nu_k\nu_j|nn^{2\lambda-2}\max_k E(\sum_{k=1}^{n}\sum_{j=1}^{n}G(e_k)G(e_j)) \\
&= n^{2\lambda-3+\tilde{m}\theta}L^{-\tilde{m}}(n)o(1)\mathrm{Var}(\sum_{k=1}^{n}G(e_k)) = n^{2\lambda-1}o(1) = o_p(1). \qquad (5.7)
\end{aligned}$$

由 (5.6)–(5.7) 式可得

$$ET_{n4}^2 \le o_p(1). \tag{5.8}$$

对 T_{n5} 求二阶矩

$$\begin{aligned}ET_{n5}^2 &= E(\lambda_n \sum_{k=1}^n \tilde{x}_k \tilde{g}_k)^2 = E(\lambda_n \sum_{k=1}^n (\tilde{f}_k + \nu_k)\tilde{g}_k)^2 = E(\lambda_n^2 (\sum_{k=1}^n \tilde{f}_k \tilde{g}_k)^2) + E(\lambda_n^2 (\sum_{k=1}^n \nu_k \tilde{g}_k)^2)\\ &:= T_{n5}^{(1)} + T_{n5}^{(2)}.\end{aligned}$$

由引理 2 有

$$\begin{aligned}T_{n5}^{(1)} &\le n^{\tilde{m}\theta-2} L^{-\tilde{m}}(n) \sum_{k=1}^n \sum_{j=1}^n \sup_{k,j} |\tilde{f}_k \tilde{f}_j| \sup_{k,j} |\tilde{g}_k \tilde{g}_j|\\ &= n^{\tilde{m}\theta-2} L^{-\tilde{m}}(n) n^2 (O(n^{-2\gamma}) + O(\tau_m^2))^2\\ &= n^{\tilde{m}\theta-4\gamma} L^{-\tilde{m}}(n) = o_p(1).\end{aligned} \tag{5.9}$$

由引理 2 及引理 6 有

$$\begin{aligned}T_{n5}^{(1)} &\le n^{\tilde{m}\theta-2} L^{-\tilde{m}}(n) \sum_{k=1}^n \sum_{j=1}^n \max_{k,j} |\nu_k \nu_j| \sup_{k,j} |\tilde{g}_k \tilde{g}_j|\\ &= n^{\tilde{m}\theta-2} L^{-\tilde{m}}(n) n^2 o(1) (O(n^{-2\gamma}) + O(\tau_m^2)) = o_p(1).\end{aligned} \tag{5.10}$$

由 (5.9)–(5.10) 式可得

$$ET_{n5}^2 \le o_p(1). \tag{5.11}$$

由 (F4) 有

$$\begin{aligned}ET_{n6}^2 &= E(\lambda_n \frac{J(\tilde{m})}{\tilde{m}!} \sum_{k=1}^n \bar{u}_k H_{\tilde{m}}(e_k))^2\\ &= \frac{J^2(\tilde{m})}{(\tilde{m}!)^2} n^{\tilde{m}\theta-2} L^{-\tilde{m}}(n) E(\sum_{k=1}^n \sum_{j=1}^n \bar{u}_k \bar{u}_j H_{\tilde{m}}(e_k) H_{\tilde{m}}(e_j))\\ &\le \frac{J^2(\tilde{m})}{(\tilde{m}!)^2} n^{\tilde{m}\theta-2} L^{-\tilde{m}}(n) \sum_{1\le j,k\le n} (E(\bar{u}_k \bar{u}_j) \tilde{m}! \rho^{\tilde{m}}(j-k))\\ &\le \frac{J^2(\tilde{m})}{\tilde{m}!} n^{\tilde{m}\theta-2} L^{-\tilde{m}}(n) \max_{k,j} |E(\bar{u}_k \bar{u}_j)| \sum_{1\le j,k\le n} \rho^{\tilde{m}}(j-k),\end{aligned}$$

