

FIXED POINT THEOREMS FOR F -CONTRACTION IN COMPLETE GENERALIZED METRIC SPACES

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Abstract: In this article, we study concepts of weak F -contraction of type (A) and weak F -contraction of type (B) in generalized metric spaces. By using the method of iteration, we obtain some fixed point theorems for these mappings in a complete generalized metric space, which generalize the results of F -contraction in complete metric spaces.

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1 Introduction and Preliminaries

It is well known that Banach contraction mapping principle [1] which was published in 1922 is one of the most important theorems in classical functional analysis. Indeed, it was widely used as the source of metric fixed point theory, for more details see [2–8]. In 2012, Wardowski [2] introduced a new concept of contraction called F -contraction and proved a fixed point theorem which generalized the Banach contraction principle. Later, Wardowski and Van Dung [3] introduced the definition of an F -weak contraction and proved a fixed point theorem for it. Dung and Hung [4] generalized an F -weak contraction to a generalized F -contraction in 2015. Recently, Piri and Kumam [5, 6] described a large class of functions by replacing condition (F3)' instead of condition (F3) in the definition of F -contraction. They introduced the concepts of modified generalized F -contraction of type (A) and modified generalized F -contraction of type (B) and proved some fixed point theorems for these contractions in a complete metric space.

Following this direction of research, in this paper, we introduce some F -contractions which satisfy different conditions in generalized metric space and then we prove some fixed point theorems for these contractions in a complete generalized metric space. Finally, we give an example to support our result.

Throughout the article \mathbb{N} , \mathbb{R}^+ and \mathbb{R} will denote the set of natural numbers, non-negative real numbers and real numbers, respectively.

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First, Wardowski [2] introduced the notion of a F -contraction and proved the Wardowski fixed point theorem as the generalization of Banach contraction principle. Let $F : (0, \infty) \rightarrow \mathbb{R}$ be a mapping satisfying

- (F1) F is strictly increasing, that is, $s < t \Rightarrow F(s) < F(t)$ for all $s, t > 0$;
- (F2) for every sequence $\{s_n\}$ in \mathbb{R}^+ , we have $\lim_{n \rightarrow \infty} s_n = 0$ if and only if $\lim_{n \rightarrow \infty} F(s_n) = -\infty$;
- (F3) there exists a number $k \in (0, 1)$ such that $\lim_{s \rightarrow 0^+} s^k F(s) = 0$.
- (F3)' F is continuous on $(0, \infty)$.

We denote with F_1 the family of all functions F that satisfy conditions (F1)–(F3). Let F denote the family of all functions $F : \mathbb{R}^+ \rightarrow \mathbb{R}$ that satisfy conditions (F1), (F3) and \mathfrak{F} denote the family of all functions $F : \mathbb{R}^+ \rightarrow \mathbb{R}$ that satisfy conditions (F1), (F3)'.

Example 1 The following functions $F : (0, \infty) \rightarrow \mathbb{R}$ belongs to F_1 :

- (i) $F(t) = \ln t$ with $t > 0$,
- (ii) $F(t) = \ln t + t$ with $t > 0$.

Definition 1 [2] Let (X, d) be a metric space. A mapping $T : X \rightarrow X$ is called an F -contraction on X if there exist $F \in F_1$ and $\tau > 0$ such that for all $x, y \in X$ with $d(Tx, Ty) > 0$, we have $\tau + F(d(Tx, Ty)) \leq F(d(x, y))$.

Theorem 1 Let (X, d) be a complete metric space and $T : X \rightarrow X$ be an F -contraction. Then T has a unique fixed point $x^* \in X$ and for every $x \in X$ the sequence $\{T^n x\}_{n \in \mathbb{N}}$ converges to x^* .

In 2000, Branciari [7] introduced the concept of rectangular metric space by replacing the sum of the right hand side of the triangle inequality in metric space by a three-term expression and proved an analog of the Banach contraction principle in such spaces.

Definition 2 [7] Let X be a nonempty set and $d : X \times X \rightarrow [0, \infty)$ satisfying the following conditions, for all $x, y \in X$ and all distinct $u, v \in X$ each of which is different from x and y :

- (GMS1) $d(x, y) = 0$ if and only if $x = y$,
- (GMS2) $d(x, y) = d(y, x)$,
- (GMS3) $d(x, y) \leq d(x, u) + d(u, v) + d(v, y)$.

