

COMPLETE CONVERGENCE FOR ARRAYS OF ROWWISE M -NSD RANDOM VARIABLES

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Abstract: In this article, we study complete convergence theorems for arrays of rowwise m -negatively superadditive-dependent (m -NSD) random variables. By using Kolmogorov-type exponential inequality for m -NSD random variables, we obtain complete convergence theorems for arrays of rowwise m -NSD random variables, which generalize those on complete convergence theorem previously obtained by Hu et al. (1998) and Sung et al. (2005) from independent distributed case to m -NSD arrays. Our results also extend the corresponding results of Chen et al.(2008), Hu et al. (2009), Qiu et al. (2011) and Wang et al. (2014).

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1 Introduction

We first introduce some concepts of dependent random variables. The concept of negatively associated (abbreviated to NA in the following) random variables was introduced by Joag-Dev and Proschan [1].

Definition 1.1 A finite family of random variables $\{X_i; 1 \leq i \leq n\}$ is said to be NA if for every pair of disjoint subsets $A, B \subset \{1, 2, \dots, n\}$,

$$\text{Cov}(f(X_i, i \in A), g(X_j, j \in B)) \leq 0,$$

whenever f and g are coordinatewise nondecreasing functions such that this covariance exists. An infinite family of random variables is NA if every finite subfamily is NA.

Definition 1.2 (see [2]) A function $\phi : \mathbb{R}^n \rightarrow \mathbb{R}$ is called superadditive if $\phi(x \vee y) + \phi(x \wedge y) \geq \phi(x) + \phi(y)$ for all $x, y \in \mathbb{R}^n$, where $\vee$ is for componentwise maximum and $\wedge$ is for componentwise minimum.

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The concept of negatively superadditive-dependent (abbreviated to NSD in the following) random variables was introduced by Hu [3] as follows.

Definition 1.3 A random vector $X = (X_1, X_2, \dots, X_n)$ is said to be NSD if

$$E\phi(X_1, X_2, \dots, X_n) \leq E\phi(X_1^*, X_2^*, \dots, X_n^*),$$

where $X_1^*, X_2^*, \dots, X_n^*$ are independent such that X_i and X_i^* have the same distribution for each i and ϕ is a superadditive function such that the expectations exist. A sequence of random variables $\{X_n; n \geq 1\}$ is said to be NSD if for any $n \geq 1$, $(X_1, X_2, \dots, X_n)$ is NSD.

Hu [3] gave an example illustrating that NSD does not imply NA. Christofides and Vaggelatos [4] indicated that NA implies NSD.

Hu et al. [5] introduced the concept of m -negatively associated random variables as follows.

Definition 1.4 Let $m \geq 1$ be a fixed integer. A sequence of random variables $\{X_n; n \geq 1\}$ is said to be m -negatively associated (abbreviated to m -NA in the following) if for any $n \geq 2$ and $i_1, \dots, i_n$ such that $|i_k - i_j| \geq m$ for all $1 \leq k \neq j \leq n$, we have that $X_{i_1}, \dots, X_{i_n}$ are NA.

The concept of m -NA random variables is a natural extension from NA random variables (wherein $m = 1$).

Similarly, we can define m -NSD random variables.

Definition 1.5 Let $m \geq 1$ be a fixed integer. A sequence of random variables $\{X_n; n \geq 1\}$ is said to be m -negatively superadditive-dependent (abbreviated to m -NSD in the following) if for any $n \geq 2$ and $i_1, \dots, i_n$ such that $|i_k - i_j| \geq m$ for all $1 \leq k \neq j \leq n$, we have that $(X_{i_1}, \dots, X_{i_n})$ is NSD.

Hsu and Robbins [6] introduced the concept of complete convergence of a sequence of random variables. Hu et al. [7] proposed the following general complete convergence of rowwise independent arrays of random variables.