$$\begin{aligned}E(\bar{u}_k \bar{u}_j)^2 &\le (\sup_s |E_m(t_k,s)|)^4 E((n^{-1} \sum_{i=1}^n u_i)^2 (n^{-1} \sum_{l=1}^n u_l)^2)\\ &\le (\sup_s |E_m(t_k,s)|)^4 E(n^{-1} \sum_{i=1}^n u_i)^4\\ &\le C(2^{2m} n^{-1})^2 (n^{-1} \sup_i E^4 u_i) = o_p(1),\end{aligned} \tag{5.12}$$

$$ET_{n6}^2 \le \frac{J^2(\tilde{m})}{\tilde{m}!} n^{\tilde{m}\theta-1} L^{-\tilde{m}}(n) n^{-1} o_p(1) n^{2-\tilde{m}\theta} L^{\tilde{m}}(n) = o_p(1). \tag{5.13}$$

对 T_{n7} 求二阶矩

$$
\begin{aligned}
ET_{n7}^2 &= E(\lambda_n \sum_{k=1}^{n} \tilde{u}_k \tilde{G}(e_k))^2 = E(\lambda_n \sum_{k=1}^{n} (u_k - \bar{u}_k) \tilde{G}(e_k))^2 \\
&\le CE(\lambda_n^2 (\sum_{k=1}^{n} u_k \tilde{G}(e_k))^2) + CE(\lambda_n^2 (\sum_{k=1}^{n} \bar{u}_k \tilde{G}(e_k))^2) \\
&:= T_{n7}^{(1)} + T_{n7}^{(2)}.
\end{aligned}
$$

由 (F2) 有

$$
\begin{aligned}
T_{n7}^{(1)} &= CE(\lambda_n^2 (\sum_{k=1}^{n} u_k \tilde{G}(e_k))^2) = Cn^{\tilde{m}\theta-2} L^{-\tilde{m}}(n) E(\sum_{k=1}^{n} \sum_{j=1}^{n} u_k u_j \tilde{G}(e_k) \tilde{G}(e_j)) \\
&= Cn^{\tilde{m}\theta-2} L^{-\tilde{m}}(n) n^{-1} \max_j E(u_j^2) E(\sum_{k=1}^{n} \tilde{G}(e_k))^2 \\
&\le Cn^{\tilde{m}\theta-2} L^{-\tilde{m}}(n) n^{-1} n^{2-\tilde{m}\theta} L^{\tilde{m}}(n) = o_p(1).
\end{aligned} \tag{5.14}
$$

由 (5.12) 式及 (F2) 有

$$
\begin{aligned}
T_{n7}^{(2)} &= CE(\lambda_n^2 (\sum_{k=1}^{n} \bar{u}_k \tilde{G}(e_k))^2) = Cn^{\tilde{m}\theta-2} L^{-\tilde{m}}(n) E(\sum_{k=1}^{n} \sum_{j=1}^{n} \bar{u}_k \bar{u}_j \tilde{G}(e_k) \tilde{G}(e_j)) \\
&= Cn^{\tilde{m}\theta-2} L^{-\tilde{m}}(n) \max_j E(\bar{u}_k \bar{u}_j) E(\sum_{k=1}^{n} \tilde{G}(e_k))^2 \\
&\le Cn^{\tilde{m}\theta-2} L^{-\tilde{m}}(n) o_p(1) n^{2-\tilde{m}\theta} L^{\tilde{m}}(n) = o_p(1).
\end{aligned} \tag{5.15}
$$

