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THE EXISTENCE OF SOLUTIONS TO CHERN-SIMONS-SCHRÖDINGER SYSTEMS WITH EXPONENTIAL NONLINEARITIES

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Abstract: In this paper, the nonlinear Chern-Simons-Schrödinger systems with exponential nonlinearities are studied. By mountain pass theorem, the existence of a solution to these systems is obtained.

Keywords: Chern-Simons-Schrödinger systems; exponential nonlinearities; variational methods; mountain pass theorem

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1 Introduction and Main Result

We study the existence of solutions to the following Chern-Simons-Schrödinger system (**CSS** system) in $H^1(\mathbb{R}^2)$

$$\begin{cases} -\Delta u + V(x)u + A_0 u + \sum\limits_{j=1}^{2} A_j^2 u = f(u), \\ \partial_1 A_0 = A_2|u|^2, \quad \partial_2 A_0 = -A_1|u|^2, \\ \partial_1 A_2 - \partial_2 A_1 = -\frac{1}{2}u^2, \quad \partial_1 A_1 + \partial_2 A_2 = 0, \end{cases} \tag{1.1}$$

where $V(x)$ and $f(u)$ satisfy

(V1) $V(x) \in C(\mathbb{R}^2, \mathbb{R})$ and $V(x) \geq V_0 > 0$ for all $x \in \mathbb{R}^2$;

(V2) the function $[V(x)]^{-1}$ belongs to $L^1(\mathbb{R}^2)$;

(F1) $f \in C(\mathbb{R}, \mathbb{R})$ and $f(0) = 0$;

(F2) $\lim\limits_{s \to +\infty} \frac{f(s)}{e^{\alpha s^2}} = 0$ for all $\alpha > 0$;

(F3) there exist $\theta > 6$ and $s_1 > 0$ such that for all $|s| \geq s_1$,

$$0 < \theta F(s) \doteq \theta \int_0^s f(t)\, dt \leq s f(s);$$

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(F4) $\lim\limits_{s\to 0} 2F(s)s^{-2} < \lambda_1$, where

$$\lambda_1 = \inf_{u\in E\setminus\{0\}} \frac{\displaystyle\int_{\mathbb{R}^2} |\nabla u|^2 + V(x)u^2\,dx}{\displaystyle\int_{\mathbb{R}^2} u^2\,dx} \geq V_0 > 0,$$

$E = \{u \in H^1(\mathbb{R}^2) : \int_{\mathbb{R}^2} V(x)u^2\,dx < \infty\}$ is a subspace of $H^1(\mathbb{R}^2)$ and also a Hilbert space endowed with the following inner product

$$\langle u, v\rangle = \int_{\mathbb{R}^2} (\nabla u\cdot\nabla v + V(x)uv)\,dx, \ \ \forall u, v \in E.$$

$f(u) = \lambda(2s + s^2)e^s$ for $0 < \lambda < \frac{\lambda_1}{2}$ is an example that $f(u)$ satisfies assumptions (F1)–(F4), which was given in [1].

The **CSS** system received much attention recently, which describes the dynamics of large number of particles in a electromagnetic field. About the detail of its physical background, we refer to the references we mentioned below and references therein.

The **CSS** system arises from the Euler-Lagrange equations which are given by

$$\begin{cases} iD_0\phi + (D_1D_1 + D_2D_2)\phi = f(\phi), \\ \partial_0A_1 - \partial_1A_0 = -\mathrm{Im}(\bar{\phi}D_2\phi), \\ \partial_0A_2 - \partial_2A_0 = \mathrm{Im}(\bar{\phi}D_1\phi), \\ \partial_1A_2 - \partial_2A_1 = -\frac{1}{2}|\phi|^2. \end{cases} \tag{1.2}$$

This system was proposed in [2–4]. Berge, De Bouard, Saut [5] and Huh [6] studied the blowing up time-dependent solutions of problem (1.2) as well as Liu, Smith, Tataru [7] considered the local wellposedness.

