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GLOBAL STABILITY ANALYSIS FOR THE SINGLE-SPECIES ECOLOGICAL MODEL WITH THE DISPERSAL AND DISCONTINUOUS CONTROL TERM BASED ON THE THRESHOLD POLICY

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Abstract: In this paper, the control problem of single-species ecological model with the dispersal is investigated via the threshold policy (TP). The positive equilibrium's existence theorem for the new model is obtained by using the uniform persistence theorem and Filippov theorem. The sufficient conditions to the uniqueness and globally asymptotical stability of the positive equilibrium are obtained based on the new model by applying graph theoretical approach of the coupled systems and constructing Lyapunov functions method. Related results of [6] are generalized.

Keywords: Filippov; single species model; patchy dispersal; global stability; threshold policy; differential inclusion

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1 Introduction

The study on the patch models became one central issue of concerns in the literature of ecology systems (see [1–5]), since it is an interesting problem to consider how the dispersal or migration of the species influences the global dynamics of the interacting ecological system.

Since the systems of discrete patchy models are usually high-dimensional, it is rather a challenge to study the uniqueness and stability of the positive equilibrium for patchy models from the mathematical aspect. The availably global dynamics criteria in the literatures mainly focus on the special case of two-patch (see [2]) or the permanence and existence of periodic solutions (see [3–6]).

Recently, Li and Shuai (see [6]) considered the following system that described growth and dispersal of a single species among n patches ($n \geq 2$),

$$\dot{x_i} = x_i f_i(x_i) + \sum_{j=1}^{n} d_{ij}(x_j - \alpha_{ij} x_i), \ \ i = 1, \cdots, n, \tag{1.1}$$

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here $x_i \in R_+$ represents population density of the species on patch i. $f_i \in C^1(R_+, R)$ represents the density dependent growth rate on patch i. Constant $d_{ij} \geq 0$ is the dispersal rate from patch j to i, and constant $\alpha_{ij} \geq 0$ can be selected to represent different boundary condition.

In [6], the authors studied the global stability of the coexistence equilibrium of system (1.1) by considering it as a coupled n sub-models on networks. A systematic approach to construct global Lyapunov functions for large-scale coupled systems was developed. Li and Shuai obtained the following sharp result for system (1.1).

Proposition 1 (see [6, Theorem 5.1]) Assume that the following assumptions hold.

(1) $(d_{ij})_{n\times n}$ is irreducible.

(2) $f_i'(x_i) \leq 0, x_i > 0, i = 1, 2, \cdots, n$, and there exists p such that $f_p'(x_p) \not\equiv 0$ in any open interval of R_+.

(3) System (1.1) is uniformly persistent.

(4) Solutions of system (1.1) are uniformly ultimately bounded.

Then system (1.1) has a positive equilibrium $E^* \in R_+^n$ which possesses globally asymptotical stability.

Although well-improved results were obtained in the above work on the single-species model with the dispersal, the model is not well studied when the discontinuous control is considered. In this paper, we will use a so-called threshold policy (TP) to control the single-species system.

In the context of fishing management, Collie and Spencer (see [8]) introduced a so-called threshold policy (TP), which was intermediate between the well-known constant escapement and constant harvest rate policies. A TP is defined as follows: if estimated species abundance is below a previously chosen threshold level, harvesting is suppressed; above the threshold, harvesting is applied. TP is also an alternative strategy used in systems such as terrestrial harvesting (see [11]), grazing (see [12]) and control of aquatic vegetation (see [13]) etc.

A lot of researchers were interested in the threshold policy in the recent years (see [14–20, 38–41]). In 2000 (see [14]), authors analyzed the dynamics of two predator-prey models(Lotka-Volterra and Leslie-Gower) via the weighted escapement policy. In 2005 (see [15]), stability of predator-prey models with TP was studied by using the idea of backstepping and control Lyapunov functions (CLF). In 2010 (see [16]), the concept of virtual equilibria was used to design three different kinds of threshold policies. In 2011 (see [17]), yield and related economic items generated by a TP were studied. In 2012 (see [18]), a specific management strategy was proposed in order to control pests. In 2013 (see [19]), a specific threshold policy was designed in order to control plant diseases and eventually maintain the number of infected plants below an economic threshold. In 2014 (see [20]), a Filippov epidemic model with media coverage was proposed to describe the real characteristics of media/psychological impact in spread of the infectious disease. Mathematical bifurcation analyses with regard to the local, global stability of equilibria and local sliding bifurcations were performed.

