

COMPOSITION OF PSEUDO AUTOMORPHIC STOCHASTIC PROCESS AND APPLICATIONS

ZHOU Hui¹, ZHOU Zong-fu², QIAO Zong-min¹

(1. *School of Mathematics and Statistics, Hefei Normal University, Hefei 230061, China*)

(2. *School of Mathematical Science, Anhui University, Hefei 230039, China*)

Abstract: In this paper, a composition theorem for pseudo-almost automorphic stochastic process is established under a weaker Lipschitz condition. The composition theorem is more general than some known results. Using these properties, we obtain the existence and uniqueness of pseudo-almost automorphic mild solutions for a class of stochastic differential equations.

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1 Introduction

Almost periodic functions were introduced in [1], and Zhang [2] introduced the concept of pseudo-almost periodic functions, which are a natural generalization of almost periodic functions. The investigation of the existence of almost periodic, pseudo-almost periodic, almost automorphic and pseudo-almost automorphic solutions is one of the most interesting topics in the qualitative theory of differential equations due to their applications [3–10] in physics, biology, dynamical systems and so on. Recently, in [11], the concept of square-mean almost periodicity was introduced and studied.

As we all know, most real problems in a real life situation are basically modeled by stochastic equations rather than deterministic, because a natural extension of a deterministic model is stochastic model. Bezandry et al. [12] studied the existence of almost periodic or pseudo almost periodic solutions to stochastic functional differential equations. However, almost automorphic functions are a natural generalization of almost periodic functions. Fu and Liu [13] introduced a new notion of a square-mean almost automorphic stochastic process and the authors studied the existence and uniqueness of square-mean almost automorphic mild solutions to a class of automorphic differential equations.

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Biography: Zhou Hui (1986–), male, born at Lixin, Anhui, major in the theory of functional differential equations and stochastics differential equations.

Huang and Yang [14] established some criteria ensuring the existence and uniqueness of almost periodic solutions for the following stochastic equations with delays

$$dx(t) = [-Ax(t) + Bf(t, x(t - \sigma))]dt + G(t, x(t)) \circ d\omega(t), t \in R. \quad (1.1)$$

In recent papers, Cao et al. [15] concerned with the existence of square-mean almost periodic mild solutions for stochastic differential equations

$$dx(t) = (Ax(t) + F(t, x(t), x_t))dt + G(t, x(t), x_t) \circ d\omega(t), t \in [0, T]. \quad (1.2)$$

Some sufficient conditions are obtained by semigroups of operators and fixed point method.

To the best of our knowledge, under weaker Lipschitz conditions, there are few results on the existence and uniqueness of pseudo-almost automorphic mild solutions of stochastic differential equations. Motivated by the works [13, 14], the main purpose of this paper is to study the composition theorem for pseudo-almost automorphic process under weaker Lipschitz conditions, and established the existence, uniqueness and stability of square-mean pseudo-almost periodic mild solutions for semilinear stochastic differential equations

$$\begin{aligned} dx(t) &= [Ax(t) + F(t, Bx_t)]dt + G(t, x(t)) \circ d\omega(t), t \in [0, T], \\ x(t) &= \varphi(t), t \in [-\sigma, 0]. \end{aligned} \quad (1.3)$$

We assume that A generates a C_0 semigroup $T(t)$ satisfying $\|T(t)\| \leq Me^{-\mu t}$ for some $M, \mu > 0$, and $B : D(B) \subseteq X \rightarrow X$ is closed linear operator, where X is a Banach space endowed with a norm $\|\cdot\|$. F, G are two appropriate functions to be specified later.

The rest of this paper is organized as follows. In Section 2, we recall some basic definitions, lemmas and preliminary facts which will be useful throughout this paper. In Section 3, we establish a composition for pseudo-almost automorphic process. In Section 4, the existence and uniqueness of square-mean pseudo-almost periodic mild solutions of (1.3) are proved.