Then the map d is called generalized metric and abbreviated as GM, here the pair (X, d) is called a generalized metric space and abbreviated as GMS.

Definition 3 [7] Let (X, d) be a generalized metric space and $\{x_n\}$ be a sequence of elements of X .

- (1) A sequence $\{x_n\}$ is said to be GMS convergent to a limit $x \in X$ if and only if $d(x_n, x) \rightarrow 0$ as $n \rightarrow \infty$.
- (2) A sequence $\{x_n\}$ is said to be GMS Cauchy sequence if and only if for every $\varepsilon > 0$ there exists a positive integer $N(\varepsilon)$ such that $d(x_n, x_m) < \varepsilon$ for all $n > m > N(\varepsilon)$.
- (3) A GMS (X, d) is called complete if every GMS Cauchy sequence in X is GMS convergent.
- (4) A mapping $T : (X, d) \rightarrow (X, d)$ is continuous if for any sequence $\{x_n\}$ in X such that $d(x_n, x) \rightarrow 0$ as $n \rightarrow \infty$, we have $d(Tx_n, Tx) \rightarrow 0$ as $n \rightarrow \infty$.

Lemma 1 Let (X, d) be a GMS and let $\{x_n\}$ be a sequence in X . Assume that $\{x_n\}$ is Cauchy sequence with $x_n \neq x_m$ for all $n, m \in \mathbb{N}$ with $n \neq m$ and $\lim_{n \rightarrow \infty} x_n = x$. Then $\lim_{n \rightarrow \infty} d(x_n, y) = d(x, y)$ for all $y \in X$.

2 Main Results

Definition 4 Let (X, d) be a GMS. A mapping $T : X \rightarrow X$ is called a weak F -contraction of type (A) on X if there exist $F \in F$ and $\tau > 0$ such that

$$\forall x, y \in X, [d(Tx, Ty) > 0 \Rightarrow \tau + F(d(Tx, Ty)) \leq F(d(x, y))]. \quad (2.1)$$

Theorem 2 Let (X, d) be a complete GMS and $T : X \rightarrow X$ be a weak F -contraction of type (A). Then T has a unique fixed point $x^* \in X$ and for every $x \in X$ the sequence $\{T^n x\}_{n \in \mathbb{N}}$ converges to x^* .

Proof Choose $x_0 \in X$ and define a sequence $\{x_n\}_{n=1}^\infty$ by

$$x_1 = Tx_0, x_2 = Tx_1 = T^2x_0, \dots, x_n = T^n x_0, \forall n \in \mathbb{N}.$$

If there exists $n \in \mathbb{N}$ such that $x_n = x_{n+1}$. Then x_n is a fixed point of T and we have nothing to prove. Now we suppose that $x_n \neq x_{n+1}$, i.e., $d(Tx_{n-1}, Tx_n) > 0$ for all $n \in \mathbb{N}$. It follows from (2.1) that, for all $n \in \mathbb{N}$,

$$\tau + F(d(x_n, x_{n+1})) = \tau + F(d(Tx_{n-1}, Tx_n)) \leq F(d(x_{n-1}, x_n)),$$

i.e.,

$$F(d(x_n, x_{n+1})) = F(d(Tx_{n-1}, Tx_n)) \leq F(d(x_{n-1}, x_n)) - \tau. \quad (2.2)$$

It follows from (2.2) and (F1) that

$$d(x_n, x_{n+1}) < d(x_{n-1}, x_n), \forall n \in \mathbb{N}.$$

Therefore $\{d(x_n, x_{n+1})\}_{n \in \mathbb{N}}$ is a nonnegative decreasing sequence of real numbers, and hence

$$\lim_{n \rightarrow \infty} d(x_n, x_{n+1}) = \gamma \geq 0.$$

Now, we claim that $\gamma=0$. Arguing by contradiction, we assume that $\gamma > 0$.

