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MINIMAL I-OPEN SETS

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Abstract: In this paper, we investigate the minimal I-open sets in ideal topological spaces. By using the theory of general topology, we obtain several characterizations and properties of minimal I-open sets. In addition, the relationships between minimal I-open sets and other types of open sets are investigated.

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1 Introduction

The notions of I-open sets in topological spaces were introduced by Janković and Hatmlett [1]. Abd EI-Monsef et al. [2] further investigated I-open sets and I-continuous functions. In 1999, Abd EI-Monsef et al. [3] introduced and investigated almost-I-open sets and almost-I-continuous functions. Subsequently, Hatir and Noiri [4] introduced the notion of β-I-open sets to obtain certain decompositions of continuity. Nasef [5] introduced the notion of fuzzy pre-I-open sets and obtained a decomposition of fuzzy I-continuity. The notion of semi-I-open sets to obtain decompositions of continuity was introduced by Hatir and Noiri [6]. In addition to this, Guler and Aslim [7] introduced the notion of b-I-sets and b-I-continuous functions. The ideal theory plays an important role in studying logical algebras. For more details of the logical algebras, we refer the reader to [8–11]. In the light of above results, the purpose of this paper is to study minimal I-open sets and to obtain several characterizations and properties of these concepts.

2 Preliminaries

An ideal $\mathcal{J}$ on a topological space $(X, \mathcal{J})$ is a nonempty collection of subsets of X which satisfies:

(1) (Heredity) If $A \in \mathcal{J}$ and $B \subset A$, then $B \in \mathcal{J}$.

(2) (Finite additivity) If $A \in \mathcal{J}$ and $B \in \mathcal{J}$, then $A \cup B \in \mathcal{J}$.

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An ideal topological space is defined to be a topological space $(X, \mathcal{J})$ with an ideal $\mathcal{J}$ on X and is denoted by $(X, \tau, \mathcal{J})$. For a subset A of X,

$$A^*(\mathcal{J}, \tau) = \{x \in X \mid U \cap A \notin \mathcal{J} \text{ for every } U \in \tau(x)\},$$

where $\tau(x) = \{U \in \tau \mid x \in U\}$, is called the local function of A with respect to $\mathcal{J}$ and τ. We simply write $A^*(\mathcal{J})$ or A^* instead of $A^*(\mathcal{J}, \tau)$ when there is no chance for confusion. For every ideal topological space $(X, \tau, \mathcal{J})$, there exists a topology $\tau^*(\mathcal{J})$, finer than τ, generated by $\beta(\mathcal{J}, \tau) = \{U \setminus I \mid U \in \tau,\ I \in \mathcal{J}\}$, but in general $\beta(\mathcal{J}, \tau)$ is not always a topology (see [12]). It is well known that $\mathrm{Cl}^*(A) = A \cup A^*$ defines a Kuratowski closure operator for $\tau^*(\mathcal{J})$.

Lemma 2.1 [12] Let (X, τ) be a topological space with ideals $\mathcal{J}$ and $\mathcal{K}$. For any subsets A and B of X, we have the following assertions:

(1) $A \subset B \Rightarrow A^* \subset B^*$.

(2) $\mathcal{J} \subset \mathcal{K} \Rightarrow A^*(\mathcal{K}) \subset A^*(\mathcal{J})$.

(3) $A^* = \mathrm{Cl}(A^*) \subset \mathrm{Cl}(A)$ (A^* is a closed subset of $\mathrm{Cl}(A)$).

(4) $(A^*)^* \subset A^*$.

(5) $(A \cup B)^* = A^* \cup B^*$.

(6) $A^* \setminus B^* = (A \setminus B)^* \setminus B^* \subset (A \setminus B)^*$.

(7) $U \in \tau \Rightarrow U \cap A^* = U \cap (U \cap A)^* \subset (U \cap A)^*$.

(8) $B \in \mathcal{J} \Rightarrow (A \cup B)^* = A^* = (A \setminus B)^*$.