Theorem A Let $\{X_{ni}; 1 \leq i \leq k_n, n \geq 1\}$ be an array of rowwise independent random variables and $\{c_n\}$ be a sequence of positive real numbers. Suppose that for every $\varepsilon > 0$ and some $\delta > 0$,

- (i) $\sum_{n=1}^{\infty} c_n \sum_{i=1}^{k_n} P\{|X_{ni}| > \varepsilon\} < \infty$;
- (ii) there exists $j \geq 1$ such that $\sum_{n=1}^{\infty} c_n \left(\sum_{i=1}^{k_n} EX_{ni}^2 I\{|X_{ni}| \leq \delta\} \right)^j < \infty$;
- (iii) $\sum_{i=1}^{k_n} EX_{ni} I\{|X_{ni}| \leq \delta\} \rightarrow 0$ as $n \rightarrow \infty$.

Then

$$\sum_{n=1}^{\infty} c_n P\left(\left|\sum_{i=1}^{k_n} X_{ni}\right| > \varepsilon\right) < \infty \text{ for all } \varepsilon > 0.$$

In this paper, we let $\{k_n, n \geq 1\}$ be a sequence of positive integers such that $\lim_{n \rightarrow \infty} k_n = \infty$.

The proof of Hu et al. [7] is mistakenly based on the fact that the assumptions of Theorem A imply convergence in probability of the corresponding partial sums. Hu and Volodin [8] and Hu et al. [9] presented counterexamples to this proof. They mentioned that whether Theorem A was true remained open. Since then many authors attempted to solve this problem. Hu et al. [9] and Kuczmazewska [10] gave partial solution to this question. Sung et al. [11] completely solved this problem by using a symmetrization procedure and Kruglov et al. [12] obtained the complete convergence for maximum partial sums by using a submartingale approach.

Recently, Chen et al. [13] extended Theorem A to the case of arrays of rowwise NA random variables and obtained the complete convergence for maximum partial sums. Hu et al. [5] obtained complete convergence for maximum partial sums similar to Theorem A for arrays of rowwise m -NA random variables. Qiu et al. [14] obtained similar result for arrays of rowwise ND random variables. Wand et al. [15] extended and improved Theorem A for NSD arrays. Qiu [16] obtained similar result for weighted sums of NA random variables. The main purpose of this article is to generalize and improve Theorem A for the case of arrays of rowwise m -NSD random variables.

2 Main Results

Theorem 2.1 Let $\{X_{ni}; 1 \leq i \leq k_n, n \geq 1\}$ be an array of rowwise m -NSD random variables and $\{c_n\}$ be a sequence of positive real numbers. Assume that for every $\varepsilon > 0$ and some $\delta > 0$,

- (i) $\sum_{n=1}^{\infty} c_n \sum_{i=1}^{k_n} P(|X_{ni}| > \varepsilon) < \infty$;
- (ii) there exists $j \geq 1$ such that $\sum_{n=1}^{\infty} c_n \left(\sum_{i=1}^{k_n} \text{Var}(X_{ni} I\{|X_{ni}| \leq \delta\}) \right)^j < \infty$.

Then

$$\sum_{n=1}^{\infty} c_n P \left(\max_{1 \leq l \leq k_n} \left| \sum_{i=1}^l (X_{ni} - EX_{ni} I\{|X_{ni}| \leq \delta\}) \right| > \varepsilon \right) < \infty \text{ for all } \varepsilon > 0.$$

From Theorem 2.1, we can obviously obtain the following corollary.