由 (5.14)–(5.15) 式可得

$$
ET_{n7}^2 \le o_p(1). \tag{5.16}
$$

对 T_{n8} 求二阶矩

$$
\begin{aligned}
ET_{n8}^2 &= E(\lambda_n \sum_{k=1}^{n} \tilde{u}_k \bar{\varepsilon}_k)^2 = E(\lambda_n \sum_{k=1}^{n} (u_k - \bar{u}_k) \bar{\varepsilon}_k)^2 \\
&= E(\lambda_n^2 (\sum_{k=1}^{n} u_k \bar{\varepsilon}_k)^2) + E(\lambda_n^2 (\sum_{k=1}^{n} \bar{u}_k \bar{\varepsilon}_k)^2) := T_{n8}^{(1)} + T_{n8}^{(2)}.
\end{aligned}
$$

由 (F2) 有

$$
\begin{aligned}
T_{n8}^{(1)} &= n^{\tilde{m}\theta-2} L^{-\tilde{m}}(n) E(\sum_{k=1}^{n} \sum_{j=1}^{n} u_k u_j \bar{\varepsilon}_k \bar{\varepsilon}_j) \le n^{\tilde{m}\theta-2} L^{-\tilde{m}}(n) n \max_j E(u_j^2) E(\bar{\varepsilon}_j^2) \\
&= n^{\tilde{m}\theta-2} L^{-\tilde{m}}(n) n \max_j E(u_j^2) E(\sum_{k=1}^{n} \int_{A_j} E_m(t_k, s) ds G(e_j))^2 \\
&\le Cn^{\tilde{m}\theta-2} L^{-\tilde{m}}(n) n n^{2\lambda-2} \max_{k,j} E(\sum_{k=1}^{n} \sum_{j=1}^{n} G(e_k) G(e_j)) \\
&= Cn^{2\lambda-1} = o_p(1).
\end{aligned} \tag{5.17}
$$

由 (5.12) 式及 (F2) 有

$$\begin{aligned}
T_{n8}^{(2)} &= n^{\tilde{m}\theta-2}L^{-\tilde{m}}(n)E(\sum_{k=1}^{n}\sum_{j=1}^{n}\bar{u}_k\bar{u}_j\bar{\varepsilon}_k\bar{\varepsilon}_j) \le n^{\tilde{m}\theta-2}L^{-\tilde{m}}(n)n\max_j E(\bar{u}_j^2)E(\bar{\varepsilon}_j^2) \\
&= n^{\tilde{m}\theta-2}L^{-\tilde{m}}(n)n\max_j E(\bar{u}_j^2)E(\sum_{k=1}^{n}\int_{A_j}E_m(t_k,s)dsG(e_j))^2 \\
&\le Cn^{\tilde{m}\theta-2}L^{-\tilde{m}}(n)no_p(1)n^{2\lambda-2}\max_{k,j}E(\sum_{k=1}^{n}\sum_{j=1}^{n}G(e_k)G(e_j)) \\
&= Cn^{2\lambda-1}o_p(1) = o_p(1).
\end{aligned} \tag{5.18}$$

由 (5.17)–(5.18) 式可得

$$ET_{n8}^2 \le o_p(1). \tag{5.19}$$

对 T_{n9} 求二阶矩

$$\begin{aligned}
ET_{n9}^2 &= E(\lambda_n\sum_{k=1}^{n}\tilde{u}_k\tilde{g}_k)^2 = E(\lambda_n\sum_{k=1}^{n}(u_k-\bar{u}_k)\tilde{g}_k)^2 \\
&= E(\lambda_n^2(\sum_{k=1}^{n}u_k\tilde{g}_k)^2) + E(\lambda_n^2(\sum_{k=1}^{n}\bar{u}_k\tilde{g}_k)^2) \\
&:= ET_{n9}^{(1)} + ET_{n9}^{(2)}.
\end{aligned}$$