The set of all I-open sets in ideal topological space $(X, \tau, \mathcal{J})$ is denoted by $IO(X, \tau, \mathcal{J})$ or written simply as $IO(X)$ when there is no chance for confusion.

Definition 2.2 [2] Let $(X, \tau, \mathcal{J})$ be an ideal topological space. A subset A of X is said to be I-open if $A \subseteq \mathrm{Int}(A^*)$.

Definition 2.3 [13] A subset A of a space X is said to be pre-open if $A \subseteq \mathrm{Int}(\mathrm{Cl}(A))$.

Remark 2.4 [2] One can deduce that I-open set $\Rightarrow$ pre-open set, and the converse is not true, in general.

Definition 2.5 [14] Let (X, τ) be a topological space. A nonempty subset U of X is said to be a minimal open set if it is an open set satisfies

$$(\forall A \in \tau)\ (A \subseteq U \ \Rightarrow\ A = \emptyset \text{ or } A = U).$$

Lemma 2.6 [14] Let (X, τ) be a topological space. We have the following assertions:

(1) Let U be a minimal open subset of X and W an open subset of X. Then $U \cap W = \emptyset$ or $U \subseteq W$.

(2) Let U and V be minimal open subsets of X. Then $U \cap V = \emptyset$ or $U = V$.

3 Minimal I-Open Sets

Definition 3.1 Let $(X, \tau, \mathcal{J})$ be an ideal topological space. A nonempty subset U of X is said to be a minimal I-open set if it is an I-open set satisfies

$$(\forall A \in IO(X))\ (A \subseteq U \ \Rightarrow\ A = \emptyset \text{ or } A = U).$$

Example 3.2 Consider an ideal topological space $(X, \tau, \mathcal{I})$, where $X = \{a, b, c, d\}$, $\tau = \{\emptyset, X, \{a, b\}, \{c\}, \{a, b, c\}\}$ and $\mathcal{I} = \{\emptyset, \{b\}\}$. We have

$$IO(X) = \{\emptyset, \{a\}, \{c\}, \{a, b\}, \{a, c\}, \{a, b, c\}, \{a, c, d\}, X\}.$$

Thus $\{a\}$ and $\{c\}$ are minimal I-open.

It is clear that minimal openness and minimal I-openness are independent concepts as shown by the following example.

Example 3.3 Consider an ideal topological space $(X, \tau, \mathcal{I})$, where $X = \{a, b, c, d\}$, $\tau = \{\emptyset, X, \{a, b\}, \{c\}, \{a, b, c\}\}$ and $\mathcal{I} = \{\emptyset, \{a\}, \{c\}, \{a, c\}\}$. We have

$$IO(X) = \{\emptyset, \{b\}, \{a, b\}\}.$$

Thus $\{b\}$ is not a minimal open set but a minimal I-open set. And $\{c\}$ is not a minimal I-open set but a minimal open set.

Theorem 3.4 Let $(X, \tau, \mathcal{J})$ be an ideal topological space in which every minimal open set is contained in $\mathcal{J}$. If U is a minimal open set of X, then $A^* \cap U = \emptyset$ for any $A \subseteq X$.

Proof Let U be a minimal open set of X and let A be a subset of X. Suppose that $A^* \cap U \neq \emptyset$. Then there exists an element $x \in A^* \cap U$. That is, $x \in A^*$ and $x \in U$. Since every minimal open set is contained in $\mathcal{J}$, $U \in \mathcal{J}$ and so $A \cap U \in \mathcal{J}$. This implies that $x \notin A^*$. This is a contradiction to $x \in A^*$. Therefore $A^* \cap U = \emptyset$.

Lemma 3.5 Let $(X, \tau, \mathcal{J})$ be an ideal topological space in which every minimal open set is contained in $\mathcal{J}$. Then $\mathrm{Int}(A^*) = \emptyset$ for any $A \subseteq X$.