Corollary 2.1 Let $\{X_{ni}; 1 \leq i \leq k_n, n \geq 1\}$ be an array of rowwise m -NSD random variables. If conditions (i) and (ii) of Theorem 2.1 and

- (iii) $\sum_{i=1}^{k_n} EX_{ni} I\{|X_{ni}| \leq \delta\} \rightarrow 0$ as $n \rightarrow \infty$

are satisfied, then

$$\sum_{n=1}^{\infty} c_n P \left(\max_{1 \leq l \leq k_n} \left| \sum_{i=1}^l X_{ni} \right| > \varepsilon \right) < \infty \text{ for all } \varepsilon > 0.$$

Theorem 2.2 Let $\{X_{ni}; 1 \leq i \leq k_n, n \geq 1\}$ be an array of rowwise m -NSD random variables with $EX_{ni} = 0$ and $EX_{ni}^2 < \infty$ for $1 \leq i \leq k_n, n \geq 1$. Let $\{c_n\}$ be a sequence of positive real numbers. Assume that for every $\varepsilon > 0$ and some $\delta > 0$,

- (i) $\sum_{n=1}^{\infty} c_n \sum_{i=1}^{k_n} P(|X_{ni}| > \varepsilon) < \infty$;
- (ii) there exists $j \geq 1$ such that $\sum_{n=1}^{\infty} c_n \left(\sum_{i=1}^{k_n} EX_{ni}^2 \right)^j < \infty$.

Then

$$\sum_{n=1}^{\infty} c_n P \left(\max_{1 \leq l \leq k_n} \left| \sum_{i=1}^l X_{ni} \right| > \varepsilon \right) < \infty \text{ for all } \varepsilon > 0.$$

Remark 1 Corollary 2.1 shows that the main results of Hu et al. [7] and Sung et al. [11] remain true for m -NSD random variables. We generalize the corresponding complete convergence theorems from the independent case to m -NSD arrays without adding any extra conditions.

Remark 2 In Theorem 2.2, we only need conditions (i) and (ii) of Corollary 2.1. Condition (iii) of Corollary 2.1 is not needed. Therefore Theorem 2.2 extends and improves the corresponding results of Hu et al. [7] and Sung et al. [11]. In addition, our results also extend the corresponding results of Chen et al. [13], Hu et al. [5], Qiu et al. [14] and Wang et al. [15]. When $m = 1$, from Theorem 2.1 and Theorem 2.2, we can obtain the results of Theorem 3.3 and Theorem 3.2 of Wand et al. [15], respectively. We mention that Theorem 2.1 of this paper not only extends the results of Wand et al. [15] but also we have a simpler proof. More precisely, we only divide the sum into two parts in our proof instead of into four parts as was in the paper of Wand et al. [15].

Throughout this paper, C denotes a positive constant which may differ from one place to another.

3 Proofs of Main Results

In order to prove our results, we need the following lemmas.

Lemma 3.1 (cf. Wand et al. [15], Lemma 2.4) Let $\{X_n; n \geq 1\}$ be a sequence of NSD random variables with $EX_n = 0$ and $EX_n^2 < \infty$, $n \geq 1$. Let

$$S_n = \sum_{i=1}^n X_i, \quad B_n = \sum_{i=1}^n EX_i^2.$$

Then for all $x > 0, a > 0$,

$$P\left(\max_{1 \leq k \leq n} S_k \geq x\right) \leq P\left(\max_{1 \leq k \leq n} |X_k| > a\right) + 2 \exp\left(-\frac{x^2}{8B_n}\right) + 2 \left(\frac{B_n}{4(xa + B_n)}\right)^{\frac{x}{12a}},$$

$$P\left(\max_{1 \leq k \leq n} |S_k| \geq x\right) \leq 2P\left(\max_{1 \leq k \leq n} |X_k| > a\right) + 4 \exp\left(-\frac{x^2}{8B_n}\right) + 4 \left(\frac{B_n}{4(xa + B_n)}\right)^{\frac{x}{12a}}.$$