We assume that the Coulomb gauge condition $\partial_0A_0 + \partial_1A_1 + \partial_2A_2 = 0$ holds, then the standing wave $\psi(x,t) = e^{i\omega t}\, u$ of problem (1.2) satisfies

$$\begin{cases} -\Delta u + \omega u + A_0u + A_1^2u + A_2^2u = f(u), \\ \partial_1A_0 = A_2u^2, \quad \partial_2A_0 = -A_1u^2, \\ \partial_1A_2 - \partial_2A_1 = -\frac{1}{2}|u|^2, \quad \partial_1A_1 + \partial_2A_2 = 0. \end{cases} \tag{1.3}$$

Under some radial assumptions, on the one hand, the existence, non-existence, and multiplicity of standing waves to the nonlinear **CSS** systems were investigated by [8–12] etc.

On the other hand, the existence of solitary waves was considered by [13–16] etc. For problem (1.1) with $f(u) = |u|^{p-2}u$, $p > 4$, without the radial assumptions we mentioned above, by the concentration compactness principle with $V(x)$ is a constant and the argument of global compactness with $V \in C(\mathbb{R}^2)$ and $0 < V_0 < V(x) < V_\infty$, the existence of nontrivial solutions to Chern-Simons-Schrödinger systems (1.1) was obtained in [17].

Inspired by [1] and [17], the purpose of the present paper is to study the existence of solutions for systems (1.1) with exponential nonlinearities. The main difficult of systems

(1.1) is that the non-local term A_j, $j = 0, 1, 2$ depend on u and there is a lack of compactness in $\mathbb{R}^2$. Using the mountain pass theorem, we have the following main result.

Theorem 1.1 Suppose (V1), (V2), (F1), (F2), (F3) and (F4) hold, then problem (1.1) has a solution.

This paper is organized as follows. In Section 2 we introduce the workframe and some technical lemmas. In Section 3 we prove the mountain pass construction and (PS) condition, which yields Theorem 1.1.

2 Mathematical Framework

In this section, we outline the variational workframe for the future study.

Let $H^1(\mathbb{R}^2)$ denote the usual Sobolev space with

$$\|u\| = \Big(\int_{\mathbb{R}^2} |\nabla u|^2 + V(x)|u|^2 \, dx \Big)^{1/2}.$$

We consider the following subspace of $H^1(\mathbb{R}^2)$,

$$E = \{u \in H^1(\mathbb{R}^2) : \int_{\mathbb{R}^2} V(x)u^2 \, dx < \infty\}.$$

Condition (V1) implies that the embedding $E \hookrightarrow H^1(\mathbb{R}^2)$ is continuous. Assumption (V2) and Hölder inequality yield that

$$\|u\|_{L^1(\mathbb{R}^2)} \le \Big(\int_{\mathbb{R}^2} V(x)^{-1} \, dx \Big)^{\frac{1}{2}} \|u\|. \tag{2.1}$$

Consequently,

$$E \hookrightarrow L^q(\mathbb{R}^2), \ \ 1 \le q < \infty \tag{2.2}$$

are continuous. Furthermore, by condition (V2), the above embeddings are compact (see [18, 19]).

Define the functional

$$J(u) = \frac{1}{2} \int_{\mathbb{R}^2} \Big(|\nabla u|^2 + V(x)|u|^2 + A_1^2|u|^2 + A_2^2|u|^2 \Big) \, dx - \int_{\mathbb{R}^2} F(u) \, dx,$$

where $F(u) = \displaystyle\int_0^u f(t) \, dt$. Note that

$$\begin{aligned} &\int_{\mathbb{R}^2} A_0|u|^2 \, dx = -2 \int_{\mathbb{R}^2} A_0(\partial_1 A_2 - \partial_2 A_1) \, dx \\ =&2 \int_{\mathbb{R}^2} (A_2 \partial_1 A_0 - A_1 \partial_2 A_0) \, dx = 2 \int_{\mathbb{R}^2} (A_1^2 + A_2^2)|u|^2 \, dx. \end{aligned} \tag{2.3}$$