Since $\{d(x_n, x_{n+1})\}_{n \in \mathbb{N}}$ is a nonnegative decreasing sequence of real numbers, for every $n \in \mathbb{N}$, we have

$$d(x_n, x_{n+1}) \geq \gamma. \quad (2.3)$$

From (2.2), (2.3) and (F1), we get

$$F(\gamma) \leq F(d(x_n, x_{n+1})) \leq F(d(x_{n-1}, x_n)) - \tau \leq \dots \leq F(d(x_0, x_1)) - n\tau \quad (2.4)$$

for all $n \in \mathbb{N}$. Since $F(\gamma) \in \mathbb{R}$ and $\lim_{n \rightarrow \infty} [F(d(x_0, x_1)) - n\tau] = -\infty$, there exists $n_1 \in \mathbb{N}$ such that

$$F(d(x_0, x_1)) - n\tau < F(\gamma), \quad \forall n > n_1. \quad (2.5)$$

It follows from (2.4) and (2.5) that $F(\gamma) < F(d(x_0, x_1)) - n\tau < F(\gamma)$, $\forall n > n_1$. It is a contraction. Therefore we have

$$\lim_{n \rightarrow \infty} d(x_n, x_{n+1}) = 0. \quad (2.6)$$

From (F3), there exists $k \in (0, 1)$ such that

$$\lim_{n \rightarrow \infty} \left((d(x_n, x_{n+1}))^k F(d(x_n, x_{n+1})) \right) = 0. \quad (2.7)$$

It follows from (2.4) that

$$(d(x_n, x_{n+1}))^k (F(d(x_n, x_{n+1})) - F(d(x_0, x_1))) \leq -(d(x_n, x_{n+1}))^k n\tau \leq 0 \quad (2.8)$$

for all $n \in \mathbb{N}$. By using (2.6), (2.7) and taking the limit as $n \rightarrow \infty$ in (2.8), we get

$$\lim_{n \rightarrow \infty} \left(n(d(x_n, x_{n+1}))^k \right) = 0. \quad (2.9)$$

Then there exists $n_2 \in \mathbb{N}$ such that $n(d(x_n, x_{n+1}))^k \leq 1$ for all $n \geq n_2$, that is,

$$d(x_n, x_{n+1}) \leq \frac{1}{n^{\frac{1}{k}}}. \quad (2.10)$$

Now, we shall prove that $x_n \neq x_m$, for all $n \neq m$. Assume on the contrary that $x_n = x_m$ for some $m, n \in \mathbb{N}$ with $n \neq m$. Since $d(x_p, x_{p+1}) > 0$ for each $p \in \mathbb{N}$, without loss of generality, we may assume that $m > n + 1$. Substitute again $x = x_n = x_m$, $y = x_{n+1} = x_{m+1}$ in (2.1), which yield

$$\tau + F(d(x_n, Tx_n)) = \tau + F(d(x_m, Tx_m)) = \tau + F(d(Tx_{m-1}, Tx_m)) \leq F(d(x_{m-1}, x_m)),$$

then

$$F(d(x_n, x_{n+1})) \leq F(d(x_{m-1}, x_m)) - \tau \leq \cdots \leq F(d(x_{n+1}, x_n)) - (m - n)\tau.$$

Since $\tau > 0$, we get a contradiction, therefore $x_n \neq x_m$, for all $n \neq m$. Now, we prove $\lim_{n \rightarrow \infty} d(x_n, x_{n+2}) = 0$ for all $n \in \mathbb{N}$. It follows from (2.1) that for all $n \in \mathbb{N}$,

$$\tau + F(d(x_n, x_{n+2})) = F(d(Tx_{n-1}, Tx_{n+1})) \leq F(d(x_{n-1}, x_{n+1})),$$

i.e.,

$$F(d(x_n, x_{n+2})) = F(d(Tx_{n-1}, Tx_{n+1})) \leq F(d(x_{n-1}, x_{n+1})) - \tau. \quad (2.11)$$

It follows from (2.11) and (F1) that

$$d(x_n, x_{n+2}) < d(x_{n-1}, x_{n+1}), \quad \forall n \in \mathbb{N}.$$

Therefore $\{d(x_n, x_{n+2})\}_{n \in \mathbb{N}}$ is a nonnegative decreasing sequence of real numbers, and hence

$$\lim_{n \rightarrow \infty} d(x_n, x_{n+2}) = \gamma \geq 0.$$

Now, we claim that $\gamma=0$. Arguing by contradiction, we assume that $\gamma > 0$.

