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CONVERGENCE THEORY ON QUASI-PROBABILITY MEASURE

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Abstract: In this paper, we investigate the relationships among the convergence concepts on quasi-probability space. By using analogy method, some new convergence concepts for quasi-random variables are proposed on quasi-probability space and the relationships among the convergence concepts are discussed. Convergence theory about fuzzy measure is obtained, and all conclusions are natural extensions of the classical convergence concepts to the case where the measure tool is fuzzy.

Keywords: quasi-probability measure; convergence theory; convergence concept; quasi-random variables

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1 Introduction

Convergence theory was well developed based on classical measure theory, and some applications can be found in [1–3]. As for convergence theory in fuzzy environments, information and data are usually vague or imprecise which is essentially different from the classical measure case [4–6]. Therefore, it is more reasonable to utilize quasi-probability measure, which is an important extension of probability measure [1–2] to deal with fuzziness, to study such convergence theory. Quasi-probability measure was introduced by Wang [6], which offered an efficient tool to deal with fuzzy information fusion, subjective judgement, decision making, and so forth [7–11].

Convergence concepts play an important role in classical measure theory. Some mathematics workers explored them for fuzzy (or non-additive) measures such as Liu [12–13], Wang [14], Zhang [4–5], Gianluca [15], and so on. While the measure tool is non-additive, the convergence concepts are very different from additive case. In order to investigate quasi-probability theory deeper, we will propose in the present paper some new convergence concepts on quasi-probability space, and discuss the relationships among the convergence con-

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cepts. Our work helps to build important theoretical foundations for the development of quasi-probability measure theory.

The paper is outlined as follows: Section 1 is for introduction. In Section 2, some preliminaries are given. In Section 3, we study convergence concepts of q-random variables sequence and, ultimately, Section 4 is for conclusions.

2 Quasi-Probability Measure

In this paper, let X be a nonempty set and $(X, \mathcal{F})$ be a measurable space, here $\mathcal{F}$ is a σ-algebra of X. If $A,\ B \in \mathcal{F}$, then the notation $A \subset B$ means that A is a subset of B, and the complement of A is denoted by A^c.

Definition 2.1 [6] Let $a \in (0, +\infty]$, an extended real function is called a T-function iff $\theta : [0, a] \to [0, +\infty]$ is continuous, strictly increasing, and such that $\theta(0) = 0,\ \theta^{-1}(\{\infty\}) = \emptyset$ or $\{\infty\}$, according to a being finite or not.

Let $a \in (0, +\infty]$, an extended real function $\theta : [0, a] \to [0, +\infty]$ is called a regular function, if θ is continuous, strictly increasing, and $\theta(0) = 0, \theta(1) = 1$ [10].

Obviously, if θ is a regular function, then θ^{-1} is also a regular function.

Definition 2.2 [6] μ is called quasi-additive iff there exists a T-function θ, whose domain of definition contains the range of μ , such that the set function $\theta \circ \mu$ defined on $\mathcal{F}$ by $(\theta \circ \mu)(E) = \theta\,[\mu(E)]$ $(\forall E \in \mathcal{F})$, is additive; μ is called a quasi-measure iff there exists a T-function θ such that $\theta \circ \mu$ is a classical measure on $\mathcal{F}$. The T-function θ is called the proper T-function of μ.

Definition 2.3 If θ is a regular T-function of μ, then μ is called a quasi-probability. The triplet $(X, \mathcal{F}, \mu)$ is called a quasi-probability space.

From Definition 2.3, we know probability is a quasi-probability with $\theta(x) = x$ as its T-function.

Example 2.1 Suppose that $X = \{1, 2, \cdots, n\}$, $\rho(X)$ is the power set of X. If

$$\mu(E) = (\frac{|E|}{n})^2,$$

where $|E|$ is the number of those points that belong to E, then μ is a quasi-probability with $\theta(x) = \sqrt{x},\ \ x \in [0, 1]$ as its T-function [6].