由引理 2 有

$$\begin{aligned}
T_{n9}^{(1)} &= n^{\tilde{m}\theta-2}L^{-\tilde{m}}(n)E(\sum_{k=1}^{n}\sum_{j=1}^{n}u_ku_j\tilde{g}_k\tilde{g}_j) \le n^{\tilde{m}\theta-2}L^{-\tilde{m}}(n)n\max_j E(u_j^2)E(\tilde{g}_j^2) \\
&= n^{\tilde{m}\theta-2}L^{-\tilde{m}}(n)n\max_j E(u_j^2)\sup_j|\tilde{g}_j|^2 \\
&\le Cn^{\tilde{m}\theta-2}L^{-\tilde{m}}(n)n(O(n^{-2\gamma})+O(\tau_m^2)) \\
&= Cn^{\tilde{m}\theta-1}L^{-\tilde{m}}(n)o_p(1) = o_p(1).
\end{aligned} \tag{5.20}$$

由 (5.12) 式及引理 2 有

$$\begin{aligned}
T_{n9}^{(2)} &= n^{\tilde{m}\theta-2}L^{-\tilde{m}}(n)E(\sum_{k=1}^{n}\sum_{j=1}^{n}\bar{u}_k\bar{u}_j\tilde{g}_k\tilde{g}_j) \le n^{\tilde{m}\theta-2}L^{-\tilde{m}}(n)n\max_j E(\bar{u}_j^2)E(\tilde{g}_j^2) \\
&= n^{\tilde{m}\theta-2}L^{-\tilde{m}}(n)n\max_j E(\bar{u}_j^2)\sup_j|\tilde{g}_j|^2 \\
&\le n^{\tilde{m}\theta-2}L^{-\tilde{m}}(n)no_p(1)(O(n^{-2\gamma})+O(\tau_m^2)) \\
&= n^{\tilde{m}\theta-1}L^{-\tilde{m}}(n)o_p(1) = o_p(1).
\end{aligned} \tag{5.21}$$

由 (5.20)–(5.21) 式可得

$$ET_{n9}^2 \le o_p(1). \tag{5.22}$$

对 T_{n10} 求二阶矩

$$\begin{aligned}ET_{n10}^2 &= E(\beta\lambda_n\sum_{k=1}^n \tilde{x}_k\tilde{u}_k)^2 = E(\beta\lambda_n\sum_{k=1}^n \tilde{x}_k(u_k-\bar{u}_k))^2 \\ &\le E(\beta\lambda_n\sum_{k=1}^n \tilde{x}_k u_k)^2 + E(\beta\lambda_n\sum_{k=1}^n \tilde{x}_k\bar{u}_k)^2 \\ &:= T_{n10}^{(1)} + T_{n10}^{(2)}.\end{aligned}$$

对于 $T_{n10}^{(1)}$ 有

$$\begin{aligned}T_{n10}^{(1)} &= E(\beta\lambda_n\sum_{k=1}^n \tilde{x}_k u_k)^2 \le \beta^2\Gamma^{-2}E(S_n^{-2}\sum_{k=1}^n \tilde{x}_k)^2 \max_k E(u_k^2) \\ &\le Cn^{\tilde{m}\theta-1}L^{-\tilde{m}}(n) = o_p(1).\end{aligned} \tag{5.23}$$

对于 $T_{n10}^{(2)}$, 由 (5.12) 式有

$$\begin{aligned}T_{n10}^{(2)} &= E(\beta\lambda_n\sum_{k=1}^n \tilde{x}_k\bar{u}_k)^2 \le \beta^2\Gamma^{-2}E(S_n^{-2}\sum_{k=1}^n \tilde{x}_k)^2 \max_k E(\bar{u}_k^2) \\ &\le Cn^{\tilde{m}\theta-1}L^{-\tilde{m}}(n)o_p(1) = o_p(1).\end{aligned} \tag{5.24}$$

由 (5.12) 式, 有

$$E(\lambda_n\frac{J(\tilde{m})}{\tilde{m}!}\sum_{k=1}^n u_k H_{\tilde{m}}(e_k))^2 = o_p(1). \tag{5.25}$$

由 (5.23)–(5.24) 式可得

$$ET_{n10}^2 \le o_p(1), \tag{5.26}$$

$$E(\mu_n^{-1})^2 = \Gamma_n^2 n^{-1}E(nS_n^{-2}) = O(1)n^{-\tilde{m}\theta}L^{\tilde{m}}(n) = O(1). \tag{5.27}$$

由 (5.12) 和 Chebyshev 不等式可得

$$\mu_n^{-1} = O_p(n^{-\tilde{m}\theta/2}L^{\tilde{m}/2}). \tag{5.28}$$

定理 1 证毕.