We have the derivative of J in $H^1(\mathbb{R}^2)$ as follows

$$\begin{aligned}&\langle J'(u),\,\eta\rangle\\&=\int_{\mathbb{R}^2}\Big(\nabla u\nabla\eta+V(x)u\eta+(A_1^2(u)+A_2^2(u))u\eta+A_0u\eta-f(u)\eta\Big)\,dx\end{aligned}$$

for all $\eta\in C_0^\infty(\mathbb{R}^2)$. Especially, from (2.3), we obtain that

$$\langle J'(u),\,u\rangle=\int_{\mathbb{R}^2}\Big(|\nabla u|^2+V(x)|u|^2+3\big(A_1^2(u)|u|^2+A_2^2(u)\big)|u|^2-f(u)u\Big)\,dx.$$

By (1.1), we have that A_j satisfy

$$\Delta A_1=\partial_2(\frac{|u|^2}{2}),\quad \Delta A_2=-\partial_1(\frac{|u|^2}{2}),\quad \Delta A_0=\partial_1(A_2|u|^2)-\partial_2(A_1|u|^2),$$

which provide

$$A_1=A_1(u)=K_2*(\frac{|u|^2}{2})=-\frac{1}{2\pi}\int_{\mathbb{R}^2}\frac{x_2-y_2}{|x-y|^2}\frac{|u|^2(y)}{2}\,dy,\tag{2.4}$$

$$A_2=A_2(u)=-K_1*(\frac{|u|^2}{2})=\frac{1}{2\pi}\int_{\mathbb{R}^2}\frac{x_1-y_1}{|x-y|^2}\frac{|u|^2(y)}{2}\,dy,\tag{2.5}$$

$$A_0=A_0(u)=K_1*(A_1|u|^2)-K_2*(A_2|u|^2),\tag{2.6}$$

where $K_j=\frac{-x_j}{2\pi|x|^2}$ for $j=1,2$ and $*$ denotes the convolution.

We know that J is well defined in $H^1(\mathbb{R}^2)$, $J\in C^1(H^1(\mathbb{R}^2))$, and the weak solution of (1.1) is the critical point of the functional J from the following properties.

Proposition 2.1 (see [17]) Let $1<s<2$ and $\frac{1}{s}-\frac{1}{q}=\frac{1}{2}$.

(i) Then there is a constant C depending only on s and q such that

$$\Big(\int_{\mathbb{R}^2}\big|Tu(x)\big|^q\,dx\Big)^{\frac{1}{q}}\le C\Big(\int_{\mathbb{R}^2}|u(x)|^s\,dx\Big)^{\frac{1}{s}},$$

where the integral operator T is given by

$$Tu(x):=\int_{\mathbb{R}^2}\frac{u(y)}{|x-y|}\,dy.$$

(ii) If $u\in H^1(\mathbb{R}^2)$, then we have that for $j=1,2$, $\|A_j^2(u)\|_{L^q(\mathbb{R}^2)}\le C\|u\|^2_{L^{2s}(\mathbb{R}^2)}$ and

$$\|A_0(u)\|_{L^q(\mathbb{R}^2)}\le C\|u\|^2_{L^{2s}(\mathbb{R}^2)}\|u\|^2_{L^4(\mathbb{R}^2)}.$$

(iii) For $q'=\frac{q}{q-1}$, $j=1,2$,

$$\|A_j(u)u\|_{L^2(\mathbb{R}^2)}\le\||A_j(u)|^2\|_{L^q(\mathbb{R}^2)}\|u\|^2_{L^{2q'}(\mathbb{R}^2)}.$$

We will use the following mountain pass theorem to obtain our main result.

Theorem 2.2 (see [20]) Let $\mathbb{E}$ be a real Banach space and suppose that $I\in C^1(\mathbb{E},\mathbb{R})$ satisfies the following conditions

(i) $I(0)=0$ and there exist $\rho>0$, $\alpha>0$ such that $I_{\partial B_\rho(0)}\geq\alpha$;

(ii) there exists $e\in E\backslash\overline{B_\rho(0)}$ such that $I(e)\leq 0$.