2 Preliminaries

Now let us state some definitions and lemmas, which will be used in what follows. Let (Ω, F, P) be a probability space. For a Banach spaces $(B, \|\cdot\|)$, the notation $L^2(P, B)$ stands for the space of all B -valued random variables x such that

$$\|x\|_{L^2(P, B)} = \left(\int_{\Omega} \|x\|^2 dp \right)^{\frac{1}{2}} < \infty.$$

Definition 2.1 (see [12]) A stochastic process $x : R \rightarrow L^2(P, B)$ is said to be continuous, provided that, for any $s \in R$, $\lim_{t \rightarrow s} E\|x(t) - x(s)\|^2 = 0$.

Definition 2.2 (see [12]) A stochastic process $x : R \rightarrow L^2(P, B)$ is said to be stochastically bounded if there exists $M > 0$ such that $E\|x(t)\|^2 \leq M$ for all $t \in R$.

Let $SBC(R, L^2(P, B))$ be the collection of all the stochastic bounded and continuous process. It is easy to see that $SBC(R, L^2(P, B))$ is a Banach space.

Definition 2.3 (see [18]) A stochastic process $x(t) \in SBC_0(R, L^2(P, B))$, if

$$x(t) \in SBC(R, L^2(P, B))$$

and

$$\lim_{r \rightarrow \infty} \frac{1}{2r} \int_{-r}^r E \|x(t)\|^2 dt = 0.$$

Definition 2.4 (see [18]) A stochastic process $x : R \rightarrow L^2(P, B)$ is said to be square-mean almost automorphic, if for every sequence real number $\{s'_n\}_{n \in N}$, there exists a subsequence $\{s_n\}_{n \in N}$ and a stochastic process $y : R \rightarrow L^2(P, B)$ such that $\lim_{n \rightarrow s} E \|x(t + s_n) - y(t)\|^2 = 0$ and $\lim_{n \rightarrow s} E \|y(t - s_n) - x(t)\|^2 = 0$ hold for each $t \in R$.

Denote the collection of all square-mean almost automorphic stochastic process by $SAA(R, L^2(P, B))$. By [18], we know that $SAA(R, L^2(P, B))$, $\|\cdot\|_\infty$ is a Banach space when it is equipped with norm $\|x\|_\infty := \sup_{t \in R} \|x(t)\| = \sup_{t \in R} (E \|x(t)\|^2)^{\frac{1}{2}}$ for $x \in SAA(R, L^2(P, B))$.

Definition 2.5 (see [18]) A stochastically continuous process $f : R \rightarrow L^2(P, B)$ is said to be square-mean pseudo-almost automorphic if it can be decomposed as $f = g + \varphi$, where $g \in SAA(R, L^2(P, B))$ and $\varphi \in SBC_0(R, L^2(P, B))$.

The collection of all the square-mean pseudo-almost automorphic processes $f : R \rightarrow L^2(P, B)$ is denoted by $SPAA(R, L^2(P, B))$. It is obvious that

$$SAA(R, L^2(P, B)) \subset SPAA(R, L^2(P, B)) \subset SBC(R, L^2(P, B)).$$

Definition 2.6 (see [17,19]) A stochastically process $f(t, x) : R \times L^2(P, B) \rightarrow L^2(P, B)$, which is jointly continuous, is said to be square-mean pseudo-almost automorphic in t for any $x \in L^2(P, B)$ if it can be decomposed as $f = g + \varphi$, where

$$g(t, x) \in SAA(R \times L^2(P, B), L^2(P, B)), \varphi(t, x) \in SBC_0(R \times L^2(P, B), L^2(P, B)).$$

3 The Composition Theorem

In this section, we establish a composition theorem for stochastically square-mean pseudo-almost automorphic processes. Firstly, we prove some lemmas which are useful to obtain the composition theorem.