推论 1 的证明 由 (3.1) 和 (5.28) 式, 有

$$\tilde{\beta}_n - \beta = n^{-1}\frac{J(\tilde{m})}{\tilde{m}!}\sum_{k=1}^n (u_k+\nu_k)H_{\tilde{m}}(e_k) + o_p(1). \tag{5.29}$$

令 $\xi_k = n^{-1}\frac{J(\tilde{m})}{\tilde{m}!}(u_k+\nu_k)H_{\tilde{m}}(e_k)$. 则 $\{\xi_k, k=1,2,\cdots,n\}$ 是独立随机变量, 且 $E\xi_k=0$,

$$\mathrm{Var}(\sum_{k=1}^n \xi_k) = \sum_{k=1}^n \mathrm{Var}(\xi_k) = \sum_{k=1}^n n^{-2}(\frac{J(\tilde{m})}{\tilde{m}!})^2\sigma_1^2 E(H_{\tilde{m}}^2(e_k)) = n^{-1}\sigma_1^2\frac{J^2(\tilde{m})}{\tilde{m}!}.$$

因为 $B^{-\frac{2+\delta}{2}} := (\sum\limits_{k=1}^{n} E\xi_k^2)^{-(2+\delta)/2} = n^{(2+\delta)/2}(C\frac{J^2(\tilde{m})}{\tilde{m}!})^{-(2+\delta)/2}$, 且

$$\sum_{k=1}^{n} E|\xi_k|^{2+\delta} \le |\frac{J(\tilde{m})}{\tilde{m}!}|^{2+\delta} n^{-2-\delta} \sum_{k=1}^{n} (|\nu_k|^{4+2\delta} E|H_{\tilde{m}}(e_k)|^{4+2\delta})^{1/2} = O(n^{-1-\delta}),$$

则有 $B^{-(2+\delta)/2} \sum\limits_{k=1}^{n} E|\varsigma_k|^{2+\delta} = O(n^{\delta/2}) = o(1)$. 因此 Lindeberg 条件成立, 由中心极限定理有

$$\sum_{k=1}^{n} \varsigma_k / (\mathrm{Var}(\sum_{k=1}^{n} \varsigma_k))^{1/2} \xrightarrow{D} N(0,1). \tag{5.30}$$

因此由 (5.29)–(5.30) 式得推论 1 成立.

推论 2 的证明 由 (3.1) 和 (5.28) 式, 有

$$\tilde{\beta}_n - \beta = n^{-1} \frac{J(\tilde{m})}{\tilde{m}!} \sum_{k=1}^{n} \nu_k H_{\tilde{m}}(e_k) + o_p(1). \tag{5.31}$$

令 $\varsigma_k = n^{-1}\frac{J(\tilde{m})}{\tilde{m}!}\nu_k H_{\tilde{m}}(e_k)$. 则 $\{\varsigma_k, k = 1, 2, \cdots, n\}$ 是独立随机变量, 且 $E\varsigma_k = 0$,

$$\mathrm{Var}(\sum_{k=1}^{n} \varsigma_k) = \sum_{k=1}^{n} \mathrm{Var}(\varsigma_k) = \sum_{k=1}^{n} n^{-2} (\frac{J(\tilde{m})}{\tilde{m}!})^2 \nu_k^2 E(H_{\tilde{m}}^2(e_k)) = n^{-1} \nu_k^2 \frac{J^2(\tilde{m})}{\tilde{m}!}.$$