In this paper, the single-species ecological system with the dispersal among n patches is studied. The specific TP is designed to control the increasing of the single species on patch k.

In this part, we will generalize model (1.1) into the new model (1.2).

First of all, the assumptions of model (1.2) are listed as follows.

(1) $x_i \in R_+$ represents population density of the species on patch i.

(2) $f_i \in C^1(R_+, R)$ represents the density dependent growth rate on patch i.

(3) Constant $d_{ij} \geq 0$ is the dispersal rate from patch j to i, and constant $\alpha_{ij} \geq 0$ can be selected to represent different boundary condition.

(4) $\theta_k > 0$ represents the roguing proportional of the species on patch k.

Second, the control aim is listed as follows.

The Control Aim Through controlling the population density of kth patch less than ET via the TP

$$I(x_k) = \begin{cases} 0, & x_k < ET, \\ 1, & x_k > ET, \end{cases}$$

the number of species on each patch will be eventually stable at some corresponding positive value.

Therefore, the following single-species ecological model with the dispersal and discontinuous control term is constructed,

$$\begin{cases} \dot{x_i} = x_i f_i(x_i) + \sum\limits_{j=1}^{n} d_{ij}(x_j - \alpha_{ij} x_i), \ \ i \in \{1, \cdots, n\} - \{k\}, \\ \dot{x_k} = x_k f_k(x_k) + \sum\limits_{j=1}^{n} d_{kj}(x_j - \alpha_{kj} x_k) - \theta_k x_k I(x_k), \ \ i = k. \end{cases} \tag{1.2}$$

The network method was applied widely in recently years (see [22–30, 35–37]). In this paper, we interpret system (1.2) as a coupled system on a network. Using the method of Li and Shuai [6] and Filippov system theory, we prove positive equilibrium's existence theorems and global stability theorems.

A mathematical description of a network is a directed graph consisting of vertices and directed arcs connecting them. At each vertex, the local dynamics are given by a system of differential equations called the vertex system. The directed arcs indicate inter-connections and interactions among vertex systems.

A digraph G with n vertices for system (1.2) can be constructed as follows. Each vertex represents a patch and $(j, i) \in E(G)$ if and only if $d_{ij} > 0$, here $E(G)$ denotes the set of arcs (i, j) leading from inial vertex i to terminal vertex j. At each vertex of G, the vertex dynamics are described by a system

$$\dot{x_i} = x_i f(x_i)(i \neq k) \ \text{ or } \ \dot{x_k} = x_k f(x_k) - \theta_k I(x_k).$$

The coupling among system (1.2) is provided by dispersal of species among patches. The dispersal network G is strongly connected if and only if the dispersal matrix $(d_{ij})_{n \times n}$ is irreducible.

From the ecology viewpoint, when the population density of the species on patch k exceeds ET, then the control is implemented to reduce the population density of the species on patch k. While, when the population density of the species on patch k is less than ET, it is not necessary to implement control.

Remark 1 The term of $\theta_k x_k$ comes from Zhao (see [19]).

On one hand, the value of the roguing rate θ_k is dependent on the number of available workers.

On the other hand, such a roguing term is reasonable in mathematics.

Remark 2 It is natural and reasonable to adopt the threshold policy in order to control the population density of the species on some patch. Besides, the control cost is reasonable.

Our contribution is listed as follows.

(1) Existence conditions of positive equilibria for system (1.2) are obtained by the uniform persistence theory and Filippov theory.

(2) Sufficient conditions that the positive coexistence equilibrium of the coupling model is unique and globally asymptotically stable are derived by using the method of constructing Lyapunov functions based on graph-theoretical approach for coupled systems.

This paper is organized as follows. We introduce preliminary results on graph theory based on coupled network models in Section 2. In Section 3, we obtain main results. Finally, the conclusions and outlooks are drawn in Section 4.

2 Preliminaries

In this section, we will list some definitions and theorems which will be used in the later sections.