Lemma 3.1 Assume that $f \in SBC_0(R \times L^2(P, B), L^2(P, B))$ satisfying that for any $\varepsilon > 0$, there exists $\delta > 0$ and stochastic process $L(t) \in SBC(R, L^2(P, B))$ such that

$$E \|f(t, x) - f(t, y)\|^2 \leq E \|L(t)\|^2 \varepsilon \quad (3.1)$$

for all $t \in R$ and $x, y \in K$ with $\|x - y\| < \delta$, where $K \subset L^2(P, B)$ is compact. Then

$$\lim_{r \rightarrow \infty} \frac{1}{2r} \int_{-r}^r \sup_{x \in K} E \|f(t, x)\|^2 dt = 0.$$

Proof For any given $\varepsilon > 0$, let δ and $L(t)$ be as in the assumptions. Take $\varepsilon' = \min\{\varepsilon, \delta\}$. Since K is compact, there exists $x_1, \dots, x_k \in K$ such that $K \subset \bigcup_{i=1}^k B(x_i, \varepsilon')$. From the definition of $SBC_0(R \times L^2(P, B), L^2(P, B))$, for the above $\varepsilon > 0$, there is constant $T^* > 0$ such that

$$\frac{1}{2r} \int_{-r}^r E \|f(t, u_i)\|^2 dt < \frac{\varepsilon}{k}, \quad r > r^*. \quad (3.2)$$

For each $x \in K$, there exists $u_{ix} \in \{u_1, u_2, \dots, u_k\}$ such that $\|u_{ix} - u\| < \varepsilon' < \delta$. Then, from (3.1), we have

$$E \|f(t, x)\|^2 \leq E \|f(t, x) - f(t, u_{ix})\|^2 + E \|f(t, u_{ix})\|^2 \leq E \|L(t)\|^2 \varepsilon + E \|f(t, u_{ix})\|^2.$$

Then for each $t \in R$,

$$\sup_{x \in K} E \|f(t, x)\|^2 \leq E \|L(t)\|^2 \varepsilon + \sum_{i=1}^k E \|f(t, u_i)\|^2. \quad (3.3)$$

By (3.2) and (3.3), for all $r > r^*$, we obtain that

$$\begin{aligned} \frac{1}{2r} \int_{-r}^r \sup_{x \in K} E \|f(t, x)\|^2 dt &\leq \frac{\varepsilon}{2r} \int_{-r}^r E \|L(t)\|^2 dt + \frac{1}{2r} \sum_{i=1}^k \int_{-r}^r E \|f(t, u_i)\|^2 dt \\ &\leq \frac{\varepsilon}{2r} \int_{-r}^r E \|L(t)\|^2 dt + \sum_{i=1}^k \frac{\varepsilon}{k} \leq (\|L\| + 1)\varepsilon, \end{aligned}$$

which implies that

$$\lim_{r \rightarrow \infty} \frac{1}{2r} \int_{-r}^r \sup_{x \in K} E \|f(t, x)\|^2 dt = 0.$$

Theorem 3.2 If the assumptions of Lemma 3.1 hold,

$$f = g + \varphi \in SPAA(R \times L^2(P, B), L^2(P, B)),$$

where $g \in SAA(R \times L^2(P, B), L^2(P, B))$ and $\varphi \in SBC_0(R \times L^2(P, B), L^2(P, B))$. Moreover, $g(t, u)$ is uniformly continuous in any compact subset $K \subset L^2(P, B)$ uniformly for $t \in R$. Then for each

$$x \in SPAA(R \times L^2(P, B), L^2(P, B)), f(\cdot, x(\cdot)) \in SPAA(R \times L^2(P, B), L^2(P, B)).$$

Proof Let $x = \alpha + \beta \in SPAA(R \times L^2(P, B), L^2(P, B))$, where

$$\alpha \in SAA(R \times L^2(P, B), L^2(P, B))$$

and

$$\beta \in SBC_0(R \times L^2(P, B), L^2(P, B)).$$

Set

$$G(t) = g(t, \alpha(t)), F(t) = f(t, x(t)) - f(t, \alpha(t)), \Phi(t) = \varphi(t, \alpha(t)).$$