Definition 2.4 Let $(X, \mathcal{F}, \mu)$ be a quasi-probability space, and $\xi = \xi(\omega), \omega \in X$, be a real set function on X. For any given real number x, if $\{\omega|\ \xi(\omega) \le x\} \in \mathcal{F}$, then ξ is called a quasi-random variable, denoted by q-random variable. The distribution function of q-random variable ξ is defined by $F_\mu(x) = \mu\{\omega \in X|\ \xi(\omega) \le x\}$.

Let ξ and η be two q-random variables. For any given real numbers $x,\ y$, if

$$\mu(\xi \le x, \eta \le y) = \theta^{-1}[(\theta \circ \mu)(\xi \le x) \cdot (\theta \circ \mu)(\eta \le y)],$$

then ξ and η are independent q-random variables [10].

Theorem 2.1 Let μ be a quasi-probability on $\mathcal{F}$. Then there exists a regular T-function θ, such that $\theta \circ \mu$ is a probability on $\mathcal{F}$ [4].

Theorem 2.2 If μ is a quasi-probability, then μ is continuous and $\mu(\emptyset) = 0$ [4].

Theorem 2.3 Let μ be a quasi-probability on $\mathcal{F}$, $A, B \in \mathcal{F}$, then we have

(1) if $A \subset B$, then $\mu(A) < \mu(B)$;

(2) if $\mu(A) = 0$, then $\mu(A^c) = 1$;

(3) $\mu(A \bigcup B) \leq \theta^{-1}[(\theta \circ \mu)(A) + (\theta \circ \mu)(B)]$.

Proof (1) Since $A \subset B$, and there exists a T-function θ such that $\theta \circ \mu$ is a probability, we have $(\theta \circ \mu)(A) < (\theta \circ \mu)(B)$, θ is continuous, strictly increasing, it is clear that $\mu(A) < \mu(B)$.

(2)Since $\theta \circ \mu$ is a probability, one can have

$$1 = (\theta \circ \mu)(X) = (\theta \circ \mu)(A \bigcup A^c) = (\theta \circ \mu)(A) + (\theta \circ \mu)(A^c),$$

which implies that

$$(\theta \circ \mu)(A^c) = 1 - (\theta \circ \mu)(A) = 1 - \theta(\mu(A)) = 1 - \theta(0) = 1,$$

namely, $\theta(\mu(A^c)) = 1$. It follows from the regularity of θ that $\mu(A^c) = 1$.

(3) $\theta \circ \mu$ is a probability, we have

$$(\theta \circ \mu)(A \bigcup B) \leq (\theta \circ \mu)(A) + (\theta \circ \mu)(B),$$

that is,

$$\mu(A \bigcup B) \leq \theta^{-1}[(\theta \circ \mu)(A) + (\theta \circ \mu)(B)].$$

3 Convergence Concepts of q-Random Variables Sequence

In the section, we introduce some new convergence concepts such as convergence almost surely, convergence in distribution, fundamental convergence almost everywhere, fundamental convergence in quasi-probability, etc., and we will investigate the relationships among the convergence concepts.

Definition 3.1 [4] Suppose that ξ, ξ_1, ξ_2, $\cdots$, ξ_n, $\cdots$ are q-random variables defined on the quasi-probability space $(X, \mathcal{F}, \mu)$. If

$$\mu\{\lim_{n\to\infty} \xi_n = \xi\} = 1,$$

then we say that $\{\xi_n\}$ converges almost surely to ξ. Denoted by

$$\lim_{n\to\infty} \xi_n = \xi \qquad (\mu - \text{a.s.}).$$

Definition 3.2 Suppose that ξ, ξ_1, ξ_2, $\cdots$, ξ_n, $\cdots$ are q-random variables defined on the quasi-probability space $(X, \mathcal{F}, \mu)$. If there exists $E \in \mathcal{F}$ with $\mu(E) = 0$ such that $\{\xi_n\}$ converges to ξ on E^c, then we say $\{\xi_n\}$ converges to ξ almost everywhere. Denoted by