Since $\{d(x_n, x_{n+2})\}_{n \in \mathbb{N}}$ is a nonnegative decreasing sequence of real numbers, for every $n \in \mathbb{N}$, we have

$$d(x_n, x_{n+2}) \geq \gamma. \quad (2.12)$$

From (2.11), (2.12) and (F1), we get

$$F(\gamma) \leq F(d(x_n, x_{n+2})) \leq F(d(x_{n-1}, x_{n+2})) - \tau \leq \cdots \leq F(d(x_0, x_2)) - n\tau \quad (2.13)$$

for all $n \in \mathbb{N}$. Since $F(\gamma) \in \mathbb{R}$ and $\lim_{n \rightarrow \infty} [F(d(x_0, x_2)) - n\tau] = -\infty$, there exists $n_3 \in \mathbb{N}$ such that

$$F(d(x_0, x_2)) - n\tau < F(\gamma), \quad \forall n > n_3. \quad (2.14)$$

It follows from (2.13) and (2.14) that $F(\gamma) < F(d(x_0, x_2)) - n\tau < F(\gamma)$, $\forall n > n_3$. It is a contraction. Therefore we have

$$\lim_{n \rightarrow \infty} d(x_n, x_{n+2}) = 0. \quad (2.15)$$

Now, we shall prove that $\{x_n\}$ is a GMS Cauchy sequence in (X, d) , that is, for $\varepsilon > 0$ and $N(\varepsilon)$ such that $d(x_n, x_m) < \varepsilon$, for all $m, n \in \mathbb{N}$ with $m > n > N(\varepsilon)$. Assume that $m = n+k$, where $k > 2$. We will consider the only two cases as follows.

Case 1 For k is odd. Let $k = 2l+1$, where $l \in \mathbb{N}$. By using the quadrilateral inequality (GMS3) and (2.10), we have

$$\begin{aligned} d(x_n, x_m) &= d(x_n, x_{n+2l+1}) \\ &\leq d(x_n, x_{n+1}) + d(x_{n+1}, x_{n+2}) + \cdots + d(x_{n+2l}, x_{n+2l+1}) \\ &< \sum_{i=n}^{\infty} d(x_{i+1}, x_i) \leq \sum_{n \geq N(\varepsilon)} \frac{1}{n^{\frac{1}{k}}} < \varepsilon. \end{aligned}$$

Case 2 For k is even. Let $k = 2l$, where $l \in \mathbb{N}$. By using the quadrilateral inequality (GMS3) and (2.10), (2.15), we have

$$\begin{aligned} d(x_n, x_m) &= d(x_n, x_{n+2l}) \\ &\leq d(x_n, x_{n+2}) + d(x_{n+2}, x_{n+3}) + \cdots + d(x_{n+2l-1}, x_{n+2l}) \\ &< d(x_n, x_{n+2}) + \sum_{i=n+2}^{\infty} d(x_{i+1}, x_i) \leq d(x_n, x_{n+2}) + \sum_{n \geq N(\varepsilon)} \frac{1}{n^{\frac{1}{k}}} < \varepsilon. \end{aligned}$$

Therefore we conclude that $\{x_n\}$ is a GMS Cauchy sequence in (X, d) . By the completeness of (X, d) , there exists $x^* \in X$ such that

$$\lim_{n \rightarrow \infty} d(x_n, x^*) = 0. \quad (2.16)$$

Also, we suppose that T is continuous, we have

$$\lim_{n \rightarrow \infty} d(x_{n+1}, Tx^*) = \lim_{n \rightarrow \infty} d(Tx_n, Tx^*) = 0. \quad (2.17)$$

By Lemma 1, it follows that x_n differs from both x^* and Tx^* for n sufficiently large. Hence, we have $d(x^*, Tx^*) \leq d(x^*, x_n) + d(x_n, x_{n+1}) + d(x_{n+1}, Tx^*)$, from (2.6), (2.16), (2.17), we have $d(x^*, Tx^*) \leq \lim_{n \rightarrow \infty} d(x^*, x_n) + d(x_n, x_{n+1}) + d(x_{n+1}, Tx^*) = 0$. So we obtain that $d(x^*, Tx^*) = 0$, that is $Tx^* = x^*$. Hence x^* is a fixed point of T . Now let us show that T has at most one fixed point. Indeed, if $x^*, y^* \in X$ are two distinct fixed points of T , that is, $Tx^* = x^* \neq y^* = Ty^*$, then $d(Tx^*, Ty^*) > 0$. So from the assumption of the theorem, we obtain

$$F(d(x^*, y^*)) = F(d(Tx^*, Ty^*)) < \tau + F(d(Tx^*, Ty^*)) \leq F(d(x^*, y^*)),$$

which is a contradiction. Thus T has a unique fixed point.