Lemma 3.2 Let $\{X_n; n \geq 1\}$ be a sequence of m -NSD random variables with $EX_n = 0$ and $EX_n^2 < \infty$, $n \geq 1$. Let

$$S_n = \sum_{i=1}^n X_i, \quad B_n = \sum_{i=1}^n EX_i^2.$$

Then for all $n \geq m, x > 0, a > 0$,

$$\begin{aligned} P\left(\max_{1 \leq k \leq n} S_k \geq x\right) &\leq mP\left(\max_{1 \leq k \leq n} |X_k| > a\right) + 2m \exp\left(-\frac{x^2}{8m^2 B_n}\right) \\ &\quad + 2m \left(\frac{B_n}{4(xa/m + B_n)}\right)^{\frac{x}{12ma}}, \\ P\left(\max_{1 \leq k \leq n} |S_k| \geq x\right) &\leq 2mP\left(\max_{1 \leq k \leq n} |X_k| > a\right) + 4m \exp\left(-\frac{x^2}{8m^2 B_n}\right) \\ &\quad + 4m \left(\frac{B_n}{4(xa/m + B_n)}\right)^{\frac{x}{12ma}}. \end{aligned}$$

Proof Given any $1 \leq k \leq n$, let $r = \lfloor \frac{n}{m} \rfloor$. Set

$$Y_i = \begin{cases} X_i, & \text{if } 1 \leq i \leq n, \\ 0, & \text{if } i > n \end{cases}$$

and $S'_{mk+j} = \sum_{i=0}^k Y_{mi+j}$ for $1 \leq j \leq m$.

$$\text{Since } \left\{ \max_{1 \leq k \leq n} S_k \geq x \right\} \subset \left\{ \max_{0 \leq k \leq r} S'_{mk+1} \geq \frac{x}{m} \right\} \cup \cdots \cup \left\{ \max_{0 \leq k \leq r} S'_{mk+m} \geq \frac{x}{m} \right\},$$

by Lemma 3.1, we have

$$\begin{aligned} P\left(\max_{1 \leq k \leq n} S_k \geq x\right) &\leq \sum_{j=1}^m P\left(\max_{0 \leq k \leq r} S'_{mk+j} \geq \frac{x}{m}\right) \\ &\leq \sum_{j=1}^m P\left(\max_{0 \leq i \leq r} Y_{mi+j} > a\right) + \sum_{j=1}^m \left[2 \exp\left(-\frac{x^2}{8m^2 \sum_{i=0}^r EY_{mi+j}^2}\right) + 2 \left(\frac{\sum_{i=0}^r EY_{mi+j}^2}{4(xa/m + \sum_{i=0}^r EY_{mi+j}^2)}\right)^{\frac{x}{12ma}} \right] \\ &\leq mP\left(\max_{1 \leq k \leq n} |X_k| > a\right) + 2m \exp\left(-\frac{x^2}{8m^2 \sum_{i=1}^n EX_i^2}\right) + 2m \left(\frac{\sum_{i=1}^n EX_i^2}{4(xa/m + \sum_{i=1}^n EX_i^2)}\right)^{\frac{x}{12ma}} \\ &\leq mP\left(\max_{1 \leq k \leq n} |X_k| > a\right) + 2m \exp\left(-\frac{x^2}{8m^2 B_n}\right) + 2m \left(\frac{B_n}{4(xa/m + B_n)}\right)^{\frac{x}{12ma}}. \end{aligned}$$

If we consider $-X_n$ instead of X_n in the arguments above, by a similar way we get

$$P(\max_{1 \leq k \leq n} (-S_k) \geq x) \leq mP\left(\max_{1 \leq k \leq n} |X_k| > a\right) + 2m \exp\left(-\frac{x^2}{8m^2 B_n}\right) + 2m \left(\frac{B_n}{4(xa/m + B_n)}\right)^{\frac{x}{12ma}}.$$

Therefore

$$P\left(\max_{1 \leq k \leq n} |S_k| \geq x\right) \leq 2mP\left(\max_{1 \leq k \leq n} |X_k| > a\right) + 4m \exp\left(-\frac{x^2}{8m^2 B_n}\right) + 4m \left(\frac{B_n}{4(xa/m + B_n)}\right)^{\frac{x}{12ma}}.$$