Let $c=\inf\limits_{\gamma\in\Lambda}\max\limits_{0\leq\tau\leq 1}I(\gamma(\tau))$, where $\Lambda=\{\gamma\in C([0,1],\mathbb{E}):\gamma(0)=0,\gamma(1)=e\}$ is the set of continuous paths joining 0 and e. Then $c\geq\alpha$ and there exists a sequence $\{u_k\}\subset\mathbb{E}$ such that

$$I(u_k)\xrightarrow{k}c \text{ and } \|I'(u_k)\|_{\mathbb{E}^*}\xrightarrow{k}0.$$

Moreover, if I satisfies (PS) condition, then c is a critical value of I in E.

The following result on A_j, $j=0,1,2$ is important to prove the compactness.

Proposition 2.3 (see [17]) Suppose that u_n converges to u a.e. in $\mathbb{R}^2$ and u_n converges weakly to u in $H^1(\mathbb{R}^2)$. Let $A_{j,n}:=A_j(u_n(x))$, $j=0,1,2$. Then

(i) $A_{j,n}$ converges to $A_j(u(x))$ a.e. in $\mathbb{R}^2$.

(ii) $\displaystyle\int_{\mathbb{R}^2}A_{i,n}^2u_nu\,dx$, $\displaystyle\int_{\mathbb{R}^2}A_{i,n}^2|u|^2\,dx$ and $\displaystyle\int_{\mathbb{R}^2}A_{i,n}^2|u_n|^2\,dx$ converge to $\displaystyle\int_{\mathbb{R}^2}A_i^2|u|^2\,dx$ for $i=1,2$; $\displaystyle\int_{\mathbb{R}^2}A_{0,n}u_nu\,dx$ and $\displaystyle\int_{\mathbb{R}^2}A_{0,n}|u_n|^2\,dx$ converge to $\displaystyle\int_{\mathbb{R}^2}A_0|u|^2\,dx$.

(iii) $\displaystyle\int_{\mathbb{R}^2}|A_i(u_n-u)|^2|u_n-u|^2\,dx=\int_{\mathbb{R}^2}|A_i(u_n)|^2|u_n|^2\,dx-\int_{\mathbb{R}^2}|A_i(u)|^2|u|^2\,dx+o_n(1)$ for $i=1,2$.

In order to prove the mountain pass construction, we need the following results in [1, 21, 22].

Proposition 2.4 (i) If $\alpha>0$ and $u\in H^1(\mathbb{R}^2)$ then $\displaystyle\int_{\mathbb{R}^2}\left(e^{\alpha u^2}-1\right)dx<\infty$. Moreover, if $\|\nabla u\|_2^2\leq 1$, $\|u\|_2\leq M<\infty$ and $\alpha<4\pi$, then there exists a constant $C=C(M,\alpha)$ such that $\displaystyle\int_{\mathbb{R}^2}\left(e^{\alpha u^2}-1\right)dx<C(M,\alpha)$.

(ii) Let $\beta>0$ and $r>1$. Then for each $\alpha>r$ there exists a positive constant $C=C(\alpha)$ such that for all $s\in\mathbb{R}$, $\left(e^{\beta s^2}-1\right)^r\leq C\left(e^{\alpha\beta s^2}-1\right)$. In particular, if $u\in H^1(\mathbb{R}^2)$, then $\left(e^{\beta u^2}-1\right)^r$ belongs to $L^1(\mathbb{R}^2)$.

(iii) If $v\in E$, $\beta>0$, $q>0$ and $\|v\|\leq M$ with $\beta M^2<4\pi$, then there exists $C=C(\beta,M,q)>0$ such that

$$\int_{\mathbb{R}^2}\left(e^{\beta v^2}-1\right)|v|^q\,dx\leq C\|v\|^q. \tag{2.7}$$

3 Proof of Main Theorem

First of all, we prove the mountain pass structure.

Lemma 3.1 Assume (F2), (F3), and (F4) hold. Then there exist $\rho>0$, $\alpha>0$ such that $J(u)>\alpha$ for all $\|u\|=\rho$.