Thus $f(t, x(t)) = G(t) + F(t) + \Phi(t)$. By [12] and the conditions, we have

$$G \in SAA(R, L^2(P, B)).$$

From the definition of F , it is easy to see that F is bounded, i.e., for all $t \in R$ there $M > 0$ such that $E\|F(t)\|^2 \leq M$. For any $\varepsilon > 0$, and set $M_{r,\varepsilon}(f) := \{t \in [-r, r] : E\|f(t)\|^2 \geq \varepsilon\}$, we have

$$\begin{aligned} & \frac{1}{2r} \int_{-r}^r E\|F(t)\|^2 dt \\ &= \frac{1}{2r} \int_{M_{r,\varepsilon}(\beta)} E\|F(t)\|^2 dt + \frac{1}{2r} \int_{[-r,r] \setminus M_{r,\varepsilon}(\beta)} E\|f(t, x(t)) - f(t, \alpha(t))\|^2 dt \\ &\leq \frac{M}{2r} \int_{M_{r,\varepsilon}(\beta)} dt + \frac{1}{2r} \int_{[-r,r] \setminus M_{r,\varepsilon}(\beta)} E\|L(t)\beta(t)\|^2 dt \\ &\leq M \frac{M_{r,\varepsilon}(\beta)}{2r} + \frac{\varepsilon}{2r} \int_{-r}^r E\|L(t)\|^2 dt \\ &\leq M \frac{M_{r,\varepsilon}(\beta)}{2r} + \|L\|\varepsilon. \end{aligned}$$

One can get $\lim_{r \rightarrow \infty} \frac{M_{r,\varepsilon}(\beta)}{2r} = 0$. Thus, it is easy to see that

$$\lim_{r \rightarrow \infty} \frac{1}{2r} \int_{-r}^r E\|F(t)\|^2 dt = 0.$$

Therefore, $F \in SBC_0(R, L^2(P, B))$. Next, we show that $\Phi \in SBC_0(R, L^2(P, B))$. Let $K = \overline{\{\alpha(t) : t \in R\}}$. Then K is compact in $L^2(P, B)$. Since $g(t, u)$ is uniformly continuous in K uniformly for $t \in R$, for any $\varepsilon > 0$, $t \in R$, and $u, v \in K$ with $\|u - v\| < \delta$, there exists a constant $\delta \in (0, \varepsilon)$ such that $E\|g(t, u) - g(t, v)\|^2 \leq \varepsilon$. Therefore, it follows that

$$E\|\varphi(t, u) - \varphi(t, v)\|^2 \leq E\|f(t, u) - f(t, v)\|^2 + E\|g(t, u) - g(t, v)\|^2 \leq (E\|L(t)\|^2 + 1)\varepsilon$$

for all $t \in R$ and $u, v \in K$ with $\|u - v\| < \delta$. From Lemma 3.1, we have

$$\lim_{r \rightarrow \infty} \frac{1}{2r} \int_{-r}^r \sup_{x \in K} E\|\varphi(t, u)\|^2 dt = 0.$$

Since for each $t \in R$,

$$E\|\Phi(t)\|^2 = E\|\varphi(t, \alpha(t))\|^2 \leq \sup_{u \in K} E\|\varphi(t, u)\|^2,$$

one can get

$$\lim_{r \rightarrow \infty} \frac{1}{2r} \int_{-r}^r E\|\Phi(t)\|^2 dt = 0,$$

which implies that $\Phi \in SBC_0(R, L^2(P, B))$. The proof is completed.

Corollary 3.3 If $f \in SAA(R \times L^2(P, B), L^2(P, B))$ and (3.1) holds, then for each $x \in SAA(R \times L^2(P, B), L^2(P, B))$, $f(\cdot, x(\cdot)) \in SAA(R \times L^2(P, B), L^2(P, B))$.

4 Application to the Composition Theorem

In this section, we study the existence and uniqueness of pseudo-almost automorphic solutions for the semilinear stochastic differential equation (1.3).