Lemma 3.3 Let $\{X_n; n \geq 1\}$ be a sequence of m -NSD random variables with $EX_n = 0$ and $EX_n^2 < \infty$, $n \geq 1$. Let $S_n = \sum_{i=1}^n X_i$, $B_n = \sum_{i=1}^n EX_i^2$. Then for all $n \geq 1, x > 0, a > 0$,

$$P\left(\max_{1 \leq k \leq n} |S_k| \geq x\right) \leq 2mP\left(\max_{1 \leq k \leq n} |X_k| > a\right) + 8m\left(\frac{2mB_n}{3xa}\right)^{\frac{x}{12ma}}.$$

Proof By the fact that $e^{-x} \leq 1/(1+x) \leq 1/x$ for $x > 0$, we have, for all $n \geq 1, x > 0, a > 0$,

$$\exp\left(-\frac{x^2}{8m^2B_n}\right) = \left[\exp\left(-\frac{3xa}{2mB_n}\right)\right]^{\frac{x}{12ma}} \leq \left(\frac{2mB_n}{3xa}\right)^{\frac{x}{12ma}}.$$

On the other hand,

$$\left(\frac{B_n}{4(xa/m + B_n)}\right)^{\frac{x}{12ma}} \leq \left(\frac{B_n}{4xa/m}\right)^{\frac{x}{12ma}} \leq \left(\frac{2mB_n}{3xa}\right)^{\frac{x}{12ma}}.$$

Therefore by Lemma 3.2, the conclusion holds.

Proof of Theorem 2.1 Let $Y_{ni} = \delta I\{X_{ni} > \delta\} + X_{ni}I\{|X_{ni}| \leq \delta\} - \delta I\{X_{ni} < -\delta\}$ and $Y'_{ni} = \delta I\{X_{ni} > \delta\} - \delta I\{X_{ni} < -\delta\}$ and $1 \leq i \leq k_n$, $n \geq 1$. $\{Y_{ni}, 1 \leq i \leq k_n, n \geq 1\}$ is an array of rowwise m -NSD random variables. Note that

$$\begin{aligned} & P\left(\max_{1 \leq l \leq k_n} \left|\sum_{i=1}^l (X_{ni} - EX_{ni}I\{|X_{ni}| \leq \delta\})\right| > \varepsilon\right) \\ & \leq \sum_{i=1}^{k_n} P(|X_{ni}| > \delta) + P\left(\max_{1 \leq l \leq k_n} \left|\sum_{i=1}^l (X_{ni}I\{|X_{ni}| \leq \delta\} - EX_{ni}I\{|X_{ni}| \leq \delta\})\right| > \varepsilon\right) \\ & = \sum_{i=1}^{k_n} P(|X_{ni}| > \delta) + P\left(\max_{1 \leq l \leq k_n} \left|\sum_{i=1}^l (Y_{ni} - EY_{ni} - Y'_{ni} + EY'_{ni})\right| > \varepsilon\right) \\ & \leq \sum_{i=1}^{k_n} P(|X_{ni}| > \delta) + P\left(\max_{1 \leq l \leq k_n} \left|\sum_{i=1}^l (Y'_{ni} - EY'_{ni})\right| > \varepsilon/2\right) \\ & \quad + P\left(\max_{1 \leq l \leq k_n} \left|\sum_{i=1}^l (Y_{ni} - EY_{ni})\right| > \varepsilon/2\right) \\ & \leq \sum_{i=1}^{k_n} P(|X_{ni}| > \delta) + C \sum_{i=1}^{k_n} P(|X_{ni}| > \delta) + P\left(\max_{1 \leq l \leq k_n} \left|\sum_{i=1}^l (Y_{ni} - EY_{ni})\right| > \varepsilon/2\right). \end{aligned}$$

Hence, by condition (i), it is sufficient to prove that

$$\sum_{n=1}^{\infty} c_n P\left(\max_{1 \leq l \leq k_n} \left|\sum_{i=1}^l (Y_{ni} - EY_{ni})\right| > \varepsilon/2\right) < \infty.$$