Now, we show another important result.

Definition 5 Let (X, d) be a GMS. A mapping $T : X \rightarrow X$ is called a weak F -contraction of type (B) on X if there exist $F \in \mathfrak{F}$ and $\tau > 0$ such that

$$\forall x, y \in X, [d(Tx, Ty) > 0 \Rightarrow \tau + F(d(Tx, Ty)) \leq F(d(x, y))]. \quad (2.18)$$

Theorem 3 Let (X, d) be a complete GMS and $T : X \rightarrow X$ be a weak F -contraction of type (B), then T has a unique fixed point $x^* \in X$ and for every $x \in X$ the sequence $\{T^n x\}_{n \in \mathbb{N}}$ converges to x^* .

Proof Choose $x_0 \in X$ and define a sequence $\{x_n\}_{n=1}^\infty$ by

$$x_1 = Tx_0, x_2 = Tx_1 = T^2x_0, \dots, x_n = T^n x_0, \forall n \in \mathbb{N}.$$

By using the similar method in the proof of Theorem 2, we have

$$\lim_{n \rightarrow \infty} d(x_n, x_{n+1}) = 0 \text{ and } \lim_{n \rightarrow \infty} d(x_n, x_{n+2}) = 0, \quad x_n \neq x_m \text{ for all } n \neq m. \quad (2.19)$$

Now, we shall prove that $\{x_n\}$ is a GMS Cauchy sequence in (X, d) . Suppose $\{x_n\}$ is not a Cauchy sequence, then there exists $\varepsilon > 0$ for which we can find two subsequences $\{x_{p(n)}\}$ and $\{x_{q(n)}\}$ of $\{x_n\}$ such that $p(n)$ is the smallest index for which

$$p(n) - 3 \geq q(n) > n, \quad d(x_{p(n)}, x_{q(n)}) \geq \varepsilon, \quad d(x_{p(n)-2}, x_{q(n)}) < \varepsilon, \quad \forall n \in \mathbb{N}.$$

Since $x_m \neq x_n$ for all $m, n \in \mathbb{N}$ with $n \neq m$, we have

$$\begin{aligned} \epsilon &\leq d(x_{p(n)}, x_{q(n)}) \leq d(x_{p(n)}, x_{p(n)-1}) + d(x_{p(n)-1}, x_{p(n)-2}) + d(x_{p(n)-2}, x_{q(n)}) \\ &\leq d(x_{p(n)}, x_{p(n)-1}) + d(x_{p(n)-1}, x_{p(n)-2}) + \varepsilon. \end{aligned}$$

By taking the limit in above inequality and using inequalities (2.19), we get

$$\lim_{n \rightarrow \infty} d(x_{p(n)}, x_{q(n)}) = \varepsilon. \quad (2.20)$$

Since

$$d(x_{p(n)}, x_{q(n)}) \leq d(x_{p(n)}, x_{p(n)+1}) + d(x_{p(n)+1}, x_{q(n)+1}) + d(x_{q(n)+1}, x_{q(n)}),$$

we have

$$\begin{aligned} \epsilon &\leq \liminf_{n \rightarrow \infty} d(x_{p(n)}, x_{q(n)}), \\ \epsilon &\leq \liminf_{n \rightarrow \infty} (d(x_{p(n)}, x_{p(n)+1}) + d(x_{p(n)+1}, x_{q(n)+1}) + d(x_{q(n)+1}, x_{q(n)})) \\ &= \liminf_{n \rightarrow \infty} d(x_{p(n)+1}, x_{q(n)+1}) \end{aligned}$$

and $d(x_{p(n)+1}, x_{q(n)+1}) \leq d(x_{p(n)+1}, x_{p(n)}) + d(x_{p(n)}, x_{q(n)}) + d(x_{q(n)}, x_{q(n)+1})$, then we have

$$\begin{aligned} &\limsup_{n \rightarrow \infty} d(x_{p(n)+1}, x_{q(n)+1}) \\ &\leq \limsup_{n \rightarrow \infty} d(x_{p(n)+1}, x_{p(n)}) + d(x_{p(n)}, x_{q(n)}) + d(x_{q(n)}, x_{q(n)+1}) = \varepsilon. \end{aligned}$$