$$\xi_n \to \xi \qquad (\text{a.e.}).$$

Definition 3.3 Suppose that $\xi_1,\ \xi_2,\ \cdots,\ \xi_n,\ \cdots$ are q-random variables defined on the quasi-probability space $(X,\mathcal{F},\mu)$. If there exists $E\in\mathcal{F}$ with $\mu(E)=0$ such that for any $x\in E^c$,

$$\lim_{n,\ m\to\infty}(\xi_n(x)-\xi_m(x))=0,$$

then we say $\{\xi_n\}$ is fundamentally convergent almost everywhere.

Definition 3.4 [4] Suppose that $\xi_1,\ \xi_2,\ \cdots,\ \xi_n,\ \cdots$ is a sequence of q-random variables. If there exists a q-random variable ξ, such that $\forall\varepsilon>0$,

$$\lim_{n\to\infty}\mu\{|\ \xi_n-\xi\ |\geq\varepsilon\}=0,$$

namely,

$$\lim_{n\to\infty}\mu\{|\ \xi_n-\xi\ |<\varepsilon\}=1,$$

then we say that $\{\xi_n\}$ converges in quasi-probability to ξ. Denoted by

$$\xi_n\to\xi\qquad(\mu).$$

Definition 3.5 Suppose that $\xi_1,\ \xi_2,\ \cdots,\ \xi_n,\ \cdots$ are q-random variables. If for any given $\varepsilon>0$,

$$\lim_{n,\ m\to\infty}\mu(|\xi_n-\xi_m|\geq\varepsilon)=0,$$

then we say $\{\xi_n\}$ fundamentally converges in quasi-probability.

Definition 3.6 Suppose that $F_\mu(x),\ F_\mu^1(x)\ F_\mu^2(x)\cdots$ are the distribution functions of q-random variables $\xi,\ \xi_1,\ \xi_2,\ \cdots$, respectively. The sequence $\{\xi_n\}$ is said to be convergent in distribution to ξ if

$$\lim_{n\to\infty}F_\mu^n(x)=F_\mu(x)$$

at any continuity point of $F_\mu(x)$.

Theorem 3.1 If $\{\xi_n\}$ converges in quasi-probability to ξ, then $\{\xi_n\}$ fundamentally converges in quasi-probability.

Proof Suppose that $\{\xi_n\}$ converges in quasi-probability to ξ, then for any given $\varepsilon>0$, we have

$$\lim_{n\to\infty}\mu\{|\ \xi_n-\xi\ |\geq\frac{\varepsilon}{2}\}=0.$$

According to [2],

$$\{|\xi_n-\xi_m|\geq\varepsilon\}\subset\{|\xi_n-\xi|\geq\frac{\varepsilon}{2}\}\bigcup\{|\xi_m-\xi|\geq\frac{\varepsilon}{2}\}.$$

It follows from Theorem 2.3 that

$$\begin{aligned}&\mu\{|\xi_n-\xi_m|\geq\varepsilon\}\leq\mu[\{|\xi_n-\xi|\geq\frac{\varepsilon}{2}\}\bigcup\{|\xi_m-\xi|\geq\frac{\varepsilon}{2}\}]\\ \leq\ &\theta^{-1}[(\theta\circ\mu)\{|\xi_n-\xi|\geq\frac{\varepsilon}{2}\}+(\theta\circ\mu)\{|\xi_m-\xi|\geq\frac{\varepsilon}{2}\}].\end{aligned}$$