A directed graph or digraph $G = (V, E)$ contains a set $V = \{1, 2, \cdots, n\}$ of vertices and a set E of arcs (i, j) leading from initial vertex i to terminal vertex j. A subgraph H of G is said to be spanning if H and G have the same vertex set. A digraph G is weighted if each arc (j, i) is assigned a positive weight. $a_{ij} > 0$ if and only if there exists an arc from vertex j to i in G.

The weight $w(H)$ of a subgraph H is the product of the weights on all its arcs. A directed path P in G is a subgraph with distinct vertices $i_1, i_2, \cdots, i_m$ such that its set of arcs is $\{(i_k, i_{k+1}) : k = 1, 2, \cdots, m\}$. If $i_m = i_1$, we call P a directed cycle.

A connected subgraph T is a tree if it contains no cycles, directed or undirected.

A tree T is rooted at vertex i, called the root, if i is not a terminal vertex of any arcs, and each of the remaining vertices is a terminal vertex of exactly one arc. A subgraph Q is unicyclic if it is a disjoint union of rooted trees whose roots form a directed cycle.

Given a weighted digraph G with n vertices, the weight matrix $A = (a_{ij})_{n\times n}$ can be defined by their entry a_{ij} equals the weight of arc (j, i) if it exists, and 0 otherwise. For our purpose, we denote a weighted digraph as (G, A). A digraph G is strongly connected if, for any pair of distinct vertices, there exists a directed path from one to the other. A weighted digraph (G, A) is strongly connected if and only if the weight matrix A is irreducible.

The Laplacian matrix of (G, A) is denoted by L. Let c_i denote the cofactor of the i-th diagonal element of L. The following results are listed.

Theorem 2.1 [6] Assume $n \geq 2$. Then

$$c_i = \sum_{\mathbf{T} \in \mathbf{T}_i} w(\mathbf{T}),$$

where T_i is the set of all spanning trees $\mathbf{T}$ of (G, A) that are rooted at vertex i, and $w(T)$ is the weight of T. In particular, if (G, A) is strongly connected, then $c_i > 0$ for $1 \leq i \leq n$.

Theorem 2.2 [6] Assume $n \geq 2$. Let c_i be given in Theorem 2.1. Then the following identity holds

$$\sum_{i,j=1}^{n} c_i a_{ij} F_{ij}(x_i, x_j) = \sum_{Q \in \mathbb{Q}} w(Q) \sum_{(s,r) \in E(C_{\mathrm{Q}})} F_{rs}(x_r, x_s),$$

here $F_{ij}(x_i, x_j), 1 \leq i, j \leq n$, are arbitrary functions, Q is the set of all spanning unicyclic graphs of (G, A), $w(Q)$ is the weight of Q, and C_Q denotes the directed cycle of Q.

Given a network represented by digraph G with n vertices ($n \geq 2$), a coupled system can be built on G by assigning each vertex its own internal dynamics and then coupling these vertex dynamics based on directed arcs in G. Assume that each vertex dynamics are described by a system of differential equations $u_i^{'} = f_i(t, u_i)$, where $u_i \in \mathbf{R}^{m_i}$ and $f_i : \mathbf{R} \times \mathbf{R}^{m_i} \to \mathbf{R}^{m_i}$. Let $g_{ij} : \mathbf{R} \times \mathbf{R}^{m_i} \times \mathbf{R}^{m_j} \to \mathbf{R}^{m_i}$ represent the influence of vertex j on vertex i, and $g_{ij} \equiv 0$ if there exists no arc from j to i in G. Then we obtain the following coupled system on graph G:

$$u_i^{'} = f_i(t, u_i) + \sum_{j=1}^{n} g_{ij}(t, u_i, u_j). i = 1, 2, \cdots, n,$$

here we assume that the initial-value problem has the unique solution.

We assume that each vertex system has a globally stable equilibrium and possesses a global Lyapunov function V_i.

Theorem 2.3 [6] Assume that the following assumptions are satisfied.

(1) There exist functions $V_i(t, u_i), F_{ij}(t, u_i, u_j)$, and constants $a_{ij} \geq 0$ such that

$$\dot{V}_i(t, u_i) \leq \sum_{i,j=1}^{n} a_{ij} F_{ij}(t, u_i, u_j), t > 0, u_i \in D_i.$$

(2) Along each directed cycle C of the weighted digraph (G, A), $A = (a_{ij})$,

$$\sum_{(s,r) \in E(C)} F_{rs}(t, u_r, u_s) \leq 0.$$

(3) Constant c_i is given by the cofactor of the i-th diagonal element of L.