So we have

$$\lim_{n \rightarrow \infty} d(x_{p(n)+1}, x_{q(n)+1}) = \varepsilon. \quad (2.21)$$

Thus $\tau + F(d(Tx_{p(n)}, Tx_{q(n)})) \leq F(d(x_{p(n)}, x_{q(n)}))$. So as $n \rightarrow \infty$ and by (F3)', (2.20), (2.21), we get $\tau + F(\varepsilon) \leq F(\varepsilon)$, which is a contraction. Hence $\{x_n\}$ is a GMS Cauchy sequence in X . Since X is complete, there exists $x^* \in X$ such that $\lim_{n \rightarrow \infty} x_n = x^*$. Therefore

$$\lim_{n \rightarrow \infty} d(x_n, x^*) = 0. \quad (2.22)$$

Finally, we claim that $x^* = Tx^*$, we only have the following two cases

- (1) $\forall n \in \mathbb{N}, \exists i_n \in \mathbb{N}, i_n > i_{n-1}, i_0 = 1$ and $x_{i_n+1} = Tx^*$;
- (2) $\exists n_4 \in \mathbb{N}, \forall n \geq n_4, d(Tx_n, Tx^*) > 0$.

In the first case, we have $x^* = \lim_{n \rightarrow \infty} x_{i_n+1} = \lim_{n \rightarrow \infty} Tx^* = Tx^*$. In the second case from (2.18), for all $n \geq n_4$, we have $\tau + F(d(Tx_n, Tx^*)) \leq F(d(x_n, x^*))$. From (F1) we obtain $d(Tx_n, Tx^*) < d(x_n, x^*)$, since $\lim_{n \rightarrow \infty} d(x_n, x^*) = 0$, so $\lim_{n \rightarrow \infty} d(Tx_n, Tx^*) < \lim_{n \rightarrow \infty} d(x_n, x^*) = 0$. This is a contraction. Hence, $x^* = Tx^*$. Again by using the similar method as used in the proof Theorem 2, we can prove that T has a unique fixed point.

Example 2 Let X be a finite set defined as $X = \{1, 2, 3, 4\}$. Define $d : X \times X \rightarrow [0, \infty)$ as

$$\begin{aligned} d(1, 1) &= d(2, 2) = d(3, 3) = d(4, 4) = 0, \\ d(1, 2) &= d(2, 1) = 3, \\ d(2, 3) &= d(3, 2) = d(1, 3) = d(3, 1) = 1, \\ d(1, 4) &= d(4, 1) = d(2, 4) = d(4, 2) = d(3, 4) = d(4, 3) = 4. \end{aligned}$$

The function d is not a metric on X . Indeed, note that

$$3 = d(1, 2) \geq d(1, 3) + d(3, 2) = 1 + 1 = 2,$$

that is, the triangle inequality is not satisfied. However d is a generalized metric on X , moreover, (X, d) is a complete generalized metric space. Define $T : X \rightarrow X$ as

$$T1 = T2 = T3 = 2, \quad T4 = 3.$$

For $x \in \{1, 2, 3\}$ and $y = 4$, we have $d(Tx, Ty) = d(2, 3) = 1 > 0$, $d(x, y) = 4$. Therefore $d(Tx, Ty) \leq \frac{1}{4}d(x, y)$. So by choosing $F(t) = \ln(t)$ and $\tau = \ln \frac{1}{4}$, we see that T is a F -contraction which satisfies Theorems 3.

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完备广义度量空间 F 压缩不动点定理

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摘要: 本文研究了广义度量空间(A)型和(B)型弱 F 压缩的问题. 利用迭代的方法, 获得了在完备广义度量空间关于这些映射的不动点定理的结果. 推广了完备度量空间 F 压缩的一些结果.

关键词: 广义度量空间; 不动点; F 压缩

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