For any $a > 0$ and set

$$d = \min\left\{1, \frac{a}{6\delta}\right\}, N_1 = \left\{n : \sum_{i=1}^{k_n} P\left(|X_{ni}| > \min\left\{\delta, \frac{a}{6}\right\}\right) > d\right\} \text{ and } N_2 = \mathbb{N} \setminus N_1.$$

Note that

$$\begin{aligned} & \sum_{n \in N_1} c_n P \left(\max_{1 \leq l \leq k_n} \left| \sum_{i=1}^l (Y_{ni} - EY_{ni}) \right| > \varepsilon/2 \right) \leq \sum_{n \in N_1} c_n \\ & \leq \frac{1}{d} \sum_{n=1}^{\infty} c_n \sum_{i=1}^{k_n} P \left(|X_{ni}| > \min \left\{ \delta, \frac{a}{6} \right\} \right) < \infty. \end{aligned}$$

Hence, it remains to prove that $\sum_{n \in N_2} c_n P \left(\max_{1 \leq l \leq k_n} \left| \sum_{i=1}^l (Y_{ni} - EY_{ni}) \right| > \varepsilon/2 \right) < \infty$. By Lemma 3.3, we get

$$\begin{aligned} & \sum_{n \in N_2} c_n P \left(\max_{1 \leq l \leq k_n} \left| \sum_{i=1}^l (Y_{ni} - EY_{ni}) \right| > \varepsilon/2 \right) \\ & \leq 2m \sum_{n \in N_2} c_n P \left(\max_{1 \leq i \leq k_n} |Y_{ni} - EY_{ni}| > a \right) + 8m \sum_{n \in N_2} c_n \left(\frac{4m \sum_{i=1}^{k_n} \text{Var}(Y_{ni})}{3a\varepsilon} \right)^{\frac{\varepsilon}{24ma}}. \end{aligned}$$

Note that

$$\begin{aligned} P \left(\max_{1 \leq i \leq k_n} |Y_{ni} - EY_{ni}| > a \right) & \leq P \left(\max_{1 \leq i \leq k_n} |X_{ni} I\{|X_{ni}| \leq \delta\} - EX_{ni} I\{|X_{ni}| \leq \delta\}| > a/2 \right) \\ & \quad + P \left(\max_{1 \leq i \leq k_n} |Y'_{ni} - EY'_{ni}| > a/2 \right). \end{aligned}$$

For $n \in N_2$, we have

$$\begin{aligned} \max_{1 \leq i \leq k_n} |EX_{ni} I\{|X_{ni}| \leq \delta\}| & \leq \max_{1 \leq i \leq k_n} E|X_{ni} I\{|X_{ni}| \leq \delta\}| \\ & \leq \max_{1 \leq i \leq k_n} (E|X_{ni} I\{|X_{ni}| \leq a/6\}| + E|X_{ni} I\{a/6 < |X_{ni}| \leq \delta\}|) \\ & \leq a/6 + \delta \sum_{i=1}^{k_n} P \left(|X_{ni}| > \min \left\{ \delta, \frac{a}{6} \right\} \right) \leq a/6 + \delta d \leq a/3. \end{aligned}$$

Therefore

$$\begin{aligned} & \sum_{n \in N_2} c_n P \left(\max_{1 \leq i \leq k_n} |Y_{ni} - EY_{ni}| > a \right) \\ & \leq \sum_{n=1}^{\infty} c_n P \left(\max_{1 \leq i \leq k_n} |X_{ni} I\{|X_{ni}| \leq \delta\}| > a/6 \right) + \sum_{n=1}^{\infty} c_n P \left(\max_{1 \leq i \leq k_n} |Y'_{ni} - EY'_{ni}| > a/2 \right) \\ & \leq \sum_{n=1}^{\infty} c_n \sum_{i=1}^{k_n} P(|X_{ni}| > a/6) + C \sum_{n=1}^{\infty} c_n \sum_{i=1}^{k_n} P(|X_{ni}| > \delta) < \infty. \end{aligned}$$

Denote $B_n = \sum_{i=1}^{k_n} \text{Var}(Y_{ni})$.