因为

$$B^{-\frac{2+\delta}{2}} := (\sum_{k=1}^{n} E\varsigma_k^2)^{-(2+\delta)/2} = n^{(2+\delta)/2} (C\frac{J^2(\tilde{m})}{\tilde{m}!})^{-(2+\delta)/2},$$

且

$$\sum_{k=1}^{n} E|\varsigma_k|^{2+\delta} \le |\frac{J(\tilde{m})}{\tilde{m}!}|^{2+\delta} n^{-2-\delta} \sum_{k=1}^{n} (|\nu_k|^{4+2\delta} E|H_{\tilde{m}}(e_k)|^{4+2\delta})^{1/2} = O(n^{-1-\delta}),$$

有 $B^{-(2+\delta)/2} \sum\limits_{k=1}^{n} E|\varsigma_k|^{2+\delta} = O(n^{\delta/2}) = o(1)$. 因此 Lindeberg 条件成立, 由中心极限定理有

$$\sum_{k=1}^{n} \varsigma_k / (\mathrm{Var}(\sum_{k=1}^{n} \varsigma_k))^{1/2} \xrightarrow{D} N(0,1). \tag{5.32}$$

因此由 (5.31)–(5.32) 式得推论 2 成立.

定理 2 的证明

$$\begin{aligned}
&\tilde{g}_n(t) - g(t) = \sum_{i=1}^{n} (y_i - X_i \tilde{\beta}_n) \int_{A_i} E_m(t,s)ds - g(t) \\
=&(\sum_{i=1}^{n} g(t_i) \int_{A_i} E_m(t,s)ds - g(t)) + (\beta - \tilde{\beta}) \sum_{i=1}^{n} x_i \int_{A_i} E_m(t,s)ds + \sum_{i=1}^{n} \varepsilon_i \int_{A_i} E_m(t,s)ds \\
&+ (\beta - \tilde{\beta}) \sum_{i=1}^{n} u_i \int_{A_i} E_m(t,s)ds - \beta \sum_{i=1}^{n} u_i \int_{A_i} E_m(t,s)ds \\
:=&T_1^{(1)} + T_2^{(1)} + T_3^{(1)} + T_4^{(1)} + T_5^{(1)}.
\end{aligned}$$

注意到

$$
\begin{aligned}
& n^{(\tilde{m}\theta-\lambda)/2}L^{-\tilde{m}/2}(n)T_1^{(1)} \\
=& O(n^{(2\gamma+\lambda)/\tilde{m}-\theta}L(n))^{-\tilde{m}/2}+O((n^{\lambda/\tilde{m}-\theta}L(n))^{-\tilde{m}/2}\tau_m) \\
=& o_p(1).
\end{aligned}
$$

因为

$$
\begin{aligned}
& E(\sum_{i=1}^n x_i\int_{A_i}E_m(t,s)ds)^2 \qquad (5.33)\\
\leq & (\sum_{i=1}^n f(t_i)\int_{A_i}E_m(t,s)ds)^2+(\sum_{i=1}^n \nu_i\int_{A_i}E_m(t,s)ds)^2 \\
\leq & \sup_{i,k}|f(t_i)f(t_k)|(\int_0^1 E_m(t,s)ds)^2+\max_{i,k}|\nu_i\nu_k|(\int_0^1 E_m(t,s)ds)^2 \\
\leq & O_p(1)+o_p(1)\leq O_p(1). \qquad (5.34)
\end{aligned}
$$

所以有

$$
\sum_{i=1}^n x_i\int_{A_i}E_m(t,s)ds=O_p(1), \tag{5.35}
$$

从而

$$
n^{(\tilde{m}\theta-\lambda)/2}L^{-\tilde{m}/2}(n)T_2^{(1)}=O_p(n^{-\lambda/2})=o_p(1). \tag{5.36}
$$