Proof Let A be a subset of X and let G be a nonempty open subset of X. Then there exists a minimal open set U such that $U \subseteq G$. We have $A^* \cap U = \emptyset$ by Theorem 3.4. Since $U \subseteq G$, $G \subsetneqq A^*$. Thus $\mathrm{Int}(A^*) = \emptyset$.

We have a question: does the every ideal topological space have the minimal I-open set? The answer to this question is negative as seen in the following theorem.

Theorem 3.6 Let $(X, \tau, \mathcal{J})$ be an ideal topological space. The following conditions are equivalent:

(1) X has at least one minimal I-open set.

(2) There exists a minimal open set which is not contained in $\mathcal{J}$.

Proof (1) $\Rightarrow$ (2) Suppose that (1) is satisfied. Then $IO(X) \neq \{\emptyset\}$. Assume that $M \in \mathcal{J}$ for all minimal open set M. Then $\mathrm{Int}(A^*) = \emptyset$ for any $A \subseteq X$ by Lemma 3.5. And so $\emptyset$ is the only I-open set in $(X, \tau, \mathcal{J})$. This is a contradiction. Therefore, there exists a minimal open set which is not contained in $\mathcal{J}$.

(2) $\Rightarrow$ (1) Suppose that (2) is satisfied. Then there exists a minimal open set M such that $M \notin \mathcal{J}$. Let x be an element of M and let G be a neighborhood of x. Then since M is minimal open, $M \subset G$. And so $G \cap M = M$. Since $M \notin \mathcal{J}$, $G \cap M = M \notin \mathcal{J}$. This implies that $x \in M^*$. Thus $M \subset M^*$. Since M is minimal open, $M = \mathrm{Int}(M) \subset \mathrm{Int}(M^*)$. That is, $M \in IO(X)$. Meanwhile, since M is minimal open, $M \neq \emptyset$. Therefore M is a nonempty I-open set and so X has at least one minimal I-open set.

Definition 3.7 Let $(X, \tau, \mathcal{J})$ be an ideal topological space. Then $(X, \tau, \mathcal{J})$ is called ideal topological space with minimal I-open sets (simply, ITSMI), if there exists minimal I-open set in $(X, \tau, \mathcal{J})$.

Example 3.8 Consider the example as presented in Example 3.2. Then $(X, \tau, \mathcal{J})$ is a minimal ideal topological space. Consider an ideal topological space $(Y, \kappa, \mathcal{J})$ where $Y = \{a, b, c, d\}$, $\kappa = \{\emptyset, X, \{b\}, \{b, c\}, \{b, c, d\}\}$ and $\mathcal{J} = \{\emptyset, \{b\}\}$. We have $IO(X) = \{\emptyset\}$. Thus $(Y, \kappa, \mathcal{J})$ is an ideal topological space with minimal I-open sets (simply, ITSMI).

Theorem 3.9 Let U be a minimal open set in an ITSMI $(X, \tau, \mathcal{J})$. If $\{x\} \notin \mathcal{J}$, then $\{x\}$ is a minimal I-open set for any $x \in U$.

Proof Let x be an element of U such that $\{x\} \notin \mathcal{J}$. Now we will show that $U \subseteq \{x\}^*$. Let a be an element of U and let H be a neighborhood of a. Then since U is minimal open and $U \cap H \neq \emptyset$, $U \subseteq H$ by Lemma 2.5. And so $\{x\} = U \cap \{x\} = H \cap \{x\}$. Since $\{x\} \notin \mathcal{J}$, $H \cap \{x\} \notin \mathcal{J}$. This implies that $a \in \{x\}^*$. And so $U \subseteq \{x\}^*$. It follows that $\{x\} \subseteq U = \mathrm{Int}(U) \subseteq \mathrm{Int}(\{x\}^*)$. That is, $\{x\}$ is an I-open set. Therefore $\{x\}$ is a minimal I-open set.