$$\begin{aligned} B_n &= \sum_{i=1}^{k_n} [\text{Var}(X_{ni}I\{|X_{ni}| \leq \delta\}) + \text{Var}(\delta I\{X_{ni} > \delta\} - \delta I\{X_{ni} < -\delta\}) \\ &\quad + 2\text{cov}(X_{ni}I\{|X_{ni}| \leq \delta\}, \delta I\{X_{ni} > \delta\} - \delta I\{X_{ni} < -\delta\})] \\ &\leq \sum_{i=1}^{k_n} \text{Var}(X_{ni}I\{|X_{ni}| \leq \delta\}) + 9\delta^2 \sum_{i=1}^{k_n} P(|X_{ni}| > \delta). \end{aligned}$$

When $n \in N_2$, we have that $\sum_{i=1}^{k_n} P(|X_{ni}| > \delta) \leq 1$. Let $a = \frac{\varepsilon}{24mj}$, we have

$$\begin{aligned} \sum_{n \in N_2} c_n \left(\frac{4mB_n}{3a\varepsilon} \right)^{\frac{\varepsilon}{24ma}} &= \sum_{n \in N_2} c_n \left(\frac{4m}{3a\varepsilon} \right)^j (B_n)^j \\ &\leq C \sum_{n \in N_2} c_n \left(\sum_{i=1}^{k_n} \text{Var}(X_{ni}I\{|X_{ni}| \leq \delta\}) + 9\delta^2 \sum_{i=1}^{k_n} P(|X_{ni}| > \delta) \right)^j \\ &\leq C \sum_{n \in N_2} c_n \left(\sum_{i=1}^{k_n} \text{Var}(X_{ni}I\{|X_{ni}| \leq \delta\}) \right)^j + C \sum_{n \in N_2} c_n \left(\sum_{i=1}^{k_n} P(|X_{ni}| > \delta) \right)^j \\ &\leq C \sum_{n=1}^{\infty} c_n \left(\sum_{i=1}^{k_n} \text{Var}(X_{ni}I\{|X_{ni}| \leq \delta\}) \right)^j + C \sum_{n=1}^{\infty} c_n \sum_{i=1}^{k_n} P(|X_{ni}| > \delta) < \infty. \end{aligned}$$

Proof of Theorem 2.2 Let $a = \frac{\varepsilon}{12mj}$. By Lemma 3.3, we have that

$$\begin{aligned} &\sum_{n=1}^{\infty} c_n P \left(\max_{1 \leq l \leq k_n} \left| \sum_{i=1}^l X_{ni} \right| > \varepsilon \right) \\ &\leq \sum_{n=1}^{\infty} c_n 2m P \left(\max_{1 \leq l \leq k_n} |X_{ni}| > a \right) + \sum_{n=1}^{\infty} c_n 8m \left(\frac{2m \sum_{i=1}^{k_n} EX_{ni}^2}{3a\varepsilon} \right)^{\frac{\varepsilon}{12ma}} \\ &\leq C \sum_{n=1}^{\infty} c_n \sum_{i=1}^{k_n} P(|X_{ni}| > a) + C \sum_{n=1}^{\infty} c_n \left(\sum_{i=1}^{k_n} EX_{ni}^2 \right)^j < \infty. \end{aligned}$$

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行 m -NSD随机变量阵列的完全收敛性

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摘要: 本文研究了行 m -NSD 随机变量阵列的完全收敛性问题. 主要利用 m -NSD 随机变量的Kolmogorov 型指数不等式, 获得了行 m -NSD 随机变量阵列的完全收敛性定理, 将Hu 等(1998) and Sung 等(2005) 的结果从独立情形推广到了 m -NSD 随机变量阵列. 本文的结论同样推广了Chen 等(2008), Hu 等(2009), Qiu 等(2011) 和Wang 等(2014)的结果.

关键词: Kolmogorov 型指数不等式; 完全收敛性; m -NSD 随机变量

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