Then the function $V(t, u) = \sum_{i=1}^{n} c_i V_i(t, u_i)$ satisfies $\dot{V}(t, u) \leq 0$ for $t > 0$, and $u \in D = D_1 \times D_2 \cdots \times D_n$. Namely, V is a Lyapunov function for the system.

3 Main Results

Filippov solutions will be used for the discontinuous system (1.2). Consider the differential inclusion as follows

$$\begin{cases} \dot{x_i} = x_i f_i(x_i) + \sum\limits_{j=1}^{n} d_{ij}(x_j - \alpha_{ij}x_i), \ \ i \neq k, \\ \dot{x_k} \in x_k f_k(x_k) + \sum\limits_{j=1}^{n} d_{kj}(x_j - \alpha_{kj}x_k) - \theta_k x_k \bar{co}[I(x_k)], \ \ i = k, \end{cases} \tag{3.1}$$

here $\bar{\text{co}}[I(x_k)]$ denotes the convex closure of $I(x_k)$.

Definition 1 $x(t) = (x_1(t), x_2(t), \cdots, x_n(t))$ is the solution of system (1.2) with initial value $x_1(0) = x_{1_0}, x_2(0) = x_{2_0}, \cdots, x_n(0) = x_{n_0}$, if

(1) $x(t)$ is defined in the interval $[0, T)$ with $T \in (0, +\infty]$.

(2) $x(t)$ is absolutely continuous in any subinterval of $[0, T)$.

(3) $x(t)$ is the solution of system (3.1) for a.e. $t \in [0, T)$.

We assume that $x^* = (x_1^*, x_2^*, \cdots, x_n^*)$ is a positive equilibrium of system (1.2). By system (3.1) and measurable selection theorem (see [21]), there is $\eta_k^* \in \bar{\text{co}}[I(x_k^*)]$ such that

$$\begin{cases} x_i^* f_i(x_i^*) + \sum\limits_{j=1}^{n} d_{ij}(x_j^* - \alpha_{ij}x_i^*) = 0, \ \ i \neq k. \\ x_k^* f_k(x_k^*) + \sum\limits_{j=1}^{n} d_{kj}(x_j^* - \alpha_{kj}x_k^*) - \theta_k \eta_k^* x_k^* = 0, \ \ i = k \end{cases} \tag{3.2}$$

for a.e. $t \in [0, T)$.

In this section, first of all, the existence of the positive equilibrium for system (1.2) is shown. The uniform persistence theory and Filippov theory are used to discuss the problem. Secondly, sufficient conditions that the positive coexistence equilibrium of the coupling model is unique and globally asymptotically stable in R_+^n as long as it exists are derived by using the method of constructing Lyapunov functions based on graph-theoretical approach for coupled systems.

3.1 The Existence Conditions of the Positive Equilibrium for System (1.2)

Define $A_1(x) = \text{diag}[f_1(x_1), f_2(x_2), \cdots, f_n(x_n)]$ and

$$A_2(x) = (d_{ij})_{n\times n} - \text{diag}[\sum_{j=1}^{n} d_{1j}\alpha_{1j}, \sum_{j=1}^{n} d_{2j}\alpha_{2j}, \cdots, \sum_{j=1}^{n} d_{nj}\alpha_{nj}].$$

Then we obtain that the existence theorem for positive solutions of system (1.2).

Theorem 3.1 The solution $x(t)$ of system (1.2) satisfies that $\forall t \in [0, t_0)$, $x(t) > 0$, if it is defined in the interval $[0, t_0)$ $(0 < t_0 \leq +\infty)$ and the initial value satisfies that

$$x(0) = (x_1(0), x_2(0), \cdots, x_n(0)) > 0.$$

Proof By the definition of $I(x_k)$, there exists $\delta > 0$ $(\delta < ET)$, such that when $|x_k| < \delta$, $I(x_k) = 0$ holds. Consider $x(t)$ with $x(0) > 0$ and $||x(t)|| < \delta$, then system (1.2) can be

simplified as follows $\dot{x} = [A_1(x)+A_2(x)]x$. In the sequel, we deduce that $x(t) = x_0 \exp[A_1(x)+A_2(x)]t$. Therefore, we obtain that $x(t) > 0$, if $x(0) > 0$ and $||x(t)|| < \delta$.