Lemma 3.10 (see [2]) Let $(X, \tau, \mathcal{J})$ be an ITSMI. If $A \in IO(X)$ and $B \in \tau$, then $A \cap B \in IO(X)$.

Theorem 3.11 Let $(X, \tau, \mathcal{J})$ be an ITSMI. Then we have the following results.

(1) Let U be a minimal I-open subset of X and W be a subset of X. If $U \cap W$ is I-open, then $U \cap W = \emptyset$ or $U \subseteq W$.

(2) Let U and W be minimal I-open subsets of X. If $U \cap W$ is I-open, then $U \cap W = \emptyset$ or $U = W$.

Proof (1) Let U be a minimal I-open subset of X and W be a subset of X such that $U \cap W$ is I-open. Since U is minimal I-open and $U \cap W \subseteq U$, we have $U \cap W = U$ or $U \cap W = \emptyset$. Hence $U \subseteq W$ or $U \cap W = \emptyset$.

(2) Let U and W be minimal I-open subsets of X such that $U \cap W$ is I-open. If $U \cap W \neq \emptyset$, then $U \subseteq W$ and $W \subseteq U$ by (1). Therefore $U = W$.

Corollary 3.12 Let $(X, \tau, \mathcal{J})$ be an ITSMI. Then we have the following results.

(1) Let U be a minimal I-open subset of X and W be an open subset of X. Then $U \cap W = \emptyset$ or $U \subseteq W$.

(2) Let U and W be a minimal I-open subsets of X. If W is open, then $U \cap W = \emptyset$ or $U = W$.

Proof By Lemma 3.10, $U \cap W$ is I-open. Thus we have the results by Theorem 3.11.

Proposition 3.13 Let U be a minimal I-open set in an ITSMI $(X, \tau, \mathcal{J})$. If x is an element of U, then $U \subseteq W$ for all $W \in \tau(x)$.

Proof Let U be a minimal I-open set in an ITSMI $(X, \tau, \mathcal{J})$. Let $x \in U$ and $W \in \tau(x)$. Then $U \cap W$ is I-open by Lemma 3.10 and $U \cap W \neq \emptyset$. Thus $U \subseteq W$ by Theorem 3.11 (1).

Proposition 3.14 Let U be a minimal I-open set in an ITSMI $(X, \tau, \mathcal{J})$. Then

$$U \subseteq \cap\{W \mid W \in \tau(x)\}$$

for any $x \in U$.

Proof The proof is straightforward.

The reverse inclusion of Proposition 3.14 is not valid as seen in the following example.

Example 3.15 Consider the example as presented in Example 3.2. Putting $U := \{a\}$. Then $U \not\supseteq \cap\{W \mid W \in \tau(a)\} = \{a, b\}$.

Theorem 3.16 Let U be a minimal I-open set in an ITSMI $(X, \tau, \mathcal{J})$. Then the following are valid.

(1) $U \subseteq \mathrm{Cl}(A)$ for any nonempty subset A of U.

(2) $\mathrm{Cl}(U) = \mathrm{Cl}(A)$ for any nonempty subset A of U.

Proof (1) Let A be an nonempty subset of U. By Proposition 3.13, for any element x of U and any open neighborhood W of x, we have $A = U \cap A \subseteq W \cap A$. Then $W \cap A \neq \emptyset$. Hence x is an element of $\mathrm{Cl}(A)$. It follows that $U \subseteq \mathrm{Cl}(A)$.

(2) For any nonempty subset A of U, we have $\mathrm{Cl}(A) \subseteq \mathrm{Cl}(U)$.

On the other hand, by (1), $\mathrm{Cl}(U) \subseteq \mathrm{Cl}(\mathrm{Cl}(A)) = \mathrm{Cl}(A)$. Therefore $\mathrm{Cl}(U) = \mathrm{Cl}(A)$.

The converses of Theorem 3.16 are not valid as seen in the following example.