And θ, θ^{-1} are strictly increasing and continuous,

$$\begin{aligned}\lim_{n,\ m\to\infty}\mu\{|\xi_n-\xi_m|\geq\varepsilon\} &\leq \lim_{n,\ m\to\infty}\theta^{-1}[(\theta\circ\mu)\{|\xi_n-\xi|\geq\frac{\varepsilon}{2}\}+(\theta\circ\mu)\{|\xi_m-\xi|\geq\frac{\varepsilon}{2}\}] \\ &= \theta^{-1}[\theta(\lim_{n\to\infty}\mu\{|\xi_n-\xi|\geq\frac{\varepsilon}{2}\})+\theta(\lim_{m\to\infty}\mu\{|\xi_m-\xi|\geq\frac{\varepsilon}{2}\})] \\ &= \theta^{-1}[\theta(0)+\theta(0)]=\theta^{-1}(0)=0.\end{aligned}$$

This means that $\{\xi_n\}$ fundamentally converges in quasi-probability.

Lemma 3.1 [2] Suppose that $\xi_n,\ \xi\in\mathcal{F}$, and for any given $\varepsilon_k>0$, $\lim\limits_{n\to\infty}\varepsilon_k=0$, then we have

(1) $\{\xi_n\to\xi\}=\bigcap\limits_{\varepsilon>0}\bigcup\limits_{m=1}^{\infty}\bigcap\limits_{n=m}^{\infty}\{|\xi_n-\xi|\geq\varepsilon\}=\bigcap\limits_{k=1}^{\infty}\bigcup\limits_{m=1}^{\infty}\bigcap\limits_{n=m}^{\infty}\{|\xi_n-\xi|<\varepsilon_k\}$;

(2) $\{|\xi_n-\xi_m|\to 0\}=\bigcap\limits_{\varepsilon>0}^{\infty}\bigcup\limits_{n=1}^{\infty}\bigcap\limits_{v=1}^{\infty}\{|\xi_{n+v}-\xi_n|<\varepsilon\}=\bigcap\limits_{k=1}^{\infty}\bigcup\limits_{n=1}^{\infty}\bigcap\limits_{v=1}^{\infty}\{|\xi_{n+v}-\xi_n|<\varepsilon_k\}$.

Theorem 3.2 Suppose that $\xi_1,\ \xi_2,\ \cdots,\ \xi_n,\ \cdots$ are q-random variables, then

$$\xi_n\to\xi \qquad \text{(a.e.)}$$

if and only if

$$\mu(\bigcap_{n=1}^{\infty}\bigcup_{v=1}^{\infty}\{|\xi_{n+v}-\xi|\geq\varepsilon\}=0,\quad\forall\varepsilon>0$$

if and only if

$$\lim_{n\to\infty}\mu(\bigcup_{v=1}^{\infty}\{|\xi_{n+v}-\xi|\geq\varepsilon\})=0,\quad\forall\varepsilon>0.$$

Proof If $\xi_n\to\xi$ a.e., then $\forall\varepsilon>0$. According to Lemma 3.1,

$$\mu(\bigcap_{n=1}^{\infty}\bigcup_{v=1}^{\infty}\{|\xi_{n+v}-\xi|\geq\varepsilon\}\leq\mu(\bigcup_{\varepsilon>0}\bigcap_{n=1}^{\infty}\bigcup_{v=1}^{\infty}\{|\xi_{n+v}-\xi|\geq\varepsilon\}=\mu(\{\xi_n\to\xi\}^c)=0.$$

On the other hand, if

$$\mu(\bigcap_{n=1}^{\infty}\bigcup_{v=1}^{\infty}\{|\xi_{n+v}-\xi|\geq\varepsilon\}=0,\quad\forall\varepsilon>0,$$

then for any given $\varepsilon_k>0$, $\lim\limits_{k\to\infty}\varepsilon_k=0$, it follows from Theorems 2.1 and 2.3 that