By applying the absolutely continuous character of the solutions, it follows that $\forall t \in [0, t_0)$, $x(t) > 0$ holds. This completes the proof.

Now we will consider the existence conditions for the equilibrium of system (1.2). The theorem is listed as follows.

Theorem 3.2 Assume that the following assumptions hold for system (1.1).

(1) The system is uniformly persistent.

(2) Solutions are uniformly ultimately bounded.

Then system (1.2) has one positive equilibrium at least, if the suitable ET is chosen.

Proof Using the Theorem 5.1 of Li and Shuai [6], we obtain that system (1.1) has one equilibrium at least. Let $x^* = (x_1^*, x_2^*, \cdots, x_n^*)$ denote the positive equilibrium of system (1.1) . Then by choosing $ET \geq x_k^*$, we have $\{0\} \in \bar{\text{co}}[I(x_k^*)]$. Therefore, we can choose $r^* = 0 \in \bar{\text{co}}[I(x_k^*)]$ such that $x_i f_i(x_i) + \sum\limits_{j=1}^{n} d_{ij}(x_j - \alpha_{ij}x_i) = 0$ for $i = 1, 2, \cdots, n$. It means that x^* is the positive equilibrium of system (1.2). This completes the proof.

Given the following system

$$\begin{cases} \dot{x_i} = x_i f_i(x_i) + \sum\limits_{j=1}^{n} d_{ij}(x_j - \alpha_{ij}x_i), \ \ i \in \{1, \cdots, n\} - \{k\}, \\ \dot{x_k} = x_k f_k(x_k) + \sum\limits_{j=1}^{n} d_{kj}(x_j - \alpha_{kj}x_k) - \theta_k x_k, \ \ i = k. \end{cases} \tag{3.3}$$

Using the uniformly persistence theory (see [34]) and Theorem 3.2, the corollary is obtained naturally.

Corollary 1 Assume that the following assumptions hold for system (3.3).

(1) The system is uniformly persistent.

(2) Solutions are uniformly ultimately bounded.

Then system (1.2) has one positive equilibrium at least, if the suitable ET is chosen.

3.2 The Stability Analysis of the Positive Equilibrium for System (1.2)

Similar to system (3.1), when $x(t) = (x_1(t), x_2(t), \cdots, x_n(t))$ is the solution of system (1.2), then for any $v \in \bar{\text{co}}[I(x_k(t))]$, there is a measurable function $\eta_k(t) \in \bar{\text{co}}[I(x_k(t))]$ (measurable selection theorem (see [21])) such that

$$\begin{cases} \dot{x_i} = x_i f_i(x_i) + \sum\limits_{j=1}^{n} d_{ij}(x_j - \alpha_{ij}x_i), \ \ i \in \{1, \cdots, n\} - \{k\}, \\ \dot{x_k} = x_k f_k(x_k) + \sum\limits_{j=1}^{n} d_{kj}(x_j - \alpha_{kj}x_k) - \theta_k \eta_k(t) x_k, \ \ i = k \end{cases} \tag{3.4}$$

for a.e. $t \in [0, T)$.

The main result is listed as follows.

Theorem 3.3 Assume that the following assumptions hold.

(1) $(d_{ij})_{n\times n}$ is irreducible.

(2) $f_i'(x_i) \leq 0, x_i > 0, i = 1, 2, \cdots, n$, and there exists p such that $f_p'(x_p) \not\equiv 0$ in any open interval of R_+.

(3) There exists a positive equilibrium $x^* = (x_1^*, x_2^*, \cdots, x_n^*)$ for system (1.2).

Then positive equilibrium x^* of system (1.2) is unique and globally asymptotically stable in R_+^n.