Proof From (F4), there exist ε, $\delta>0$, such that

$$|F(s)|\leq\frac{\lambda_1-\varepsilon}{2}|s|^2,\ \ \forall|s|\leq\delta. \tag{3.1}$$

By (F2) and (F3), we have $\forall q > 2$, there exists $C = C(q, \delta)$ such that

$$|F(s)| \leq C|s|^q(e^{\alpha s^2 - 1}), \ \ \forall |s| \geq \delta. \tag{3.2}$$

By (3.1) and (3.2), we have

$$|F(s)| \leq \frac{\lambda_1 - \varepsilon}{2}|s|^2 + C|s|^q(e^{\alpha s^2 - 1}), \ \ \forall s \in \mathbb{R},\ q > 2. \tag{3.3}$$

From (iii) of Proposition 2.4, the definition of λ_1, and the continuous embeddings (2.2), we obtain

$$J(u) \geq \frac{1}{2}\|u\|^2 - \frac{\lambda_1 - \varepsilon}{2}\|u\|^2 - C\|u\|^q \geq \frac{1}{2}\big(1 - \frac{\lambda_1 - \varepsilon}{\lambda_1}\big)\|u\|^2 - C\|u\|^q.$$

Hence, we have

$$J(u) \geq \|u\|[\frac{1}{2}\big(1 - \frac{\lambda_1 - \varepsilon}{\lambda_1}\big)\|u\| - c\|u\|^{q-1}].$$

By $\varepsilon > 0$ and $q > 2$, we can choose $\rho > 0$ and $\alpha > 0$ such that $J_{\partial B_\rho(0)} \geq \alpha > 0$.

Lemma 3.2 Assume that f satisfies (F3). Then there exists $e \in E$ with $\|e\| > \rho$ such that $J(e) < 0$.

Proof Let $u \in H^1(\mathbb{R}^2)$ such that $u \equiv s_1$ in B_1, $u \equiv 0$ in B_2^c and $u \geq 0$. Define $k = \mathrm{supp}(u)$. From (F3), there exist positive constants C_1, C_2 such that for all $s \in \mathbb{R}$,

$$F(s) \geq C_1|s|^\theta - C_2. \tag{3.4}$$

Then we have for $t > 1$,

$$J(tu) \leq \frac{t^2}{2}\|u\|^2 + Ct^6\|u\|^6 - Ct^\theta \int\limits_{\{x:\, t|u(x)| \geq s_1\}} u^\theta\, dx + C_1|k|.$$

Since $\theta > 6$, we have $J(tu) \to -\infty$ as $t \to +\infty$. Let $e = tu$ with t sufficiently large, the proof is completed.

By Theorem 2.2, the functional J has a $(\mathrm{PS})_c$ sequence. Next, we show this $(\mathrm{PS})_c$ sequence is bounded.

Lemma 3.3 Assume (F2) and (F3) hold. Let (u_n) is a $(\mathrm{PS})_c$ sequence of J in E, that is, $J(u_n) \to c$ and $J'(u_n) \to 0$. Then $\|u_n\| \leq C$ for some positive constant C.

Proof We know

$$\frac{1}{2}\|u_n\|^2 + \frac{1}{2}\int\limits_{\mathbb{R}^2} \big(A_{1,n}^2|u_n|^2 + A_{2,n}^2|u_n|^2\big)\, dx - \int\limits_{\mathbb{R}^2} F(u_n)\, dx = c + o_n(1)$$

and for any $\varphi \in E$, we have

$$\int\limits_{\mathbb{R}^2} \big(\nabla u_n \nabla\varphi + V(x)u_n\varphi\big)\, dx + \int\limits_{\mathbb{R}^2} \big(A_{1,n}^2 + A_{2,n}^2 + A_{0,n}\big)u_n\varphi\, dx - \int\limits_{\mathbb{R}^2} f(u_n)\varphi\, dx = o_n(\|\varphi\|).$$

By (F3) and $\theta > 6$, we get

$$\begin{aligned}\theta c + \varepsilon_n \|u_n\| \geq & (\frac{\theta}{2} - 1)\|u_n\|^2 + (\frac{\theta}{2} - 3)\int\limits_{\mathbb{R}^2} \big(A_{1,n}^2|u_n|^2 + A_{2,n}^2|u_n|^2\big)\,dx \\ & - \int\limits_{\mathbb{R}^2} \big(\theta F(u_n) - f(u_n)u_n\big)\,dx \\ \geq & (\frac{\theta}{2} - 1)\|u_n\|^2 - \int\limits_{\{x:|u_n(x)|<s_1\}} \big(\theta F(u_n) - f(u_n)u_n\big)\,dx,\end{aligned}$$

where $\varepsilon_n \to 0$ as $n \to \infty$. From $|f(s)s - F(s)| \leq c_1|s|$ for all $|s| \leq s_1$ and inequality (2.1), we obtain $\theta c + \varepsilon_n\|u_n\| \geq (\frac{\theta}{2} - 1)\|u_n\|^2 - c_1\|u_n\|$, which implies that $\|u_n\| \leq C$.