$$\begin{aligned}&\mu(\{\xi_n\to\xi\}^c)=\mu(\bigcup_{k=1}^{\infty}\bigcap_{n=1}^{\infty}\bigcup_{v=1}^{\infty}\{|\xi_{n+v}-\xi|\geq\varepsilon_k\}) \\ =\ &\theta^{-1}[(\theta\circ\mu)(\bigcup_{k=1}^{\infty}\bigcap_{n=1}^{\infty}\bigcup_{v=1}^{\infty}\{|\xi_{n+v}-\xi|\geq\varepsilon_k\})] \\ \leq\ &\theta^{-1}[\sum_{k=1}^{\infty}(\theta\circ\mu)(\bigcap_{n=1}^{\infty}\bigcup_{v=1}^{\infty}\{|\xi_{n+v}-\xi|\geq\varepsilon_k\})] \\ =\ &\theta^{-1}[\sum_{k=1}^{\infty}\theta(\mu(\bigcap_{n=1}^{\infty}\bigcup_{v=1}^{\infty}\{|\xi_{n+v}-\xi|\geq\varepsilon_k\}))]=\theta^{-1}(0)=0,\end{aligned}$$

that is $\xi_n \to \xi$ a.e.. And since

$$\bigcup_{v=1}^{\infty}\{|\xi_{n+v}-\xi|\geq\varepsilon\}$$

is decreasing for n, it follows from the continuity of μ that

$$\mu(\bigcap_{n=1}^{\infty}\bigcup_{v=1}^{\infty}\{|\xi_{n+v}-\xi|\geq\varepsilon\}=\lim_{n\to\infty}\mu(\bigcup_{v=1}^{\infty}\{|\xi_{n+v}-\xi|\geq\varepsilon\}).$$

Now the theorem is proved.

Theorem 3.3 [4] Suppose that $\xi,\ \xi_1,\ \xi_2,\ \cdots,\ \xi_n,\ \cdots$ are q-random variables defined on the quasi-probability space $(X,\mathcal{F},\mu)$. If $\{\xi_n\}$ converges almost surely to ξ, then $\{\xi_n\}$ converges in quasi-probability to ξ.

Example 3.1 [4] Suppose that $\xi_1,\ \xi_2,\ \cdots,\ \xi_n,\ \cdots$ are independent q-random variables defined on the quasi-probability space $(X,\mathcal{F},\mu)$. If

$$\mu\{\xi_n=\frac{1}{n}\}=1-\frac{1}{n},\ \ \mu\{\xi_n=n+1\}=\frac{1}{n},\ \ n=1,2,\cdots,$$

then $\{\xi_n\}$ converges in quasi-probability to 0. However, $\{\xi_n\}$ does not converges almost surely to 0.

Theorem 3.3 shows that convergence almost surely implies convergence in quasi-probability. Example 3.1 shows that convergence in quasi-probability does not imply convergence almost surely. But for independent q-random series, convergence almost surely is equivalent to convergence in quasi-probability.

Theorem 3.4 [4] If $\{\xi_n\}$ is a sequence of independent q-random variables, then $\sum\limits_{n=1}^{\infty}\xi_n$ converges almost surely if and only if $\sum\limits_{n=1}^{\infty}\xi_n$ converges in quasi-probability.

Theorem 3.5 Suppose that $\xi,\ \xi_1,\ \xi_2,\ \cdots,\ \xi_n,\ \cdots$ are q-random variables defined on the quasi-probability space $(X,\mathcal{F},\mu)$. If $\{\xi_n\}$ converges in quasi-probability to ξ, then $\{\xi_n\}$ converges in distribution to ξ.

Proof Assume that $F_\mu^n(x), F_\mu(x)$ are the distribution functions of ξ_n,ξ, respectively. Let x,y,z be the given continuity points of the distribution function $F_\mu(x)$.