Proof After tedious calculation, we obtain that the positive equilibrium x^* satisfies

$$\begin{cases} f_i(x_i^*) = -\sum\limits_{j=1}^{n} d_{ij}(\frac{x_j^*}{x_i^*} - \alpha_{ij}), i \neq k, \\ f_k(x_k^*) = \theta_k\eta_k^* - \sum\limits_{j=1}^{n} d_{kj}(\frac{x_j^*}{x_k^*} - \alpha_{kj}), i = k. \end{cases} \tag{3.5}$$

Set $V_i(x_i) = x_i - x_i^* + x_i^* ln\frac{x_i}{x_i^*}$. It can be verified that $V_i(x_i) > 0$ for all $x_i > 0$ and $V_i(x_i) = 0$ if and only if $x_i = x_i^*$. After direct calculation, we have $(i \neq k)$

$$\dot{V}_i(x_i) = (x_i - x_i^*)[f_i(x_i) - f_i(x_i^*)] + \sum_{j=1}^{n} d_{ij}x_j^*(\frac{x_j}{x_j^*} - \frac{x_i}{x_i^*} + 1 - \frac{x_i^*x_j}{x_ix_j^*}).$$

Let $a_{ij} = d_{ij}x_j^*$, $F_{ij}(x_i, x_j) = \frac{x_j}{x_j^*} - \frac{x_i}{x_i^*} + 1 - \frac{x_i^*x_j}{x_ix_j^*}$ and $G_i(x_i) = -\frac{x_i}{x_i^*} + ln\frac{x_i^*}{x_i}$. Then we obtain that $\dot{V}_i(x_i) \leq \sum\limits_{j=1}^{n} a_{ij}F_{ij}(x_i, x_j)$, and

$$F_{ij}(x_i, x_j) = G_i(x_i) - G_j(x_j) + 1 - \frac{x_i^*x_j}{x_ix_j^*} + ln\frac{x_i^*x_j}{x_ix_j^*}.$$

In the sequel, we obtain that $F_{ij}(x_i, x_j) \leq G_i(x_i) - G_j(x_j)$. When $i = k$, we have

$$\begin{aligned} \dot{V}_k(x_k) &= (x_k - x_k^*)[(f_k(x_k) - f_k(x_k^*)) + \theta_k(\eta_k^* - \eta_k(t))] \\ &\quad + \sum_{j=1}^{n} d_{kj}x_j^*(\tfrac{x_j}{x_j^*} - \tfrac{x_k}{x_k^*} + 1 - \tfrac{x_k^*x_j}{x_kx_j^*}) \end{aligned}$$

for a.e. $t \in [0, T)$.

Now the sign of $(x_k - x_k^*)(\eta_k^* - \eta_k(t))$ will be discussed. By applying the monotonic property of $I(x_k)$, we obtain that

When $x_k \geq x_k^*$, $\eta_k(t) \geq \eta_k^*$ holds. When $x_k \leq x_k^*$, $\eta_k(t) \leq \eta_k^*$ holds. It means that $\theta_k(x_k - x_k^*)(\eta_k^* - \eta_k(t)) \leq 0$. In the sequel,

$$\dot{V}_k(x_k) \leq \sum_{j=1}^{n} a_{kj}F_{kj}(x_k, x_j)$$

for a.e. $t \in [0, T)$.

Let c_i denote the cofactor of the i-th diagonal element of Laplacian Matrix of G. Let Lyapunov function $V(x) = V(x_1, x_2, \cdots, x_n) = \sum\limits_{i=1}^{n} c_iV_i(x_i)$. Then we have $\dot{V}(x(t)) \leq 0$ for

a.e. $t \in [0,T)$. If $\dot{V}=0$, we obtain that $(x_p - x_p^*)[f_p(x_p) - f_p(x_p^*)] = 0$ for a.e. $t \in [0,T)$. Therefore, we deduce that $x_p = x_p^*$ for a.e. $t \in [0,T)$. By applying the absolutely continuous character of the solutions, it follows that $x_p \equiv x_p^*$. By using the strong connectivity of (G,A) and

$$a_{pj}(1 - \frac{x_p^* x_j}{x_p x_j^*} + \ln \frac{x_p^* x_j}{x_p x_j^*}) = 0 \ (\forall j \neq p)$$

for a.e. $t \in [0,T)$, we obtain that $\frac{x_p^* x_j}{x_p x_j^*} = 1$ for a.e. $t \in [0,T)$. By applying the absolutely continuous character of the solutions and $x_p \equiv x_p^*$, it follows that $x_j = x_j^*$ for any $j = 1,2,\cdots,n$. Furthermore, we obtain that the maximum weak invariant subset of $Z_V^{2.1} \bigcap L_l$ is the set of unique point $M = \{x^*\}$. By applying the invariance principle of differential inclusion (see [21]), we obtain that x^* is globally asymptotically stable in R_+^n. Here