Definition 4.1 A continuous stochastic process $u : R \rightarrow L^2(P, B)$ is called a mild solution to eq.(1.3) on $[-\sigma, T]$ if for all $t \in [-\sigma, 0]$, $x(t) = \varphi(t)$ and for all $t \in [0, T]$,

$$x(t) = T(t)\varphi(0) + \int_0^t T(t-s)F(s, Bx_s)ds + \int_0^t T(t-s)G(s, x(s)) \circ d\omega(s).$$

In order to establish the result, we need the following assumptions:

(H1) A generates a C_0 semigroup $T(t)$ satisfying $\|T(t)\| \leq Me^{-\mu t}$, $t > 0$ for some $M, \mu > 0$;

(H2) the stochastic processes $F : R \times L^2(P, B) \rightarrow L^2(P, B)$ and $G : R \times L^2(P, B) \rightarrow L^2(P, B)$ are square-mean pseudo-almost automorphic;

(H3) the stochastic processes F and G satisfy weaker Lipschitz conditions in second argument for t , that is there are two stochastic processes $L_F(t)$ and $L_G(t)$ such that, for any $x, y \in L^2(P, B)$,

$$E\|F(t, x) - F(t, y)\|^2 \leq E(L_F(t)\|x - y\|)^2$$

and

$$E\|G(t, x) - G(t, y)\|^2 \leq E(L_G(t)\|x - y\|)^2$$

for all $t \in R$.

Theorem 4.2 Assume that (H1), (H2) and (H3) hold. Then equation (1.3) has a unique square-mean pseudo-almost automorphic mild solution whenever

$$\Theta = M\left[\left(\frac{2L_F^2}{\mu^2} + \frac{L_G^2}{\mu}\right)\right]^{\frac{1}{2}} < 1,$$

where $L_F = \sup_{t \in [0, T]} L_F(t)$ and $L_G = \sup_{t \in [0, T]} L_G(t)$.

Proof Let us define the operator P on $SPAA(R \times L^2(P, B), L^2(P, B))$ by

$$(Px)(t) := T(t)\varphi(0) + \int_0^t T(t-s)F(s, Bx_s)ds + \int_0^t T(t-s)G(s, x(s)) \circ d\omega(s). \quad (4.1)$$

By Theorem 3.2 and B is closed linear operator, for each

$$x \in SPAA(R \times L^2(P, B), L^2(P, B)),$$

we have

$$F(\cdot, Bx(\cdot)), G(\cdot, Bx(\cdot)) \in SPAA(R \times L^2(P, B), L^2(P, B)).$$

Thanks to [15], one can easily show that P maps $SPAA(R \times L^2(P, B), L^2(P, B))$ into itself. Next, we prove that P is a contraction mapping and has a unique fixed point,

$$\begin{aligned} \|(Px)(t) - (Py)(t)\| &= \left\| \int_0^t T(t-s)(F(s, Bx_s) - F(s, By_s))ds \right. \\ &\quad \left. + \int_0^t T(t-s)(G(s, x(s)) - G(s, y(s))) \circ d\omega(s) \right\| \\ &\leq M \int_0^t e^{-\mu(t-s)} \|F(s, Bx_s) - F(s, By_s)\| ds \\ &\quad + \left\| \int_0^t T(t-s)(G(s, x(s)) - G(s, y(s))) \circ d\omega(s) \right\|. \end{aligned}$$

Since $(a - b)^2 \leq 2a^2 + 2b^2$, one can get

$$\begin{aligned} E\|(Px)(t) - (Py)(t)\|^2 &\leq 2M^2 E \left(\int_0^t e^{-\mu(t-s)} \|F(s, Bx_s) - F(s, By_s)\| ds \right)^2 \\ &\quad + 2E(\|T(t-s)[G(s, x(s)) - G(s, y(s))] \circ d\omega(s)\|)^2 \\ &:= P_1 + P_2. \end{aligned}$$