Now we are going to prove (PS) condition.

Lemma 3.4 The functional J satisfies (PS) condition.

Proof Let $\{u_n\}$ be a $(\mathrm{PS})_c$ sequence of J, that is, $J(u_n) \to c$ and $J'(u_n) = 0$. By Lemma 3.3, $\{u_n\}$ is bounded, up to a subsequence, we may assume that $u_n \rightharpoonup u_0$ in E, $u_n \to u_0$ in $L^q(\mathbb{R}^2)$ for all $q \geq 1$ and $u_n \to u_0$ almost everywhere in $\mathbb{R}^2$, as $n \to \infty$. If $f(s)$ satisfies (F2) and (F4), we have for each $\alpha > 0$, there exist b_1, $b_2 > 0$ such that for all $s \in \mathbb{R}$,

$$|f(s)| \leq b_1|s| + b_2\big(e^{\alpha s^2} - 1\big).$$

Then we have

$$|f(u_n) - f(u_0)||u_n - u_0| \leq C[|u_n| + |u_0| + (e^{\alpha u_n^2} - 1) + (e^{\alpha u_0^2} - 1)]|u_n - u_0|.$$

By (i) and (ii) of Proposition 2.4 and Hölder inequality, we obtain

$$\lim_{n\to\infty} \int_{\mathbb{R}^2} (f(u_n) - f(u_0))(u_n - u_0)\,dx = 0. \tag{3.5}$$

By Proposition 2.3, we have

$$\lim_{n\to\infty} \int_{\mathbb{R}^2} [(A_1^2(u_n) + A_2^2(u_n))u_n - (A_1^2(u_0) + A_2^2(u_0))u_0](u_n - u_0)\,dx = 0, \tag{3.6}$$

and

$$\lim_{n\to\infty} \int_{\mathbb{R}^2} [(A_0(u_n)u_n + A_0(u_0)u_0)](u_n - u_0)\,dx = 0. \tag{3.7}$$

From (3.5), (3.6), and (3.7), we have

$$\begin{aligned}\|u_n - u_0\|_E^2 = & \langle J'(u_n) - J'(u_0), u_n - u_0\rangle \\ & + \int_{\mathbb{R}^2} [(A_1^2(u_0) + A_2^2(u_0))u_0 - (A_1^2(u_n) + A_2^2(u_n))u_n](u_n - u_0)\,dx \\ & + \int_{\mathbb{R}^2} [(A_0(u_0)u_0 - A_0(u_n)u_n)](u_n - u_0)\,dx \\ & + \int_{\mathbb{R}^2} (f(u_n) - f(u_0))(u_n - u_0)\,dx.\end{aligned}$$

We obtain that $u_n \to u_0$ as $n \to \infty$ in E.

Proof of Theorem 1.1 By Theorem 2.2, Lemma 3.1, Lemma 3.2 and Lemma 3.4, we obtain that functional J has a critical point u_0 at the minimax level

$$c = \inf_{\gamma \in \Lambda} \max_{0 \leq \tau \leq 1} I(\gamma(\tau)),$$

where $\Lambda = \{\gamma \in C([0,1], \mathbb{E}) : \gamma(0) = 0, \gamma(1) = e\}$.

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含指数增长非线性项的Chern-Simons-Schrödinger方程组解的存在性

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摘要: 本文研究了带指数增长的非线性项的非线性Chern-Simons-Schrödinger方程组. 利用山路引理的方法, 得到该方程组解的存在性.

关键词: Chern-Simons-Schrödinger方程组; 指数增长的非线性项; 变分法; 山路引理

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