$$Z_V^{2.1} = \{x \in R^n | 0 \in \dot{\bar{V}}|^{(2.1)}(x)\}, \ \ L_l = \{x \in R^n | V(x) \leq l\}$$

and

$$\begin{aligned} \dot{\bar{V}}|^{(2.1)}(x) &= \{< \nabla_x V(x), v > | v \in F(x)\}, \\ F(v) &= \begin{pmatrix} x_1 f_1(x_1) + \sum\limits_{j=1}^{n} d_{1j}(x_j - \alpha_{1j} x_1) \\ x_2 f_2(x_2) + \sum\limits_{j=1}^{n} d_{2j}(x_j - \alpha_{2j} x_2) \\ \vdots \\ x_k f_k(x_k) - \theta_k x_k \bar{\text{co}}[I(x_k)] + \sum_{j=1}^{n} d_{kj}(x_j - \alpha_{kj} x_k) \\ \vdots \\ x_n f_n(x_n) + \sum\limits_{j=1}^{n} d_{nj}(x_j - \alpha_{nj} x_n) \end{pmatrix}. \end{aligned}$$

This completes the proof.

Furthermore, the control can be used on all of patches for system (1.1). Consider the following system

$$\dot{x_i} = x_i f_i(x_i) + \sum_{j=1}^{n} d_{ij}(x_j - \alpha_{ij} x_i) - \theta_i x_i I(x_i), \ \ i = 1,\cdots,n, \tag{3.6}$$

here

$$I(x_i) = \begin{cases} 0, & x_i < ET, \\ 1, & x_i > ET, \end{cases}$$

$\theta_i > 0$ represents the roguing proportional for the species on the ith patch.

In the sequel, Corollary 2 is obtained.

Corollary 2 Assume that the following assumptions hold.

(1) $(d_{ij})_{n\times n}$ is irreducible.

(2) $f_i'(x_i) \leq 0, x_i > 0, i = 1,2,\cdots,n$, and there exists p such that $f_p'(x_p) \not\equiv 0$ in any open interval of R_+.

(3) There exists a positive equilibrium $x^* = (x_1^*, x_2^*, \cdots, x_n^*)$ for system (3.6).
Then positive equilibrium x^* of system (3.6) is unique and globally asymptotically stable in R_+^n.

Remark 3 Corollary 2 can be seen as the development of the Theorem 2.3. It means that network method can be applied in the Filippov system.

4 Conclusions and Outlooks

In this paper, we generalize the single-species ecological model (1.1) to the general model (1.2) with the discontinuous control term for the kth patch. Firstly, the uniform persistence and Filippov theory are used to prove the existence of positive equilibrium. We obtain the existence condition for the positive equilibrium which can be seen as the development of the Theorem 5.1 in [6]. Second, the globally asymptotical stability of positive equilibria of system (1.2) and (2.6) is proved based on the network method for coupled systems of differential equations, Filippov theory and differential inclusion. Our main theorems generalize Theorem 5.1 and 3.1 in [6].

Biologically, our result Theorem 3.3 implies that, if we consider to control the population density of kth patch less than ET by using the TP, the single-species ecological model with dispersal is dispersing among strongly-connected patches (which is equivalent to the irreducibility of the dispersal matrix), and if the system has an equilibrium at least, then the number of species in each patch will be eventually stable at some corresponding positive value.

Further studies on this subject are being carried out by the presenting authors in the two aspects: one is to study the TP with time delay; the other is to discuss the method to design control term via the TP.

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基于阈值策略带有扩散和不连续控制项的单物种生态模型的全局稳定性分析

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摘要: 本文通过阈值策略 (TP)研究了带有扩散的单物种生态模型的控制问题. 利用一致持久性理论和 Filippov理论方法, 得到了新模型的正平衡点的存在性定理. 通过使用耦合系统的图论方法和构建Lyapunov函数思想, 得到了新模型的正平衡点唯一且全局渐进稳定的充分条件. 推广了文献[6]的相关结果.

关键词: 菲利波夫; 单物种模型; 斑块的扩散; 全局稳定; 阈值策略; 微分包含

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