First, we evaluate P_1 as follows:

$$\begin{aligned} P_1 &= 2M^2 E \left(\int_0^t e^{-\mu(t-s)} \|F(s, Bx_s) - F(s, By_s)\| ds \right)^2 \\ &\leq 2M^2 E \left[\left(\int_0^t e^{-\mu(t-s)} ds \right) \left(\int_0^t e^{-\mu(t-s)} E \|F(s, Bx_s) - F(s, By_s)\|^2 ds \right) \right] \\ &= 2M^2 \left(\int_0^t e^{-\mu(t-s)} ds \right)^2 E \|F(s, Bx_s) - F(s, By_s)\|^2 \\ &\leq 2M^2 \left(\int_{-\infty}^t e^{-\mu(t-s)} ds \right)^2 L_F^2 \|x - y\|_\infty^2 \leq \frac{2M^2 L_F^2}{\mu^2} \|x - y\|_\infty^2. \end{aligned}$$

For P_2 , one can obtain

$$\begin{aligned} P_2 &= 2E \left(\left\| \int_0^t T(t-s)[G(s, x(s)) - G(s, y(s))] \circ d\omega(s) \right\| \right)^2 \\ &= 2E \left(\int_0^t \|T(t-s)[G(s, x(s)) - G(s, y(s))]\|^2 ds \right) \\ &\leq 2M^2 \int_0^t e^{-2\mu(t-s)} E \|G(s, x(s)) - G(s, y(s))\|^2 ds \\ &\leq 2M^2 L_G^2 E \|x - y\|_\infty^2 \int_{-\infty}^t e^{-2\mu(t-s)} ds \leq \frac{M^2 L_G^2}{\mu} \|x - y\|_\infty^2. \end{aligned}$$

Therefore, by P_1 and P_2 , it follows that

$$E\|(Lx)(t) - (Ly)(t)\|^2 \leq M^2\left(\frac{2L_F^2}{\mu^2} + \frac{L_G^2}{\mu}\right)\|x - y\|_\infty^2. \quad (4.2)$$

By (4.2), we have that

$$\|(Lx)(t) - (Ly)(t)\| \leq M\left[\left(\frac{2L_F^2}{\mu^2} + \frac{L_G^2}{\mu}\right)\right]^{\frac{1}{2}}\|x - y\|_\infty = \Theta\|x - y\|_\infty. \quad (4.3)$$

Since $\Theta < 1$, from (3.5), we obtain that P is a contraction mapping. Therefore, P has a unique fixed point $x(t)$, which clearly is the unique square-mean pseudo-almost automorphic mild solution to eq. (1.3). The proof is completed.

Corollary 4.3 If (H1), (H2) and (H3) hold with $L_F(t) = L_F$ and $L_G(t) = L_G$. Then equation (1.3) has a unique square-mean pseudo-almost automorphic mild solution whenever $\Theta = M\left[\left(\frac{2L_F^2}{\mu^2} + \frac{L_G^2}{\mu}\right)\right]^{\frac{1}{2}} < 1$.

5 Conclusion

In this paper, we study a composition theorem for pseudo-almost automorphic stochastic process is established under a weaker Lipschitz condition, the result improves some known results, such as [12, 15, 18]. So the result and its application are valuable. However, there are still many interesting and challenging questions that need to be studied for eq.(1.3). For example, whether the stability of system (1.3) can be considered by constructing an appropriate Lyapunov function? We will leave this for future work.

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伪概自首随机过程分解及应用

周 辉¹, 周宗福², 乔宗敏¹

(1.合肥师范学院数学与统计学院, 安徽 合肥 230061)

(2.安徽大学数学科学学院, 安徽 合肥 230039)

摘要: 在较弱Lipschitz 条件下, 本文建立伪概自首随机过程分解定理, 该定理推广了已知结果. 利用所建立理论的性质, 本文获得了一类随机微分方程的伪概自首弱解的存在唯一性条件.

关键词: 伪概自首; 分解; 均方; 弱解

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