On the one hand, for any $y<x$, we have

$$\{\xi\leq y\}=\{\xi_n\leq x,\ \xi\leq y\}\bigcup\{\xi_n>x,\ \xi\leq y\}\subset\{\xi_n\leq x\}\bigcup\{|\xi_n-\xi|\geq x-y\}.$$

It follows from Theorem 2.3 that

$$\begin{aligned}&\mu\{\xi\leq y\}\leq\mu[\{\xi_n\leq x\}\bigcup\{|\xi_n-\xi|\geq x-y\}]\\ \leq\ &\theta^{-1}[(\theta\circ\mu)\{\xi_n\leq x\}+(\theta\circ\mu)\{|\xi_n-\xi|\geq x-y\}].\end{aligned}$$

Since $\{\xi_n\}$ converges in quasi-probability to ξ, and $\theta,\ \theta^{-1}$ are continuous, we have

$$\begin{aligned}&\mu\{\xi\leq y\}\leq\theta^{-1}[\theta(\lim_{n\to\infty}\mu\{\xi_n\leq x\})+\theta(\lim_{n\to\infty}\mu\{|\xi_n-\xi|\geq x-y\})]\\ =\ &\theta^{-1}[\theta(\lim_{n\to\infty}\mu\{\xi_n\leq x\})+\theta(0)]=\theta^{-1}[\theta(\lim_{n\to\infty}\mu\{\xi_n\leq x\})],\end{aligned}$$

which implies that

$$F_\mu(y) \le \lim_{n\to\infty} F_\mu^n(x)$$

for any $y < x$.

Let $y \to x$, we obtain

$$F_\mu(x) \le \lim_{n\to\infty} F_\mu^n(x).$$

On the other hand, for any $z > x$, we have

$$\{\xi_n \le x\} = \{\xi_n \le x,\ \xi \le z\} \bigcup \{\xi_n \le x,\ \xi > z\} \subset \{\xi \le z\} \bigcup \{|\xi_n - \xi| \ge z - x\}.$$

Since $\mu\{|\xi_n - \xi| \ge z - x\} \to 0$ as $n \to \infty$, and $\theta,\ \ \theta^{-1}$ are continuous, we get

$$\begin{aligned} & \lim_{n\to\infty} \mu\{\xi_n \le x\} \le \theta^{-1}[\theta(\lim_{n\to\infty} \mu\{\xi \le z\}) + \theta(\lim_{n\to\infty} \mu\{|\xi_n - \xi| \ge z - x\})] \\ = \ & \theta^{-1}[\theta(\lim_{n\to\infty} \mu\{\xi \le z\}) + \theta(0)] = \theta^{-1}[\theta(\lim_{n\to\infty} \mu\{\xi \le z\})] = \mu\{\xi \le z\}. \end{aligned}$$

It means that

$$\lim_{n\to\infty} F_\mu^n(x) \le F_\mu(z)$$

for any $z > x$. Let $z \to x$, we get

$$\lim_{n\to\infty} F_\mu^n(x) \le F_\mu(x).$$

Finally, one can see that

$$\lim_{n\to\infty} F_\mu^n(x) = F_\mu(x),$$

that is to say $\{\xi_n\}$ converges in distribution to ξ.

According to Theorems 3.3 and 3.5, we conclude that convergence almost surely implies convergence in quasi-probability; convergence in quasi-probability implies convergence in distribution.

4 Conclusions

This paper proposed some new convergence concepts for quasi-random variables. Firstly, the properties of quasi-probability measure were further discussed. Secondly, the concepts of convergence in quasi-probability, convergence almost surely, convergence in distribution and convergence almost everywhere were introduced on quasi-probability space. Finally, the relationships among the convergence concepts were investigated in detail. All investigations helped to lay important theoretical foundations for the systematic and comprehensive development of quasi-probability measure theory.

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关于拟概率测度的收敛理论

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摘要: 本文研究了拟概率空间上收敛概念之间的关系这一问题. 利用类比的方法, 在拟概率空间上提出了一些新的关于拟-随机变量的收敛概念并讨论了这些收敛概念之间的关系, 获得了模糊测度下的收敛理论, 推广了关于经典测度的收敛概念.

关键词: 拟概率测度; 收敛理论; 收敛概念; 拟-随机变量

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