Example 3.17 Consider an ideal topological space $(X, \tau, \mathcal{I})$ where $X = \{a, b, c, d\}$, $\tau = \{\emptyset, X, \{a, b\}, \{a, b, c\}\}$ and $\mathcal{I} = \{\emptyset\}$. Putting $U := \{a, b\}$. Then $\{a\}, \{b\}, U$ are nonempty subsets of U. We know that $\mathrm{Cl}(\{a\}) = \mathrm{Cl}(\{b\}) = \mathrm{Cl}(U) = X$. Since $\{a\} \subseteq \mathrm{Int}(\{a\}^*)$, $\{a\}$ is a I-open set. Hence U is not a minimal I-open set.

Theorem 3.18 Let U be a minimal I-open set in an ITSMI $(X, \tau, \mathcal{J})$ and let M be a nonempty subset of X. If there exists an open neighborhood W of M such that $W \subseteq \mathrm{Cl}(M \cup U)$, then $M \cup S$ is a pre-open set for any nonempty subset S of U.

Proof $\mathrm{Cl}(M \cup S) = \mathrm{Cl}(M) \cup \mathrm{Cl}(S) = \mathrm{Cl}(M) \cup \mathrm{Cl}(U) = \mathrm{Cl}(M \cup U)$ by Theorem 3.16. Since $W \subseteq \mathrm{Cl}(M \cup U)$ by assumption, we have $\mathrm{Int}(W) \subseteq \mathrm{Int}(\mathrm{Cl}(M \cup U)) = \mathrm{Int}(\mathrm{Cl}(M \cup S))$. Since W is an open neighborhood of M, namely W is an open set such that $M \subseteq W$, we have $M \subseteq W = \mathrm{Int}(W) \subseteq \mathrm{Int}(\mathrm{Cl}(M \cup S))$. Moreover, since U is I-open, $S \subseteq U \subseteq \mathrm{Int}(U^*) \subset \mathrm{Int}(\mathrm{Cl}(U)) \subseteq \mathrm{Int}(\mathrm{Cl}(M \cup U)) = \mathrm{Int}(\mathrm{Cl}(M \cup S))$. It follows that $M \cup S \subseteq \mathrm{Int}(\mathrm{Cl}(M \cup S))$. Therefore $M \cup S$ is pre-open.

Corollary 3.19 Let U be a minimal I-open set in an ITSMI $(X, \tau, \mathcal{J})$ and let M be a nonempty subset of X. If there exists an open neighborhood W of M such that $W \subseteq Cl(U)$, then $M \cup S$ is a pre-open set for any nonempty subset S of U.

Proof By assumption, we have $W \subseteq \mathrm{Cl}(U) \subseteq Cl(M) \cup \mathrm{Cl}(U) = \mathrm{Cl}(M \cup U)$. So by Theorem 3.18, we see that $M \cup S$ is pre-open.

The condition of Theorem 3.19, namely, $W \subseteq \mathrm{Cl}(M \cup U)$, does not necessarily imply the condition of Corollary 3.19, namely, $W \subseteq \mathrm{Cl}(U)$. We have the following example.

Example 3.20 Consider the example as presented in Example 3.2. Then $(X, \tau, \mathcal{I})$ is a minimal ideal topological space. Let $U = \{a\}$ and $M = W = \{c\}$. Then U is a minimal I-open. And $W = \{c\} \subseteq \mathrm{Cl}(\{c\} \cup \{a\}) \subseteq \mathrm{Cl}(M \cup U)$ and $W = \{c\} \not\subseteq \mathrm{Cl}(\{a\}) = \mathrm{Cl}(U)$.

Theorem 3.21 Let U be a minimal I-open set in an ITSMI $(X, \tau, \mathcal{J})$ and x an element of $X \setminus U$. Then $W \cap U = \emptyset$ or $U \subseteq W$ for any open neighborhood W of x.