由上式和引理 2 有

$$
\begin{aligned}
\mathrm{Var}(T_3^{(1)}) &= \mathrm{Var}(\sum_{i=1}^n G(e_i)\int_{A_i}E_m(t,s)ds) \\
&\leq (\int_{A_i}E_m(t,s)ds)^2\mathrm{Var}(\sum_{i=1}^n G(e_i)) \\
&\leq \max_i|\int_{A_i}E_m(t,s)ds|^2 n^{2-\tilde{m}\theta}L^{\tilde{m}}(n) \\
&= O_p(n^{\lambda-\tilde{m}\theta}L^{\tilde{m}}(n)). \qquad (5.37)
\end{aligned}
$$

由引理 6 和引理 7 有

$$
n^{(\tilde{m}\theta-\lambda)/2}L^{-\tilde{m}/2}(n)T_3^{(1)}\xrightarrow{D}C_{\tilde{m},\theta}(t)Z_{\tilde{m}}. \tag{5.38}
$$

由引理 1 有

$$
\mathrm{Var}(T_4^{(1)})=\beta^2\sigma_1^2\sum_{i=1}^n(\int_{A_i}E_m(t,s)ds)^2\leq Cn^{-1}2^m=O_p(n^{\lambda-1}), \tag{5.39}
$$

因此由 Markov 不等式和 (F_3) 可得

$$n^{(\tilde{m}\theta-\lambda)/2}L^{-\tilde{m}/2}(n)T_4^{(1)}=O_p(n^{-(1-\tilde{m}\theta)}L^{-\tilde{m}/2}(n))=o_p(1). \tag{5.40}$$

由引理 1, 可得到

$$n^{(\tilde{m}\theta-\lambda)/2}L^{-\tilde{m}/2}(n)T_5^{(1)}=o_p(1). \tag{5.41}$$

定理 2 证毕.

6 模拟例子

为了对本文的证明结果做进一步的解释和验证, 选取 2013 年 5 月到 2018 年 8 月之间的全国居民消费价格指数, 城市居民消费价格指数和农村居民消费价格指数, 使用 Mathmatic 来做模拟应用 (具体数值见东方财富网《中国居民消费价格指数》).

使用下述的式子

$$\begin{cases} \tilde{\beta}_n=(\tilde{X}^2-n\Sigma_1^2)^{-1}\tilde{X}\tilde{y}, \\ \tilde{g}_n(t)=(y-X\tilde{\beta}_n)\int_A E_m(t,s)ds, \end{cases}$$

其中设全国居民消费价格指数为 y, 城市居民消费价格指数和农村居民消费价格指数为矩阵 X, 通过 X, 由 Mathmatic 算得

$$\tilde{\beta}_n=\begin{pmatrix}0.725815\\0.274299\end{pmatrix}.$$

$\tilde{g}_n(t)$ 的散点图如下图 1

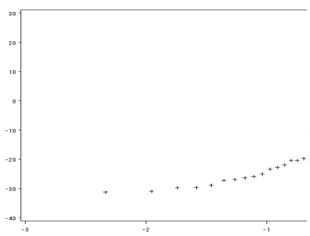

图 1: $\tilde{g}_n(t)$ 的散点图

相对误差 H 由下式算得

$$H=\frac{|y_t-y|}{y}\cdot 100\%.$$

64 个数据中有 55 个在 0.05% 以内, 占总数的 85.94%; 有 9 个在 0.1% 以内, 占总数的 14.06%, 在一定程度上说明估计出的 $\tilde{\beta}_n$ 和 $\tilde{g}_n(t)$ 是有效的. 这个例子也直接说明了本文前面所证明的结论是正确的, 对于数据处理预测也有实际的作用.

具体计算的结果如下表 1.