Proof Let U be a minimal I-open set in an ITSMI $(X, \tau, \mathcal{J})$ and x an element of $X \setminus U$. Then since W is an open set, $W \cap U$ is an I-open set by Lemma 3.10. We have the result by Theorem 3.11.

Corollary 3.22 Let U be a minimal I-open set in an ITSMI $(X, \tau, \mathcal{J})$ and x an element of $X \setminus U$. Define $U_x =: \cap\{W : W$ is an open neighborhood of $x\}$. Then $U_x \cap U = \emptyset$ or $U \subseteq U_x$.

Proof Let U be a minimal I-open set in an ITSMI $(X, \tau, \mathcal{J})$ and x an element of $X \setminus U$. Then $W \cap U = \emptyset$ or $U \subseteq W$ for any open neighborhood W of x by Theorem 3.21. If $U \subseteq W$ for any open neighborhood W of x, then $U \subseteq \cap\{W : W$ is an open neighborhood of $x\} \equiv U_x$. Therefore $U \subseteq U_x$. Otherwise there exists an open neighborhood W of x such that $W \cap U = \emptyset$, then we have $U \cap U_x = \emptyset$.

Theorem 3.23 Let U be a minimal I-open set in an ITSMI $(X, \tau, \mathcal{J})$. Then any nonempty subset A of U is a pre-open set.

Proof Let A be an nonempty subset of U. Then since U is a minimal I-open set, $U \subseteq \mathrm{Int}(U^*)$. It follows that $A \subseteq U \subseteq \mathrm{Int}(U^*) \subseteq \mathrm{Int}(\mathrm{Cl}(U)) = \mathrm{Int}(\mathrm{Cl}(A))$ by Theorem 3.16 (2). Hence A is a pre-open set.

Theorem 3.24 Let $(X, \tau, \mathcal{J})$ be an ITSMI. Let V be a nonempty finite I-open set. Then there exists at least one (finite) minimal I-open set U such that $U \subseteq V$.

Proof If V is a minimal I-open set, we may set $U = V$. If V is not a minimal I-open set, then there exists an (finite) I-open set V_1 such that $\emptyset \neq V_1 \subsetneqq V$. If V_1 is a minimal I-open set, we may set $U = V_1$. If V_1 is not a minimal I-open set, then there exists an (finite) I-open set V_2 such that $\emptyset \neq V_2 \subsetneqq V_1 \subsetneqq V$. Continuing this process, we have a sequence of I-open sets

$$V \supsetneqq V_1 \supsetneqq V_2 \cdots \supsetneqq V_k \supsetneqq \cdots.$$

Since V is a finite set, this process repeats only finitely. Then, finally we get a minimal open set $U = V_n$ for some positive integer n.

Definition 3.25 A topological ideal space is said to be a locally finite ideal topological space if each of its elements is contained in a finite I-open set.

Example 3.26 Consider the example as presented in Example 3.2. It is easy to check that $(X, \tau, \mathcal{J})$ is a locally finite ideal topological space.

Corollary 3.27 Let $(X, \tau, \mathcal{J})$ be an ITSMI. Let X be a locally finite ideal topological space and V a nonempty open set. Then there exists at least one (finite) minimal I-open set U such that $U \subseteq V$.

Proof Let $(X, \tau, \mathcal{J})$ be an ITSMI. Let X be a locally finite ideal topological space and V a nonempty open set. Then since V is a nonempty set, there exists an element x of V. Since X is a locally finite ideal topological space, we have a finite I-open set V_x such that $x \in V_x$. It follows that $V \cap V_x$ is a finite I-open set by Lemma 3.10. So we get a minimal I-open set U such that $U \subseteq V \cap V_x \subseteq V$ by Theorem 3.24.

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极小I-开集

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摘要: 本文在理想拓扑空间中研究了极小I-开集. 利用一般拓扑学的理论, 获得了极小I-开集的一些刻画和性质, 同时研究了极小I-开集和其他类型开集之间的关系.

关键词: 理想拓扑空间; I-开集; 极小开集; 极小I-开集

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