表 1: 相对误差表

日期	观测值 y	预测值 y_t	相对误差 H	日期	观测值 y	预测值 y_t	相对误差 H
2018.08	102.3	102.312	0.01173%	2015.12	101.6	101.664	0.06299%
2018.07	102.1	102.084	0.01567%	2015.11	101.5	101.461	0.03842%
2018.06	101.9	101.839	0.05986%	2015.10	101.3	101.287	0.01283%
2018.05	101.8	101.783	0.01670%	2015.09	101.6	101.584	0.01575%
2018.04	101.8	101.783	0.01670%	2015.08	102.0	101.957	0.04216%
2018.03	102.1	102.056	0.04310%	2015.07	101.6	101.657	0.05610%
2018.02	102.9	102.929	0.02818%	2015.06	101.4	101.359	0.04043%
2018.01	101.5	101.510	0.00985%	2015.05	101.2	101.224	0.02372%
2017.12	101.8	101.855	0.05403%	2015.04	101.5	101.525	0.02463%
2017.11	101.7	101.727	0.02655%	2015.03	101.4	101.352	0.04734%
2017.10	101.9	101.855	0.04416%	2015.02	101.4	101.424	0.02367%
2017.09	101.6	101.628	0.02756%	2015.01	100.8	100.754	0.04563%
2017.08	101.8	101.799	0.00098%	2014.12	101.5	101.521	0.02069%
2017.07	101.4	101.373	0.02663%	2014.11	101.4	101.454	0.05325%
2017.06	101.5	101.517	0.01675%	2014.10	101.6	101.623	0.02264%
2017.05	101.5	101.545	0.04433%	2014.09	101.6	101.626	0.02559%
2017.04	101.2	101.177	0.02273%	2014.08	102.0	101.977	0.02255%
2017.03	100.9	100.907	0.00694%	2014.07	102.3	102.250	0.04888%
2017.02	100.8	100.832	0.03175%	2014.06	102.3	102.325	0.02444%
2017.01	102.5	102.509	0.00878%	2014.05	102.5	102.452	0.04683%
2016.12	102.1	102.064	0.03526%	2014.04	101.8	101.824	0.02358%
2016.11	102.3	102.234	0.06452%	2014.03	102.4	102.398	0.00195%
2016.10	102.1	102.108	0.00784%	2014.02	102.0	101.996	0.00392%
2016.09	101.9	101.912	0.01178%	2014.01	102.5	102.497	0.00293%
2016.08	101.3	101.312	0.01185%	2013.12	102.5	102.506	0.00585%
2016.07	101.8	101.740	0.05894%	2013.11	103.0	103.033	0.03204%
2016.06	101.9	101.919	0.01865%	2013.10	103.2	103.236	0.03488%
2016.05	102.0	102.046	0.04510%	2013.09	103.1	103.089	0.01067%
2016.04	102.3	102.352	0.05083%	2013.08	102.6	102.616	0.01559%
2016.03	102.3	102.293	0.00684%	2013.07	102.7	102.689	0.01071%
2016.02	102.3	102.296	0.00391%	2013.06	102.7	102.661	0.03797%
2016.01	101.8	101.743	0.05599%	2013.05	102.1	102.129	0.02840%

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ASYMPTOTIC PROPERTY OF WAVELET ESTIMATORS IN FIXED-DESIGN PARTIALLY LINEAR ERRORS-IN-VARIABLES MODELS WITH LONG-RANGE DEPENDENT ERRORS

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Abstract: In this paper, we consider that the random errors are the function of Gaussian random variables with stationary and long-range dependence, and we investigate a partially linear errors-in-variables(EV) model in fixed-design by the wavelet method. Under several conditions, we obtain asymptotic representation of the parametric estimator, and asymptotic distribution and weak convergence rates of the parametric and nonparametric estimators.

Keywords: long-range dependence; partially linear errors-in-variables models; wavelet estimators; asymptotic representation; convergence rate

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